National Research University Higher School of Economics

International Laboratory of Dynamical Systems and Applications

as a manuscript

Nozdrinova Elena Vyacheslavovna

On classes of stable isotopic connectivity of surfaced gradient-like diffeomorphisms

Summary of the PhD thesis
for the purpose of obtaining academic degree
Doctor of Philosophy in Mathematics

Academic supervisor:
Doctor of Physical and Mathematical sciences,
Professor
Pochinka Olga Vitalievna

Nizhny Novgorod – 2021

The problem of the existence of an arc with no more than a countable (finite) number of bifurcations connecting structurally stable systems (Morse-Smale systems) on manifolds is on the list of fifty Palis-Pugh problems [26] under number 33.

In 1976, S. Newhouse, J. Palis, F. Takens [16] introduced the concept of a stable arc connecting two structurally stable systems on a manifold. Following [16], a smooth arc φ_t is called *stable* if it is an inner point of the equivalence class with respect to the following relation: two arcs φ_t, φ'_t are called *conjugate* if there are homeomorphisms $h : [0,1] \to [0,1]$, $H_t : M \to M$ such that $H_t\varphi_t = \varphi'_{h(t)}H_t, t \in [0,1]$ и H_t continuously depend on t.

Denote by $\mathcal{Q}$ the set of smooth arcs $\{\varphi_t\}$, that start and end in Morse-Smale diffeomorphisms and any diffeomorphism φ_t has a finite limit set. In [15] also established that the arc $\{\varphi_t\} \in \mathcal{Q}$ consisting of diffeomorphisms with a finite limit set, is stable iff all its points are structurally stable diffeomorphisms with the exception of a finite number of bifurcation points, $\varphi_{b_i}, i = 1, \dots, q$ such that:

1) the limit set of the diffeomorphism φ_{b_i} contains a unique nonhyperbolic periodic orbit, which is a saddle-node or a flip;

2) the diffeomorphism φ_{b_i} has no cycles;

3) the invariant manifolds of all periodic points of the diffeomorphismφ_{b_i} intersect transversally;

4) the transition through φ_{b_i} is a generically unfolded saddle-node or period doubling bifurcation, wherein the saddle-node point is non-critical.

In 1976, S. Newhouse and M. Peixoto [17] proved the existence of a simple arc between any two Morse-Smale flows. Simplicity means that the entire arc consists of Morse-Smale systems, with the exception of a finite set of points at which the vector field in a certain sense deviates least from the Morse-Smale system, namely, either contains a single non-hyperbolic saddle-node point, or a single trajectory non-transversal intersection of invariant saddle manifolds (heteroclinic tangency).

However, the results of S. Newhouse and M. Peixoto cannot be directly used to construct stable arcs between Morse-Smale diffeomorphisms. There are several reasons for this. First, generic Morse-Smale diffeomorphisms are not included in Morse-Smale flows (see, for example, [3], [5] and the review [4]). Second, the discretization of an arc with heteroclinic tangency is not a stable arc. The second problem can be avoided by virtue of the result obtained by J. Flaytas, namely, she showed that a simple arc constructed by Newhouse and Peixoto can always be replaced by a stable one. In this case, the discretization of such an arc is a stable arc connecting the one-time shift of the original gradient-like flows.

For Morse-Smale diffeomorphisms defined on manifolds of any dimension, examples of systems are known that cannot be connected by a stable arc.

Obstructions to the existence of a stable arc appear already for orientation-preserving diffeomorphisms of the circle S^1. The Morse-Smale diffeomorphisms on the circle were studied in detail by A.G. Mayer [13]. He showed that these diffeomorphisms exhaust the class of rough transformations of the circle and are characterized by a finite set of periodic points and a rational rotation number. Moreover, there exist Morse-Smale diffeomorphisms with any rational rotation number. Since the rotation number changes continuously as the homeomorphism changes continuously (see, for example, [11]), any arc connecting Morse-Smale diffeomorphisms with different rotation numbers on the circle contains a continuum of bifurcations and, therefore, is not stable.

In dimension two, additional obstructions appear to the existence of stable arcs between isotopic diffeomorphisms.

