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Turbulent Capillary Cascade near the Edge of the Inertial Range on the Surface of a Quantum Liquid

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In the dissipative range at frequencies above the inertial frequency range, the turbulent cascade of capillary waves on the surface of liquid helium and hydrogen decays according to an exponential law. The characteristic frequency of the quasi-Planck distribution is determined by the spectral characteristic of an exciting force. In the case of harmonic pumping on the surface of superfluid helium in the discrete turbulence regime, energy condensation is observed near the high-frequency edge of the inertial range. The effect is due to the influence of discreteness in the spectrum of the eigenfrequencies of surface excitations and in the turbulence distribution on the energy transfer through the cascade.

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1. INTRODUCTION

The frequency of capillary waves on the surface of a liquid ω is determined by the wave vector k, surface tension coefficient σ, and density of the liquid ρ:

$$\omega = \left(\frac{\sigma}{\rho}\right)^{1/2} k^{3/2}. \tag{1}$$

The dispersion law of capillary waves given by Eq. (1) is of the decay type: three-wave processes of the decay of one wave into two waves and merging of two waves into one wave are allowed with the conservation of energy and momentum:

$$\begin{aligned} \omega_1 \pm \omega_2 &= \omega_3, \\ k_1 \pm k_2 &= k_3. \end{aligned} \tag{2}$$

When the surface of the liquid is excited at low frequencies by an external force, the turbulent state can be formed in the system of capillary waves, where the energy flux P in the k space is directed from the pumping region toward higher wave vectors (higher frequencies); i.e., a direct cascade is developed. Wave (weak) turbulence theory [1] predicts that the main contribution to the energy transfer through the turbulent capillary cascade comes from the three-wave processes of wave merging. In this case, the energy of the waves is distributed in frequency range according to a power law $E_\omega \sim \omega^{-\alpha}$. It is most convenient to experimentally study the pair correlation function $I(\tau) = \langle \eta(r, t+\tau)\eta(r, t)\rangle$ of the deviation of the surface from the equilibrium state at the point r rather than the energy distribution E_ω, because the deviations of the surface from the plane state $\eta(r, t)$ can be directly measured. Wave turbulence theory [1] for a system of capillary waves on the liquid surface predicts the formation of a turbulent cascade in the inertial range bounded by the pumping region at low frequencies and by the dissipative range at high frequencies. The pair correlation function $I(r, t)$ within the inertial range in the Fourier representation is described by the power-law function of the frequency (turbulent cascade)

$$I_\omega \sim \omega^{-m}. \tag{3}$$

In the case of capillary waves, $E_\omega \sim \omega^{4/3} I_\omega$. The exponent m depends on the spectral characteristic of the exciting force. For broadband pump, $m = 17/6$. Our preliminary experimental studies on the surface of liquid hydrogen show that the spectral characteristic of the exciting force determines the exponent of the power-law function [2]. When the surface is excited by a low-frequency harmonic force, the correlation function I_ω includes a number of narrow peaks with frequencies multiple of the pump frequency ω_p. The positions of the peaks maxima are well described by the power-law function ω^{-m} with the exponent $m = 3.7 \pm 0.3$. If pumping at one resonance frequency is supplemented by excitation by a harmonic force at another resonance frequency, the exponent decreases to $m = 2.8 \pm 0.2$. When the surface is excited by broadband low-frequency noise, the exponent is $m = 3.0 \pm 0.3$. In those experiments, we qualitatively demonstrated that the exponent m in the case of pumping by a harmonic force at one resonance frequency of the cell is larger than that in the case of the excitation of the surface by broadband noise in agreement with [3]. Computer simulations [3] showed that, when the width of the pump noise band is decreased, a number of equidistant peaks appear on the turbulent cascade. The width of these peaks depends linearly on the frequency. In the case of pumping in a narrow band, a decrease in the height of the peaks with an increase in the frequency is described by a power law with the exponent that is one unit larger than the exponent for the case of broadband pump noise, i.e., $m = 23/6$. Detailed information on the evolution of the turbulent cascade when modifying the pumping spectral characteristic from a broadband exciting force to a narrowband one can be found in [4]. In the steady-state turbulence spectrum in the system of capillary waves, the energy is transferred toward higher frequencies, where it is transformed to heat owing to viscous losses, and the turbulent cascade decays. For this reason, to maintain the turbulent cascade in the steady state, it is necessary to continuously introduce energy at low frequencies. The frequency ω_b of the high-frequency edge of the inertial range can be estimated under the assumption that the time of the nonlinear wave interaction τ_{nl} at this frequency is on the order of viscous damping time τ_ν [2]:

$$\omega_b \sim (P^{1/2}/\nu)^{6/5} \sim (\eta_0^2\omega_0^{17/6}/\nu)^{6/5}, \tag{4}$$

where η_0^2 is the wave amplitude squared at the pump frequency ω_{p} and ν is the kinematic viscosity of the liquid. The behavior of the spectrum at high frequencies is determined by the features of energy dissipation and nonlinear wave interaction. When waves in the dissipative range interact mainly with the nearest neighbors (broadband pump) rather than with waves from the inertial range, the distribution of waves at high frequencies becomes close to a Boltzmann distribution [5]. Comprehensive analysis gives a quasi-Planck spectrum of the correlation function in the dissipative range:

$$P_\omega \sim \omega^s e^{-\omega/\omega_{\mathrm{d}}}, \tag{5}$$

where ω_{d} is the characteristic frequency of the distribution and s is the pre-exponential exponent. A numerical simulation for capillary waves confirmed the exponential dependence in the distribution of waves in the dissipative range. At the same time, when the surface of the liquid is excited by a harmonic force, interactions with modes from the inertial range can dominate for high-frequency harmonics of the turbulent cascade [6]. Under finite size geometry conditions, the spectrum of capillary waves is transformed from a continuous distribution to a discrete one, where the distance between resonance modes increases with the frequency. For the case of monochromatic excitation of the surface of the liquid, the turbulent cascade consists of harmonics whose frequencies are multiple of the frequency of the exciting force. Simple consideration indicates that the system of equations (3) has no solutions in the case of such an excitation of the surface [7, 8]. However, as was shown in [9], this constraint is eliminated if the nonlinear broadening of the resonance peaks is taken into account. In this case, the conservation laws should be rewritten in the form

