On-line «Master in Computer Vision» educational program.
Entrance examination test

Variant 0.

1. For what values of parameter '*a*' the system of linear equations has a unique solution in positive natural numbers (x = 1, 2, …)? Find this solution for all feasible values of '*a*'.

$$\begin{cases} x_1 + ax_2 = 3 \\ x_1 + 2x_2 = 3 \end{cases}$$

2. Independent random variables *X* and *Y* have the following distributions:

$$X \sim \begin{array}{c|c} 0 & 1 \\ \hline 1/3 & 2/3 \end{array}, \quad Y \sim \begin{array}{c|c|c} 0 & 1 & 2 \\ \hline 1/3 & 1/3 & 1/3 \end{array}$$

Find the expectation and dispersion of the random variable $Z = (Y - X)^2$.

In the following problems it is necessary to suggest the most efficient algorithms. Full points are given for the most efficient algorithm having the lowest computational complexity. The lower the efficiency of the suggested solution, the lower are the points. For example, for two problems below 10 points are given for an algorithm of complexity O(n), 7 points are given for an algorithm of complexity O(n logn), and 4 points are given for an algorithm of complexity O(n^2)

3. Write a pseudo-code (or code on any programming language) of an algorithm, which finds two elements in an array of integer numbers (positive and negative), such that the product of these elements is maximal. Determine the computational complexity of your algorithm.

4. Write a pseudo-code (or code on any programming language) of an algorithm, which finds a leaf in a tree, which is the farthest leaf from the head of this tree. A tree is given by its *head* and every tree node has a list of its children referenced as *node.children*. For example, *head.children* contains and array of the child nodes of the tree head. Determine the computational complexity of your algorithm.