D. Pixton [27] established the existence of the Morse energy function $\Phi_f : M^2 \to \mathbb{R}$ for any Morse-Smale diffeomorphism f on the surface M^2. Using the level sets of this function, P. Blanchard [1] constructed a special partition of the supporting surface by the level lines of the function Φ_f, connected with the notion of oddness of a periodic orbit and proved that the consistency of such partitions for different diffeomorphisms is a necessary condition for the existence of a stable arc between them. Sufficient conditions for the existence of such an arc were not considered in the paper [1].

The presence of heteroclinic intersections can also serve as an obstruction. In the paper [14] S. Matsumoto showed that the two-dimensional torus $\mathbb{T}^2$ admits isotopic Morse-Smale diffeomorphisms that cannot be connected by a stable arc. This result is based on the following concept.

Periodic points p, q of a diffeomorphism $f : M^n \to M^n$ are called it trivially connected if there exists a curve $c \subset M^n$ such that $\partial c = \{q\} - \{p\}$ and for some integer N such that $f^N(p) = p$ and $f^N(q) = q$, the closed curve $f^N(c) - c$ is trivial. Otherwise, the points p, q are called *non-trivially connected*. If all periodic points of a diffeomorphism f are trivially connected, then f is called *trivial*, otherwise *is non-trivial*.

Sh. Matsumoto constructed two Morse-Smale diffeomorphisms isotopic to the identity $f_0, f_1 : \mathbb{T}^2 \to \mathbb{T}^2$. One of them, f_0, is the one-time shift of the gradient flow of a generic Morse function. Another f_1 is a superposition of f_0 with two oppositely directed Dehn rotations. It is easy to see that the diffeomorphism f_0 is trivial, and f_1 is non-trivial. Matsumoto's result is that the diffeomorphisms f_0, f_1 of the two-dimensional torus $\mathbb{T}^2$ are not connected by a stable arc.

Generalizing Matsumoto's result, in the paper [9] trivial f_0 and non-trivial f_1 isotopic Morse-Smale diffeomorphisms on the manifold $S^{n-1} \times S^1, n \geq 3$ were constructed. As in Matsumoto's example, the diffeomorphism f_0 is the Cartesian product of source-sink diffeomorphisms on the sphere S^{n-1} and on the circle S^1. The diffeomorphism f_1 is obtained from f_0 by taking its composition with the multidimensional Dehn rotation around $cl(W^u_{\sigma_1})$, which is diffeotopic to the identity map. The resulting diffeomorphisms f_0, f_1 of the manifold $S^{n-1} \times \mathbb{S}^1, n \geq 3$ are not connected by a stable arc.

In dimension $n \geq 3$, there are other obstructions to the existence of a stable arc between isotopic Morse-Smale diffeomorphisms associated with such effects of multidimensional dynamics as the wild embedding of saddle separatrices (see the papers [7], [2]), the existence of several smooth structures on the manifold (see the paper [2]).

In connection with the presence of obstructions to the existence of stable arcs between isotopic Morse-Smale diffeomorphisms, a natural problem of describing the components of a stable isotopic connection arises. The present work presents a classification of certain classes of diffeomorphisms on surfaces up to a stable isotopic connection.

The presentation of the material is divided into seven chapters. **Chapter 1** introduces necessary concepts and facts. **Chapter 2** provides an overview of the results available on this topic. Other chapters contain a detailed presentation of results on the classification of gradient-like diffeomorphisms of surfaces up to a stable isotopic connection.

The dynamics of such diffeomorphisms is closely related to periodic transformations of surfaces classified by J. Nielsen [18] for surfaces of genus greater than zero and B. Kerekjarto [12] for the sphere. It follows from the results of B. Kerekyarto that the classification of periodic transformations of a two-dimensional sphere is based on the properties of homeomorphisms of a circle with a rational rotation number, which include the Morse-Smale diffeomorphisms on the circle.

In Chapter 3 a classification of Morse-Smale diffeomorphisms on a circle with respect to the relation of stable isotopic connection is obtained. From the results of A.G. Mayer [13] it follows that these diffeomorphisms (we denote their set by G^1) exhaust the class of rough transformations of the circle and have simple dynamics, the classification of which up to topological conjugacy is described as follows.