$$\begin{gathered} \left||k_1|^{3/2} \pm |k_2|^{3/2} - |k_3|^{3/2}\right| < \delta\omega, \\ k_1 \pm k_2 - k_3 = 0, \end{gathered} \tag{6}$$

where $\delta\omega$ is the characteristic nonlinear broadening of the resonance peak. Furthermore, it should be taken into account that the discrete spectrum for a classical liquid at high frequencies becomes quasi-continuous because of the viscous broadening of resonance peaks. The kinematic viscosity coefficients for liquid hydrogen and helium are one and two orders of magnitude smaller than that for water, respectively. For this reason, the discreteness conditions can play a significant role in energy transfer in the cascade [10] in the case of monochromatic excitation. In [11], we reported our results obtained on the surface of superfluid helium in the case of harmonic pump when discreteness is significant and discrete turbulence is developed. At the same time, for the case of broadband pump of the surface of liquid hydrogen and helium, kinetic turbulence is implemented, which is the closest to a model system theoretically developed in [1]. The features of the interaction between waves from the dissipative range and inertial range can influence the shape of the turbulence distribution near the edge of the inertial range. As was shown in [6], if energy dissipation at high frequencies does not ensure the absorption of energy from the turbulent cascade, this can lead to the deviation of the distribution in the inertial range from the power law. Our preceding experiments [12] indicated that the use of liquid helium and hydrogen for investigations of turbulence has advantages compared to traditional liquids owing to their low density and kinematic viscosity.

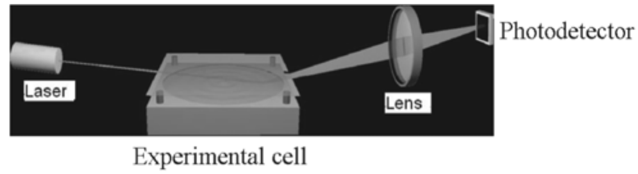

Fig. 1. Layout of the detection of waves on the surface of liquid helium and hydrogen.

2. EXPERIMENTAL PROCEDURE

Significant advances in the investigation of capillary turbulence have been achieved in recent years owing to the fast development of experimental techniques and computer methods for processing of rap-

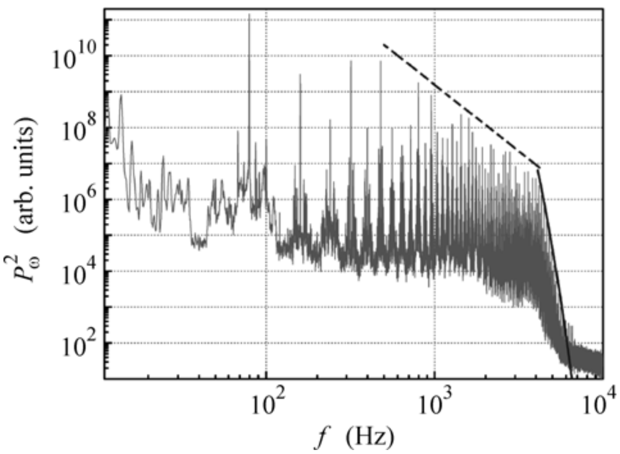

Fig. 2. Turbulence spectrum of capillary waves P^2_ω on the surface of He-II in the case of harmonic pump at a frequency of $f_p \approx 80$ Hz by an ac voltage with an amplitude of 11 V. The dashed line corresponds to the power law $P^2_\omega \sim \omega^{-3.7}$. The high-frequency edge of the inertial range f_b is at a frequency of about 4 kHz. The solid line is the exponential function $P^2_\omega \sim \exp(-f/f_d)$ with a characteristic frequency of $f_d \approx 170$ Hz close to the pump frequency f_p.

idly varying signals. In our investigations, we used the procedure [13] based on the measurement of changes in the power of a laser beam reflected from the oscillating surface of the liquid. The layout of the measurements is shown in Fig. 1. The measurements were performed on optical cells placed in the vacuum cavity of a helium cryostat. A plane horizontal capacitor was placed inside the cells. Gaseous hydrogen (or helium) was condensed into a cylindrical copper cup. The diameter of the cup was 60 and 30 mm in the experiments with hydrogen and helium, respectively. The height of the cup was 4–6 mm. The upper horizontal metallic plate was placed at a distance of 3.5 mm over the cup. The liquid was collected until the surface of the liquid reached the edge of the cup. The measurements with hydrogen and helium were performed at temperatures $T = 15.5$ and 1.7 K, respectively. A radioactive source, which emitted β-electrons with an average energy of 5 keV, was placed on the lower plate of the capacitor. An ionized liquid layer was formed near the surface of the source plate. A voltage of about 1000 V applied to the capacitor plates extracted positive ions from the ionized layer and pulled them to the surface of the liquid. Thus, the charged surface of the liquid and the upper metallic plate formed a planar capacitor. Waves on the charged surface of the liquid were excited by an alternating electric field when an ac voltage with an amplitude of 1–100 V was applied to the metallic cup in addition to the dc voltage. Surface was pumped by a harmonic force at frequencies close to the resonance frequencies of the cylindrical cell or by broadband noise. The pump noise signal was synthesized by the inverse Fourier transform using a specified power spectrum and a random set of phases. The use of the electric field to excite the surface of the liquid has a number of advantages, because the disturbance can be applied only to the surface, the symmetry of the disturbance can be controllably changed, and the spectral characteristic of the exciting force can be varied. A variation of the power of the reflected laser beam was measured by a Hamamatsu s3590-08 semiconductor detector. The ac voltage from the photodetector $P(t)$, which was proportional to the power of the reflected beam, was amplified by an SR570 amplifier and stored in computer memory at a sampling rate of up to 102.4 kHz using a Leonardo-II high-speed 24-bit analog-to-digital converter. The recording time of the signal $P(t)$ ranged from 3 to 100 s. The $P(t)$ dependences were processed by the fast Fourier transform algorithm. As a result, we obtained the frequency distribution of the squares of the amplitudes of harmonics P^2_ω; for a "wide beam" case, this distribution, as was shown in [13], is proportional to the pair correlation function of deviations of the surface from the equilibrium state: $I_\omega \sim P^2_\omega$.

3. EXPERIMENTAL RESULTS

3.1. Two Different Decay Regimes of the Turbulent Cascade in the Dissipative Frequency Range

The spectrum of capillary waves on the surface of superfluid helium-4 at a temperature of $T = 1.7$ K is shown in Fig. 2 on the log–log scale. The surface was excited by a harmonic (sinusoidal) force at one of the resonance frequencies of the cell ($f_p \approx 80$ Hz), $f = \omega/2\pi$, where ω is the angular frequency. The amplitude of the pump ac voltage was 11 V. It can be seen that the turbulence distribution consists of a set of equidistant harmonics. The first harmonic is at the frequency f_p and corresponds to oscillations forced by the external sinusoidal pump. The other harmonics are formed at frequencies multiple to the pump frequency f_p due to three-wave processes of the nonlinear interaction. The inertial range is clearly observed in the frequency range from 800 Hz to 4 kHz, where the turbulence distribution is described by a power-law function $P^2_\omega \sim \omega^{-3.7}$. The exponent of −3.7 is in agreement with the theoretical predictions for capillary turbulence in the case of narrowband pump [3]. At frequencies above 4 kHz, the turbulence distribution decreases rapidly under the effect of viscous dissipation and approaches the instrumental noise level. The frequency $f_b \approx 4$ kHz can obviously be interpreted as the position of the high-frequency edge of the inertial range. The turbulence distribution in the dissipative range (at frequencies above f_b) can be described by an exponential function $P^2_\omega \sim \exp(-f/f_d)$ with a charac-

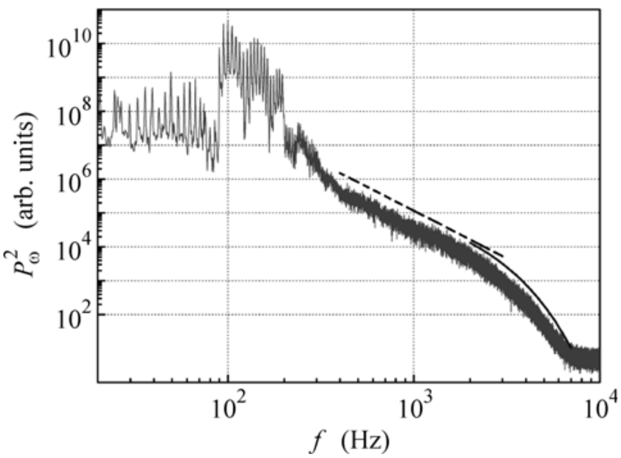

Fig. 3. Turbulence spectrum of capillary waves P_ω^2 on the surface of He-II in the case of broadband pump in the frequency range of 90–200 Hz. The dashed line corresponds to the power law $P_\omega^2 \sim \omega^{-2.8}$. The high-frequency edge of the inertial range f_b is at a frequency of about 2 kHz. The solid line is the exponential function $P_\omega^2 \sim \exp(-f/f_d)$ with a characteristic frequency of $f_d \approx 700$ Hz. According to theory, f_d is close to the boundary frequency f_b.

teristic frequency of $f_d \approx 170$ Hz. Thus, f_d appears to be close to the pump frequency f_p. Moreover, it was found that the frequencies f_d and f_p are close in the experiments with various pump frequencies and amplitudes. It is worth noting that it was shown theoretically [6] that the characteristic frequency f_d in the case of broadband pump should be close to the boundary frequency f_b. Indeed, when waves on the surface were excited by pump noise, the turbulence distribution had a qualitatively different shape. Figure 3 shows the spectrum obtained when the surface was pumped by noise in the frequency range of 90–200 Hz. It can be seen that the spectrum is smooth and the inertial range in which the spectrum is described by the Kolmogorov–Zakharov power law $P_\omega^2 \sim \omega^{-2.8}$ in agreement with theoretical predictions, is rather narrow (from about 400 Hz to $f_b \approx 2$ kHz). The wave distribution in the dissipative range can be described by an exponential $\exp(-f/f_d)$ with the frequency $f_d = 700$ Hz. It is important that f_d is much higher than the pump frequencies and is fairly close to the boundary frequency f_b of the Kolmogorov–Zakharov spectrum (this qualitative relation between f_d and f_b in the case of broadband pump is independent of the pump amplitude).

Thus, we have observed two different decay regimes of wave turbulence in the dissipative frequency range: in the case of harmonic pump, the characteristic frequency f_d of exponential decay is much lower than the boundary frequency f_b and is close to the pump frequency f_p; in the case of broadband pump, f_d is close to the boundary frequency f_b.

It is noteworthy that the presence of two different decay regimes of the turbulent cascade is not attributed to the specificity of superfluid helium-4, because qualitatively similar results were obtained in experiments with normal helium-4 and liquid hydrogen.