We divide the set G^1 into two subclasses G^1_+ and G^1_-, which consist of orientation-preserving and orientation-reversing diffeomorphisms, respectively. Then:

1. For every diffeomorphism $f \in G^1_+$ the set of periodic points $Per(f)$ consists of $2n, n \in \mathbb{N}$ periodic orbits, each of which has period m and rotation number $\frac{k}{m}$, where $k = 0$ for $m = 1$, or $k \in \{1, \dots, m-1\}$ for $m > 1$ and the numbers (m, k) are coprime. Diffeomorphisms $f; f' \in G^1_+$ with parameters $n, m, k; n', m', k'$ are topologically conjugate if and only if $n = n', m = m'$ and one of the following is true:
   - $k = k'$,
   - $k = m' - k'$.
2. For every diffeomorphism $f \in G^1_-$ the set of periodic points $Per(f)$ consists of $2q, q \in \mathbb{N}$ periodic points, two of which are fixed, while others have period 2. Assuming that $\nu = -1$; $\nu = 0; \nu = +1$, if its fixed points are sources; sink and source; sinks, respectively. Diffeomorphisms $f; f' \in G^1_-$ with parameters $q, \nu; q', \nu'$ are topologically conjugate if and only if $q = q'$ and $\nu = \nu'$.

The main result of Chapter 3 is the following theorem.

Theorem 1. *All rough orientation-reversing diffeomorphisms of the circle lie in the same component of the stable isotopic connection, whereas the stable isotopic class of the rough transformation of a circle that preserves orientation is completely determined by the Poincare rotation number.*

The idea of the proof of the theorem is to construct model diffeomorphisms $\Phi_{n,m,k}$, $\Psi_{q,\nu}$ in each topological conjugacy class of systems from G^1_+, G^1_-, respectively. Next, an arc without bifurcations which connects an arbitrary diffeomorphism in a given topological conjugacy class with the corresponding model is constructed. Thus, the problem is reduced to finding classes of stable isotopic connection of model diffeomorphisms.

For an orientation-preserving diffeomorphism $\Phi_{n,m,k}$, $n > 1$, the number of periodic orbits can be reduced by one pair by constructing an arc that unfolds generically through a non-critical saddle-node bifurcation. Thus the diffeomorphism $\Phi_{n,m,k}$ is connected by a stable arc with the diffeomorphism $\Phi_{1,m,k}$, which has the same rotation number. Since the rotation number is a topological invariant of a circle diffeomorphism that continuously depends on the arc parameter, any arc connecting orientation-preserving diffeomorphisms f, f' with different rotation numbers is not stable, since it contains a continuum of bifurcations, which contradicts the definition of the stable arc.

For the orientation-reversing diffeomorphism $\Psi_{q,0}$, the number of periodic orbits is even, which, as in the orientable case, allows us to connect it to the source-sink diffeomorphism $\Psi_{2,0}$ by an arc with $(q-2)$ unfolding generically non-critical saddle-node bifurcations. For the diffeomorphism $\Psi_{q,\pm 1}$, the number q is odd and $q > 2$. The technique described above allows one to connect any such diffeomorphism with the diffeomorphism $\Psi_{3,\pm 1}$, which in turn is connected by a stable arc with the source-sink diffeomorphism $\Psi_{2,0}$ by an arc with unfolding generically doubling period bifurcation.

The complete summary of the results of this chapter is published in [21].

Chapter 4 gives general dynamical properties of gradient-like diffeomorphisms of surfaces. The central place in this chapter is occupied by a result on representing the dynamics of any such diffeomorphism in the form of a global dual attractor-repeller pair for which the space of wandering orbits is connected.

Namely, consider an orientation-preserving gradient-like diffeomorphism f defined on a smooth orientable closed surface M^2.

We denote by Ω^0_f, Ω^1_f, Ω^2_f the set of sinks, saddles and sources of the diffeomorphism f. For any (possibly empty) f-invariant set $\Sigma \subset \Omega^1_f$ let

$$A_\Sigma = \Omega^0_f \cup W^u_\Sigma,\ R_\Sigma = \Omega^2_f \cup W^s_{\Omega^1_f \setminus \Sigma}.$$

According to [6] they are attractor and repeller, which are called *dual*. Let

$$V_\Sigma = M^2 \setminus (A_\Sigma \cup R_\Sigma),$$

it is called *characteristic space*. We denote by $\hat{V}_\Sigma$ the orbit space of the action of the diffeomorphism f on the characteristic space V_Σ. According to [8], each connected component of the manifold $\hat{V}_\Sigma$ is homeomorphic to a two-dimensional torus.

Theorem 2. *For every orientation-preserving gradient-like diffeomorphism* $f : M^2 \to$

M^2 there exists a set Σ, such that the orbit space $\hat{V}_\Sigma$ is connected.

In the framework of the proof, the case is considered separately when the diffeomorphism contains a unique sink orbit, then the theorem is true for the empty set Σ. When the space of orbits in sink basins consists of several connected components $\hat{V}_i, i = 1, \dots, l$, namely l two-dimensional tori, then, up to their renumbering, one can find a sequence of saddle points $\sigma_1, \dots, \sigma_{l-1}$ such that the unstable separatrices of the saddle point σ_j belong to $\hat{V}_j, \hat{V}_{j+1}$.

For any diffeomorphism f and a set Σ, satisfying the conditions of Theorem 2, put

$$A_f = A_\Sigma,\ R_f = R_\Sigma,\ V_f = V_\Sigma.$$

For the class G of gradient-like diffeomorphisms on the two-dimensional sphere S^2 the attractor and repeller A_f, R_f can be described in more detail. To do this, note that the space V_f consists of m_f pairwise disjoint cylinders and a set of non-contractible closed curves, taken one on each component, divides the sphere S^2 into two disjoint parts U and V such that

$$f(U) \subset U,\ A_f = \bigcap_{j\in\mathbb{N}} f^j(U);\ f^{-1}(V) \subset V,\ R_f = \bigcap_{j\in\mathbb{N}} f^{-j}(V).$$

Lemma 4.1 *For any diffeomorphism $f \in G$ (up to a consideration of the diffeomorphism f^{-1}) the following is true:*

1) the set U consists of $m_f \in \mathbb{N}$ pairwise disjoint disks $D_f, f(D_f), \dots, f^{m_f-1}(D_f)$ such that $f^{m_f}(cl\, D_f) \subset int\, D_f$;

2) the attractor A_f consists of m_f connected components $A, f(A), \dots, f^{m_f-1}(A)$ such that $A = \bigcap_{j\in\mathbb{N}} f^{jm_f}(D_f)$ and $f^{m_f}(A) = A$;

3) repeller R_f is connected.

Denote by G^+ the subset of G, consisting of diffeomorphisms all of whose saddle points have a positive orientation type. Let $G^- = G \setminus G^+$ and denote by G_1 the subset of G, consisting of diffeomorphisms f, for which there exists a fixed pair A_f, R_f ($m_f = 1$). Using the topology of a two-dimensional sphere, we can establish the following facts.

Lemma 4.4 *$G^- \subset G_1$.*

Lemma 4.5 *For any diffeomorphism $f \in G^+$ the number m_f is uniquely determined, that is, it does not depend on the choice of the pair A_f, R_f.*

Thus, the set $G^+ \setminus G_1$ is represented as a union of pairwise disjoint subsets

$$G^+ \setminus G_1 = G_2 \dots \cup G_m \cup \dots \qquad (*)$$

such that $m_f = m$ for any diffeomorphism $f \in G_m$, $m > 1$. This representation plays a key role in the classification of gradient-like diffeomorphisms up to the stable isotopic connection obtained in Chapter 6.

In Chapter 4 also establishes a number of important properties of *Palis's diffeomorphisms.* They constitute the class P of orientation-preserving gradient-like diffeomorphisms defined on an orientable surface M^2 under the assumption that all non-wandering points f are fixed and have positive orientation type. This class of diffeomorphisms was introduced in the work of J. Palis [25] as the class of Morse-Smale diffeomorphisms on surfaces that are included in a topological flow. For diffeomorphisms of the class under consideration, we construct a special energy function.