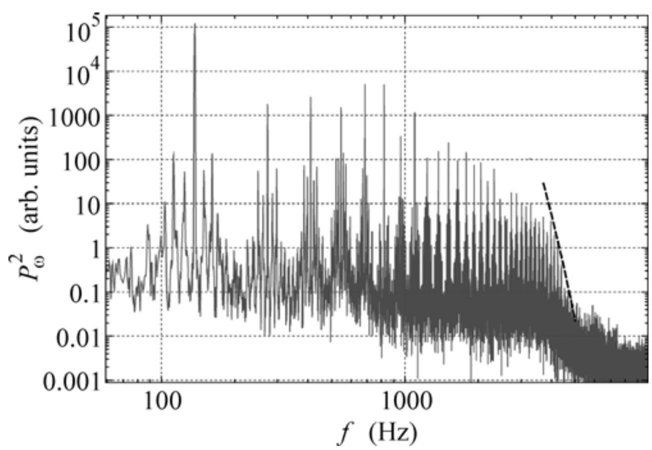

Fig. 4. Turbulence spectrum of capillary waves on the surface of liquid hydrogen in the case of harmonic pump at a frequency of about 120 Hz. The high-frequency edge of the inertial range f_b is at a frequency of about 4 kHz. The line is the exponential function $P_\omega^2 \sim \exp(-f/f_d)$ with a characteristic frequency of $f_d = 180$ Hz.

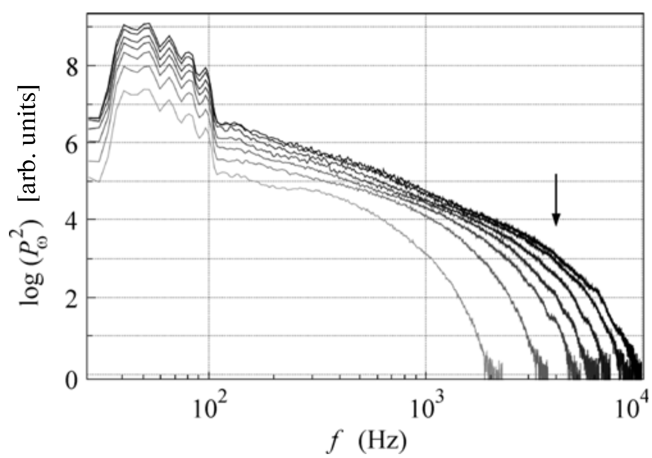

Fig. 5. Spectrum of surface oscillations P_ω^2 of liquid hydrogen excited by a random force in the frequency range of 39–103 Hz at various pump amplitudes. The rms pump voltage V_p varied from 4 to 30 V. Darker lines correspond to more intense pump. The arrow indicates the high-frequency edge of the inertial range $\omega_b/2\pi \approx 4$ kHz at a pump voltage of $V_p = 30$ V.

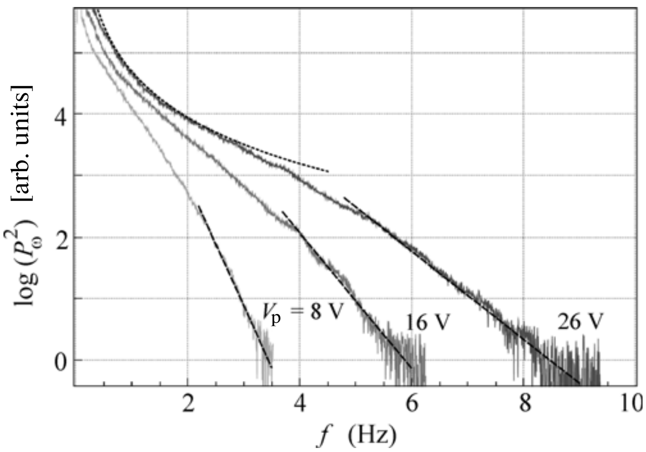

Fig. 6. Spectra P_ω^2 at pump voltages $V_p = 8$, 16, and 26 V on a semilogarithmic scale (they are also plotted in Fig. 4 on a log–log scale). The dotted line is the power law $\omega^{-2.8}$. The dashed lines are exponentials $\exp(-\omega/\omega_d)$ with $\omega_d/2\pi \approx$ 0.2, 0.4, and 0.6 kHz for $V_p = 8$, 16, and 26 V, respectively.

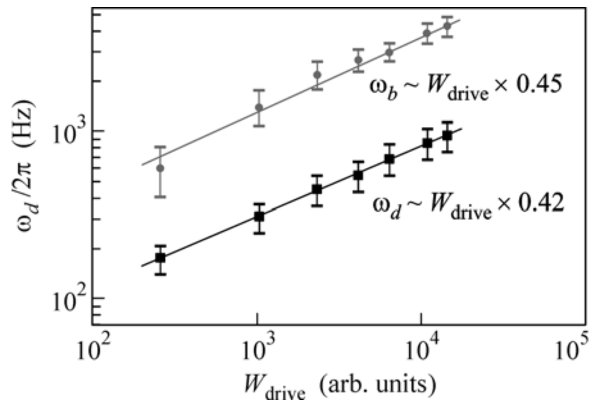

Fig. 7. Boundary frequency of the inertial range ω_b and the characteristic frequency of the exponential decay in the dissipative range ω_d versus the amplitude of the exciting force in the case of broadband pump (experiments with liquid hydrogen).