Let $f \in P$. Let L_p be a frame of saddle separatrices going to the node p, denote k_p their number.

Denote $L_k \subset \mathbb{R}^2$ a frame of rays $l_1, \dots, l_k$, which in polar coordinates (ρ, θ) has a form $l_i = \{(\rho, \theta) \in \mathbb{R}^2 : \theta = \theta_i\}$, $\theta_i \in [0, 2\pi)$.

A diffeomorphism $f \in P$ is called a *canonical* if every fixed point p of a diffeomorphism f has a local chart (U_p, ψ_p) such that $p \in U_p, \psi_p(p) = O$ and

1) $\psi_p f \psi_p^{-1}(x, y) = \left(\frac{1}{2}x, \frac{1}{2}y\right)$ for $p \in \Omega_f^0$,
$\psi_p f \psi_p^{-1}(x, y) = \left(\frac{1}{2}x, 2y\right)$ for $p \in \Omega_f^1$,
$\psi_p f \psi_p^{-1}(x, y) = (2x, 2y)$ for $p \in \Omega_f^2$;

2) $\psi_p(L_p) \subset L_{k_p}$ for any nodal point p.

Denote by $P_0 \subset P$ a class of all canonical diffeomorphisms.

Lemma 4.7 *For any diffeomorphism $g \in P_0$ there is an energy function Φ, whose level lines intersect every saddle separatrices at most one point.*

The idea of constructing such a function is based on the existence of local Morse energy functions in the neighborhood of fixed hyperbolic points and the regular behavior of saddle separatrices in basins of nodal points.

The complete summary of the results of this chapter is published in [19], [23], [22], [20], [24].

Chapter 5 is devoted to the construction of arcs without bifurcations within the same topological conjugacy class of a Morse-Smale diffeomorphism. Let us formulate the conception of obtained results which is stable isotopic classification fund.

- **Lemma 5.1** Any Morse-Smale diffeomorphism $f : M^n \to M^n$ with global attractor and repeller A and R is connected by an arc with any diffeomorphism f_1, coinciding with f in some neighborhoods $U_A \supset A$, $U_R \supset R$ and having the projection of unstable saddle separatrices into the orbit space $(U_A \setminus f(U_A))/f$ which is isotopic to the corresponding projection for the diffeomorphism f.

- **Lemma 5.2** Any Morse-Smale diffeomorphism $f : M^n \to M^n$ is connected with any diffeomorphism f_1, that coincides with f on the non-wandering set and is linear in some neighborhood of it.

- **Lemma 5.3** Any gradient-like diffeomorphism $f : M^2 \to M^2$, that is linear in some neighborhood of the non-wandering set is joined to a diffeomorphism f_1, that coincides with f in this neighborhood and such that the closures of the invariant manifolds of all its saddle points are smooth submanifolds.

- **Lemma 5.4** Any gradient-like diffeomorphism $f : M^2 \to M^2$ with attractor A, which is a smooth submanifold, is connected by an arc with any diffeomorphism f_1 topologically conjugate to f on attractor A and in some of its neighborhood and coinciding with f out of some neighborhood of A.

- **Lemma 5.5** Any gradient-like diffeomorphism $f : M^2 \to M^2$, whose union of unstable saddle manifolds with sinks is a smoothly embedded attractor A, is connected to any diffeomorphism f_1 coinciding with f in some neighborhood of A and in a neighborhood of sources.

The complete summary of the results of this chapter is published in [20], [24].

In Chapter 6 presents a complete classification of gradient-like diffeomorphisms of the 2-sphere up to a stable isotopic connection.

Consider S^1 as the equator of the sphere S^2. Then the structurally stable diffeomorphism of the circle with exactly two periodic orbits of the period $m \in \mathbb{N}$ and the rotation number $\frac{k}{m}$ can be extended to the diffeomorphism $\phi_{k,m} : S^2 \to S^2$, which has two fixed sources at the north and south poles.

We denote by $C_{k,m}$ the component of stable connectedness of the diffeomorphism $\phi_{k,m}$ and by $C^-_{k,m}$ the component of stable connectedness of the diffeomorphism $\phi^{-1}_{k,m}$. Denote by C_0 the component of stable connectedness of the source-sink diffeomorphism $\phi_0 \in G$ with a non-wandering set consisting of exactly one source and one sink.

The main result of the chapter is the following theorem.

Theorem 3. *Any orientation-preserving gradient-like diffeomorphism of the two-dimensional sphere S^2 belongs to one of the components C_0, $C_{k,m}$, $C^-_{k,m}$, $k,\ m \in \mathbb{N}$, $k < m/2$, $(k,m) = 1$. Wherein:*

- *components C_0, $C_{k,m}$, $C^-_{k,m}$, $k,\ m \in \mathbb{N}$, $k < m/2$, $(k,m) = 1$ are pairwise disjoint;*

- $C_{k,m} = C_{m-k,m}$, $C^-_{k,m} = C^-_{m-k,m}$, $C_{1,2} = C^-_{1,2} = C_{0,1} = C^-_{0,1} = C_0$.

Notice that belonging to different classes of stable isotopic connection of diffeomorphisms $\phi_{k,m}, \phi_{k',m'}$ for $m = 2^r \cdot q, m' = 2^{r'} \cdot q', q \neq q'$ for integers $r, r' \geq 0$ and natural numbers $q \neq q'$ follows from [1]. However, a complete classification is not given in that work.

The proof of Theorem 3 is based on the decomposition $(*)$ obtained earlier. Using the lemmas of Chapter 5 and the connectedness of the attractor A_f and the repeller R_f for the diffeomorphism $f \in G_1$, a stable arc connecting it with the diffeomorphism ϕ_0 is constructed. Also, using the connectedness of the attractor A_f of the diffeomorphism $f \in G_m$, $m > 1$ and the lemmas of Chapter 5, we can trivialize its attractor, that is, connect it by a stable arc with the diffeomorphism g from the class $f \in G_m$, $m > 1$ consisting of diffeomorphisms g for which the attractor A_g consists of one sink orbit of period m.

Further, it is established that for the diffeomorphism g there exists a saddle orbit $\mathcal{O}_\sigma$ of period m such that $cl\, W^u_{\mathcal{O}_\sigma}$ is a g-invariant closed curve C_σ and the map $g|_{C_\sigma}$ is topologically conjugate to a rough circle transformation with the rotation number $\frac{k}{m}$. Moreover, the rotation numbers for all such circles are the same. This allows us to connect the diffeomorphism g by a stable arc with a diffeomorphism whose non-wandering set consists of one saddle orbit $\mathcal{O}_\sigma = \{\sigma, f(\sigma), \ldots, f^{m-1}(\sigma)\}$, one sink orbit $\mathcal{O}_\omega = \{\omega, f(\omega), \ldots, f^{m-1}(\omega)\}$ and fixed sources α_1, α_2. Using the lemmas of Chapter 5, the constructed diffeomorphism is connected by an arc without bifurcations with the model diffeomorphism $\phi_{k,m}$.

The complete classification of model diffeomorphisms $\phi_{k,m}$ with respect to the relation of stable isotopic connection is essentially based on the fact that the saddle-node point is non-critical.

The complete summary of the results of this chapter is published in [22], [23].

In Chapter 7 the complete classification of Palis diffeomorphisms up to a stable isotopic connection is obtained. The main result of the chapter is the following fact.

Theorem 4. *Any diffeomorphisms $f, f' \in P$ of the surface M^2 are connected by a stable arc with a finite number of unfolding generically non-critical saddle-node bifurcations.*

The proof of this result is based on the construction of an arc without bifurcations connecting the diffeomorphism $f \in P$ with some canonical diffeomorphism $g \in P_0$. By Lemma 4.7, for the diffeomorphism $g \in P_0$, there exists an energy function Φ, whose inverse gradient vector field generates a gradient-like flow ϕ^τ_f. Using the level lines of this function, an arc connecting g with ϕ_f without bifurcations is constructed. Due to the existence of a stable arc between Morse-Smale flows on any manifold, the diffeomorphisms $\phi_f, \phi_{f'}$ are connected by an arc with a finite number of saddle-node bifurcations.