Figure 4 shows the spectrum of capillary waves on the surface of liquid hydrogen in the case of intense harmonic pump at a frequency of about 120 Hz. The characteristic frequency f_d of the exponential decay of the turbulence distribution in the dissipative range is much lower than the high-frequency edge of the inertial range and is close to the pump frequency range. Another decay regime is demonstrated in Fig. 5, where the spectra of capillary waves on the surface of liquid hydrogen in the case of pump noise with various amplitudes in the frequency range of 39–103 Hz are shown. The pump frequency range can be clearly seen at low frequencies. The pump range is followed by the inertial range, which is a relatively wide frequency range where the spectrum P_ω^2 satisfies the Kolmogorov–Zakharov power law. The width of the inertial range depends on the pump amplitude. When the surface is excited by a weak force at $V_p = 4$ V, dissipation begins immediately after the pump range and no inertial range is observed. With increasing amplitude of the pumping the inertial range widens. Therewith, the upper edge $\omega_b/2\pi$ of the inertial range is shifted toward higher frequencies. The widest inertial range (from approximately 0.3 kHz to $\omega_b/2\pi \approx 4$ kHz) is observed at the maximum pump voltage ($V_p = 30$ V). At frequencies above the high-frequency edge of the inertial range, surface oscillations decay owing to viscous losses and the spectrum P_ω^2 smoothly approaches the instrumental noise level. In turbulence spectra plotted on a semilogarithmic scale (Fig. 6), it can be clearly seen that a decrease in the amplitudes of the waves in a certain frequency range above the high-frequency edge of the inertial range can be well described by an exponential law $P_\omega^2 \sim \exp(-\omega/\omega_d)$. For example, the spectrum of the waves obtained in the experiments with a pump amplitude of $V_p = 26$ V can be approximated in the range of 5–9 kHz by the exponential $\exp(-\omega/\omega_d)$ with $\omega_d/2\pi \approx 0.6$ kHz. It is noteworthy that the frequency $\omega_d/2\pi$ is several times lower than the frequency of the visible boundary between the inertial and dissipative ranges (see Fig. 5). However, this discrepancy can be attributed to some freedom in the definition of the boundary frequency, which has the meaning of the characteristic frequency and can be normalized by a certain constant depending on the chosen procedure of determination of the edge of the inertial range. This assumption is confirmed by the fact that the pump-amplitude dependences of ω_d and ω_b are identical (see Fig. 7). (It is interesting that the exponent of the power-law dependence of ω_b on the pump amplitude is inconsistent with the predictions of the wave turbulence theory (see [15]).) Thus, it can be concluded that ω_d in the case of broadband pump corresponds to the high-frequency edge of the inertial range. The experimental results confirm the theoretical conclusion [6] that the local interaction of waves plays the crucial role in the dissipative range for the case of broadband pump. The characteristic frequency of the quasi-Planck distribution increases with the pump intensity according to a law different from the theoretically predicted dependence. Unfortunately, the measurement accuracy is insufficient to estimate the exponent s in Eq. (5).

To summarize, in the experiments on the surface of liquid hydrogen in the case of harmonic pump, as well as in the experiments on the surface of superfluid helium-4, the characteristic frequency $f_d = \omega_d/2\pi$ of the exponential decay is close to the pump frequency f_p, whereas the characteristic frequency in the case of

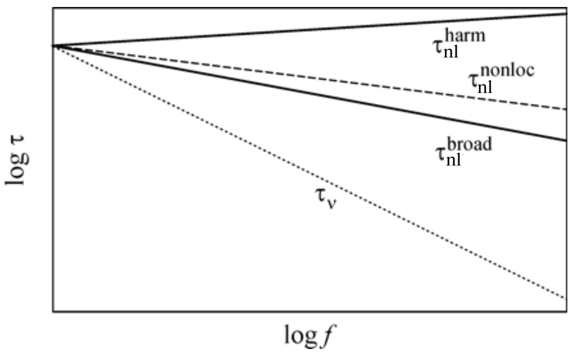

Fig. 8. Schematic frequency dependences of the characteristic times of wave processes in the dissipative range: viscous damping time $\tau_\nu \sim \omega^{-4/3}$, local-interaction time in the case of harmonic pump $\tau_{nl}^{harm} \sim \omega^{1/6}$ and in the case of broadband pump $\tau_{nl}^{broad} \sim \omega^{-1/2}$, and nonlocal-interaction time $\tau_{nl}^{nonloc} \sim \omega^{-1/3}$.

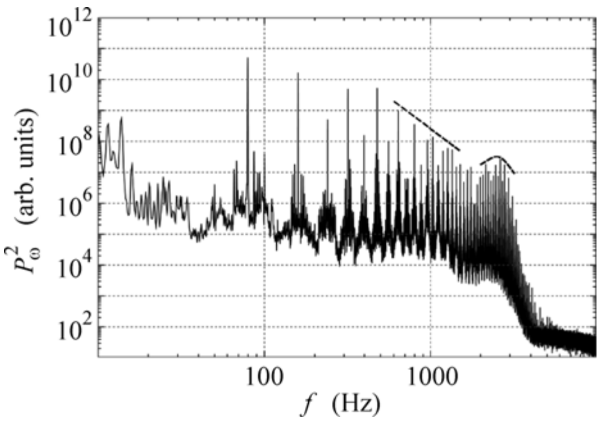

Fig. 9. Turbulence spectrum of capillary waves P_ω^2 on the surface of He-II at moderate amplitudes of harmonic pumping at a frequency of 80 Hz. Pumping voltage amplitude is 10 V. A local maximum (marked by the dashed line) is observed near 2.5 kHz. The dashed straight line is $P_\omega^2 \sim \omega^{-3.7}$.

broadband pump is close to the upper edge $f_b = \omega_b/2\pi$. We explain the results obtained for the case of harmonic pump by the nonlocality of the three-wave nonlinear interaction between waves in the dissipative range. We consider the frequency range near the high-frequency edge of the inertial range f_b, where the transition from the nonlinear transfer of the wave energy to viscous dissipation occurs. Although viscous damping with the characteristic time $\tau_\nu = (2\nu k^2)^{-1} \sim \omega^{-4/3}$ becomes the main process, some nonlinear interaction between waves also occurs. Furthermore, we take into account that, in addition to local interaction with waves from the high-frequency part of the inertial range, waves from the transient region near f_b can nonlocally interact with waves from the entire inertial range. Indeed, the characteristic time of the local nonlinear interaction within the inertial range (three-wave processes with close wave vectors $k_1 \approx k_2 \approx k_3$) in the case of harmonic and broadband pump is $\tau_{nl}^{harm} \approx \omega^{1/6}$ and $\tau_{nl}^{broad} \approx \omega^{-1/2}$, respectively [16]. We assume that the times of the local nonlinear interaction in the dissipative range near the high-frequency edge of the inertial range f_b have the same frequency dependence. At the same time, the characteristic time of the interaction between waves with strongly different wave vectors (nonlocal interaction, $k_1 \ll k_2$, $k_2 \approx k_3$) is $\tau_{nl}^{nonloc} \sim k^{-1/2} \sim \omega^{-1/3}$ [6]. Thus, the condition $\tau_{nl}^{nonloc} < \tau_{nl}^{harm}$ can be satisfied in the case of harmonic pump at high frequencies (above f_b) (see Fig. 8). Consequently, the nonlocal interaction with waves from the inertial range can be the dominant nonlinear interaction of waves in the dissipative range (this does not evidently mean that locality within the inertial range is violated). Since the wave energy is concentrated at low frequencies in the pump region, the nonlocal interaction of waves from the dissipative range with waves from the low-frequency part of the inertial range determines the exponential decay of the wave spectrum with the characteristic frequency close to the pump frequency.

3.2. Accumulation of the Wave Energy near the High-Frequency Edge of the Inertial Range

In a number of the experiments on the surface of He-II with harmonic pump at moderate amplitudes, a feature was revealed in the high-frequency range in the spectra of capillary waves. In particular, when the voltage amplitude of harmonic pump at a frequency of 80 Hz is changed from 11 V (see Fig. 2) to 10 V, the shape of the turbulence spectrum changes noticeably (Fig. 9). In agreement with preceding works [17, 18], the high-frequency edge of the turbulent cascade was shifted toward lower frequencies. However, as can be seen in Fig. 9, deviation from the power-law spectrum is observed near the high-frequency edge of the inertial range: a local maximum (shown by the dashed line) is formed near the frequency ω_b. With a further decrease in the pump amplitude, the local maximum is shifted toward lower frequencies. Finally, at very small pump amplitudes, the spectrum consists only of several harmonics and no local maximum is observed.

As was mentioned above, the formation of a local maximum was observed in a number of experiments with harmonic pump at certain frequencies. The distributions of pump frequencies at which the local maximum is formed in different series of experiments are different and apparently depend very strongly on

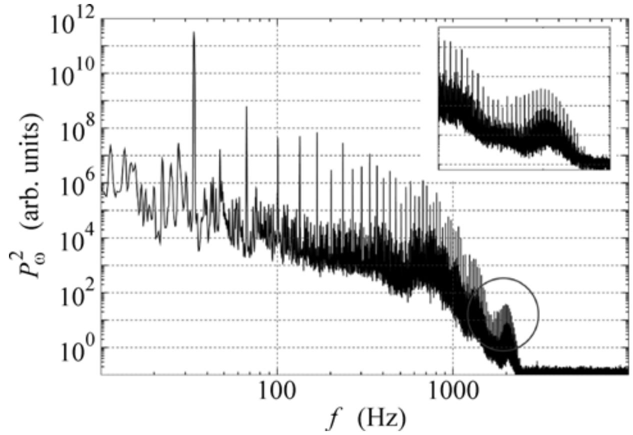

Fig. 10. Turbulence spectrum in the case of pumping at a frequency of $\omega_{pump}/2\pi = 34$ Hz. A local maximum is clearly seen in the spectrum (it is enclosed by the circle and is shown on a magnified scale in the inset).

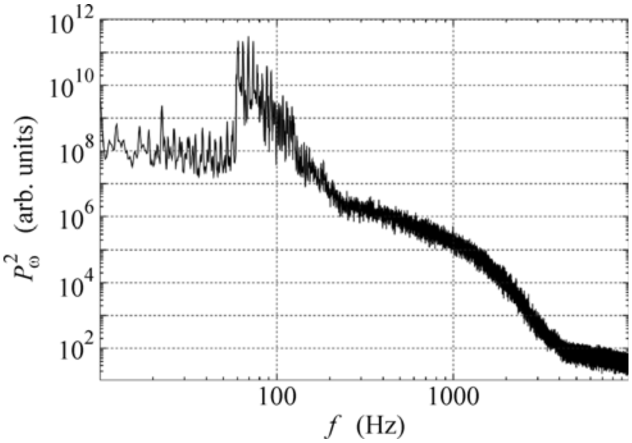

Fig. 11. Turbulence spectrum of capillary waves in the case of pump noise in the frequency range of 60–130 Hz.

the operating parameters of the system such as the temperature and level of He-II in an experimental cell. A common feature of all these experiments is that the local maximum is formed near the high-frequency edge of the turbulence distribution, but the shape of the local maximum and its exact position depend on the frequency and amplitude of pump. For example, when the surface was excited by a sinusoidal force at a frequency of 34 Hz, a spectrum with a pronounced local maximum was observed (Fig. 10). However, the local maximum was located in the dissipative range of the turbulence distribution, rather than in the inertial range, as is shown in Fig. 9.

It is noteworthy that no local maximum was observed in experiments with pump noise. For example, Fig. 11 shows the turbulence distribution of waves obtained in the experiments where the surface was excited by a noise signal in the frequency range of 60–130 Hz. The pump amplitude was chosen such that the high-frequency edge of the inertial range was approximately at the same frequencies as in the experiments presented in Figs. 9 and 10.

The formation of the local maximum can be interpreted as the accumulation of the wave energy near the high-frequency edge of the turbulent cascade, where the transition from the nonlinear transfer of the wave energy to the viscous dissipation of the energy occurs. It was theoretically shown in [6] that the energy accumulation can be due to the presence of viscous dissipation in the system that limits the inertial range owing to the "bottle neck" effect. However, the dependence of the formation of the local maximum on the pump amplitude observed in our experiments cannot be explained by the effect of viscous dissipation. At the same time, Kartashova [7, 8] showed that certain features in the processes of nonlinear interaction between surface waves can be observed in vessels of finite sizes owing to the discreteness of the wave-vector space. The discrete wave turbulence theory was more recently developed in a number of theoretical works. In particular, the model of "frozen" wave turbulence was proposed in [9]. According to this model, the discreteness of the wave-vector space can lead to the appearance of oscillations on the turbulence spectrum at small pump amplitudes. It can be assumed that the formation of the local maximum is also determined by the discreteness of the k space. However, only the case of waves on the surface with a square boundary, where the wave-vector space is two-dimensional, was considered in the mentioned work [9]. Our experiments correspond to cylindrical boundary conditions: surface oscillations are described by Bessel functions and the wave-vector space is one-dimensional. Since the density of resonance in the k space depends strongly on the boundary geometry [8], we obtained some qualitative estimates in order to understand the role of discreteness in our experiments in the cylindrical geometry. These estimates will be presented below.

We believe that the local maximum is formed owing to the discreteness of the system, namely, to the detuning of the frequencies of harmonics (shown in Figs. 2, 9, 10) from the resonance frequencies of the experimental cell. Indeed, in the case of surface waves in a cylindrical cavity with the diameter D, the wave vectors are the roots of the equation $J_1(kD/2) = 0$, where $J_1(x)$ is the Bessel function of the first order. According to the asymptotic expression for the Bessel function $J_1(x) \approx \sqrt{2/(\pi x)}\cos(x - 3\pi/4)$ in the limit $x \longrightarrow \infty$, the resonance wave vectors in the region of high k values are equidistant with the step $\Delta k \approx 2\pi/D$. However, owing to the dispersion relation, the distance between

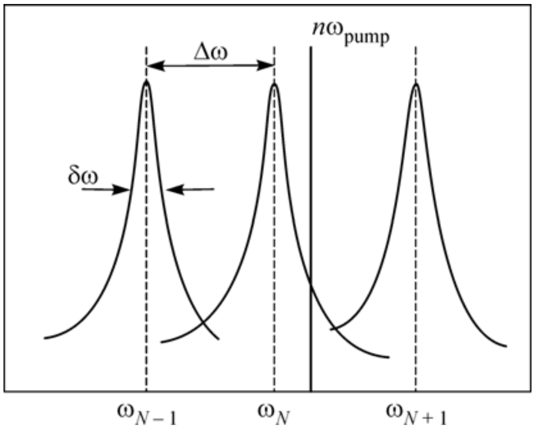

Fig. 12. Illustration of the possibility of strong suppression of some harmonics for the case where the broadening of resonance $\delta\omega$ is smaller than the distance between the two nearest resonances $\Delta\omega$ if these harmonics are far from the nearest resonance frequencies of surface oscillations in the experimental cell.

the two nearest resonance frequencies increases with the frequency as

$$\Delta\omega = \frac{\partial\omega}{\partial k}\Delta k \approx \frac{\partial\omega}{\partial k}\frac{2\pi}{D} = \frac{3\pi}{D}\left(\frac{\sigma}{\rho}\right)^{1/3}\omega^{1/3}. \quad (7)$$

Thus, the resonance frequencies in the case of capillary waves are not equidistant. On the contrary, the frequencies of nonlinear harmonics are multiple to the pump frequency ω_{pump}. For this reason, although $\Delta\omega < \omega_{\text{pump}}$ in our experiments, the sets of resonance frequencies and frequencies of nonlinear harmonics overlap insignificantly (in other words, frequency detuning always exists).

This detuning is obviously important only at frequencies where the broadening of resonance $\delta\omega$ is smaller than the distance $\Delta\omega$ between the two nearest resonances (Fig. 12),

$$\delta\omega/\Delta\omega < 1. \quad (8)$$

Resonance broadening $\delta\omega$ can be represented as the sum of viscous broadening $\delta\omega_{\nu}$ and nonlinear broadening $\delta\omega_{\text{nl}}$:

$$\delta\omega = \delta\omega_{\nu} + \delta\omega_{\text{nl}}. \quad (9)$$

The viscous broadening of the resonance corresponds to viscous damping [19],

$$\delta\omega_{\nu} = 4\nu k_{\omega}^2 = 4\nu\left(\frac{\rho}{\sigma}\right)^{2/3}\omega^{4/3} \quad (10)$$

and is related to the characteristic viscous-damping time τ_{ν} as $\delta\omega_{\nu} = \tau_{\nu}^{-1}$. The nonlinear broadening of the resonance corresponds to the nonlinear transfer of the

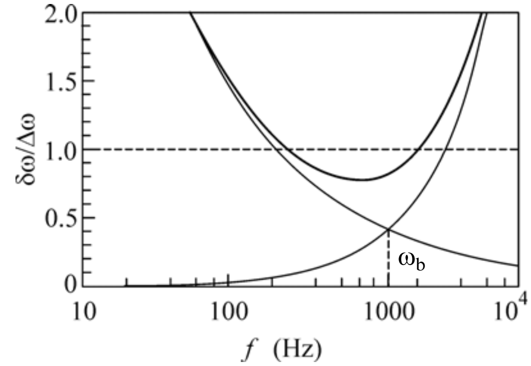

Fig. 13. (Thick line with a minimum) Estimated total relative broadening $\delta\omega/\Delta\omega$, which is the sum of (monotonically increasing thin line) viscous relative broadening $\delta\omega_{\nu}/\Delta\omega$ and (monotonically decreasing thin line) nonlinear relative broadening $\delta\omega_{\text{nl}}/\Delta\omega$.

wave energy in the inertial range from a given frequency to other frequencies and is determined by the characteristic nonlinear-interaction time τ_{nl} as $\delta\omega_{\text{nl}} = \tau_{\text{nl}}^{-1}$. The characteristic nonlinear time for the case of harmonic pump depends on the frequency as $\tau_{\text{nl}} \sim \omega^{1/6}$ [16] and, therefore, $\delta\omega_{\text{nl}} \sim \omega^{-1/6}$. Since nonlinearity is assumed to increase with the pump amplitude A, nonlinear broadening can be represented in the final form

$$\delta\omega_{\text{nl}} \approx \epsilon(A)\omega^{-1/6}, \quad (11)$$

where $\epsilon(A)$ is an increasing function of the pump amplitude A.

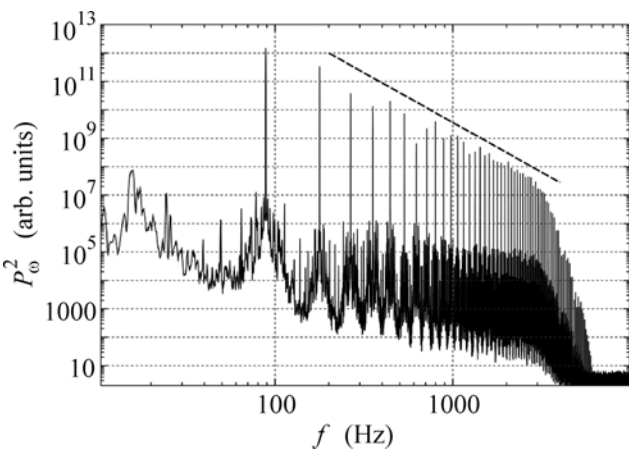

Fig. 14. Turbulence spectrum of capillary waves P_{ω}^2 on the surface of He-II in the square cell in the case of intense harmonic pump at a frequency of $f_{\text{p}} \approx 90$ Hz. The spectrum in the inertial range is described by a power law (dashed straight line).

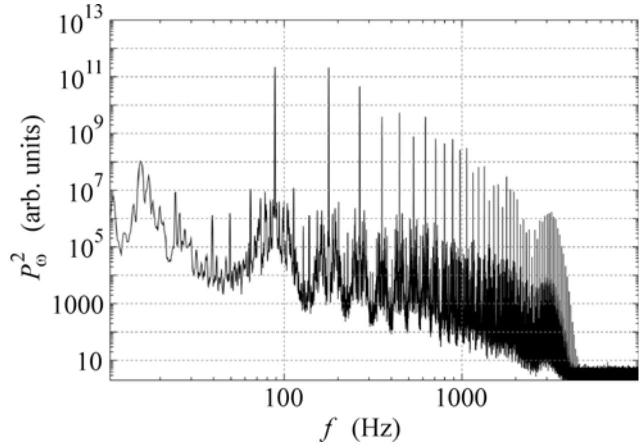

Fig. 15. Turbulence spectrum of capillary waves P_ω^2 on the surface of He-II in the square cell in the case of harmonic pump at a frequency of $f_p \approx 90$ Hz with a moderate amplitude. The local maximum is formed near the high-frequency edge of the inertial range.

After the substitution of Eqs. (7) and (9)–(11) into Eq. (8), we can conclude that detuning is important at frequencies satisfying the condition

$$\frac{4\nu(\rho/\sigma)^{2/3}\omega^{4/3} + \epsilon(A)\omega^{-1/6}}{(3\pi/D)(\sigma/\rho)^{1/3}\omega^{1/3}} < 1. \qquad (12)$$

At the high-frequency edge of the inertial range ω_b, where the damping of the turbulent cascade is observed, the transition from the nonlinear energy transfer to viscous dissipation occurs. Correspondingly, the "viscous" time at this frequency is close to the "nonlinear" time, $\tau_\nu(\omega_b) \approx \tau_{nl}(\omega_b)$. Thus, although the function $\epsilon(A)$ is unknown, its value at a given pump amplitude in a given experiment can be estimated from the condition of equality of nonlinear broadening $\delta\omega_{nl} = \tau_{nl}^{-1}$ and viscous broadening $\delta\omega_\nu = \tau_\nu^{-1}$ at the frequency ω_b:

$$\delta\omega_\nu(\omega_b) \approx \delta\omega_{nl}(\omega_b). \qquad (13)$$

According to our estimates, condition (12) is satisfied in our experiments at moderate pump amplitudes in a frequency range around the high-frequency edge of the inertial range ω_b (see Fig. 13). In this frequency range, the detuning of the frequencies of harmonics from the resonance frequencies plays an important role and is responsible for the formation of the local maximum in the turbulence distribution P_ω^2. According to the presented estimates, for the observation of the wave energy on the surface of an arbitrary liquid, the wave system should have small viscous broadening (small viscosity of the liquid), small nonlinear broadening (harmonic pump with a moderate amplitude), and a relatively large distance between the resonance frequencies (relatively small cell). These conditions are satisfied in our experiments. The use of superfluid ^{4}He, whose viscosity is much smaller than the viscosity of classical liquids, made it possible to detect the accumulation of the wave energy in the turbulent cascade.

The above estimates and conclusions are qualitative. Further experimental and theoretical investigations are necessary to explain in detail the mechanism of the formation of the local maximum. For example, we recently performed experiments on the study of the turbulence of capillary waves on the surface of superfluid helium-4 in a 42 × 42-mm cell. Figures 14 and 15 show turbulence spectra for the case of harmonic pump at a frequency of about 90 Hz with various pump amplitudes. It can be seen that, as well as in the case of the cylindrical cell, a decrease in the pump amplitude is accompanied by the formation of the local maximum near the high-frequency edge of the inertial range. However, the model presented above for the cylindrical geometry is developed for wave processes in the one-dimensional k space and strictly speaking inapplicable for a square cell, because waves in this case are characterized by the two-dimensional k space.

4. CONCLUSIONS

In summary, it has been shown experimentally that a turbulent cascade in the dissipative range decays exponentially. The characteristic decay frequency is determined by the interaction of waves from the dissipative range with each other or with waves from the inertial range, depending on the spectral characteristic of the exciting force at low frequencies. The discrete turbulence regime, which is due to discreteness in the spectrum of surface excitations and discreteness in the turbulent cascade, is developed near the high-frequency edge of the inertial range only on the surface of superfluid helium in the case of harmonic pump. The energy is accumulated in a narrow frequency range near the edge of the inertial range because a "bottle neck" is formed in the discrete regime.

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