To visualize the constructed arc the class $Q \subset P$ of polar gradient-like diffeomorphisms on the two-dimensional torus $\mathbb{T}^2$ was considered. Conceptually, this visualization is a discrete analogue of the method used by G. Fleitas in [10].

The complete summary of the results of this chapter is published in [20], [24].

The results of the thesis are published at eight papers

- Nozdrinova E., Pochinka O. Stable arcs connecting polar cascades on a torus // Russian Journal of Nonlinear Dynamics. 2021. Vol. 17. No. 1. P. 23-37.

- Nozdrinova E., Pochinka O. Solution of the 33rd Palis-Pugh problem for gradient-like diffeomorphisms of a two-dimensional sphere // Discrete and Continuous Dynamical Systems. 2021. Vol. 41. No. 3. P. 1101-1131.

- Nozdrinova E., Pochinka O. Class of Stable Connectivity of Source-Sink Diffeomorphism on Two-Dimensional Sphere // Journal of Mathematical Sciences. 2020. No. 250. P. 94-108.

- Nozdrinova E., Pochinka O. On the solution of the 33rd Palis–Pugh problem for gradient-like diffeomorphisms of a 2-sphere // Uspekhi Mat. Nauk. 2020. T. 75. № 2. C. 195-196
- Medvedev T. V., Nozdrinova E., Pochinka O., Shadrina E. V. On a class of isotopic connectivity of gradient-like maps of the 2-sphere with saddles of negative orientation type // Russian Journal of Nonlinear Dynamics. 2019. Vol. 15. No. 2. P. 199-211.
- Nozdrinova E., Pochinka O., On the existence of a smooth arc without bifurcations joining source-sink diffeomorphisms on the 2-sphere // Journal of Physics: Conference Series. 2018. Vol. 990. No. 1. P. 1-7.
- Nozdrinova E. Rotation number as a complete topological invariant of a simple isotopic class of rough transformations of a circle // Nelineinaya Dinamika. 2018. Vol. 14. No. 4. P. 543-551.
- Loginova A., Nozdrinova E., Pochinka O. One-dimensional reaction-diffusion equations and simple source-sink arcs on a circle // Nelineinaya Dinamika. 2018. Vol. 14. No. 3. P. 325-330.

Список литературы

[1] P. R. Blanchard, *Invariants of the NPT isotopy classes of Morse-Smale diffeomorphisms of surfaces*, Duke Mathematical Journal, 47:1 (1980), 33–46.

[2] C. Bonatti, V. Z. Grines, V. S. Medvedev, O. V. Pochinka, *Bifurcations of Morse–Smale Diffeomorphisms with Wildly Embedded Separatrices*, Dynamical systems and optimization, Collected papers. Dedicated to the 70th birthday of academician Dmitrii Viktorovich Anosov, Trudy MIAN, 256, Nauka, MAIK "Nauka/Inteperiodika", M., (2007), 54–69; Proc. Steklov Inst. Math., 256 (2007), 47–61

[3] V. Z. Grines, E. Ya. Gurevich, V. S. Medvedev, O. V. Pochinka, *Embedding in a Flow of Morse–Smale Diffeomorphisms on Manifolds of Dimension Higher than Two*, Mat. Zametki, 91:5 (2012), 791–794; Math. Notes, 91:5 (2012), 742–745.

[4] V. Z. Grines, E. Ya. Gurevich, O. V. Pochinka, *On embedding of the Morse–Smale diffeomorphisms in a topological flow*, Proceedings of the Crimean autumn mathematical school-symposium, CMFD, 66:2 (2020), 160–181

[5] V. Grines, E. Gurevich, O. Pochinka, *On Embedding of Multidimensional Morse-Smale Diffeomorphisms into Topological Flows*, Moscow Mathematical Journal, 19:4 (2019), 739–760.

[6] V. Z. Grines, E. V. Zhuzhoma, V. S. Medvedev, O. V. Pochinka, *Global attractor and repeller of Morse–Smale diffeomorphisms*, Proc. Steklov Inst. Math., 271 (2010), 103–124.

[7] V. Z. Grines, O. V. Pochinka,*On a simple isotopic class of a source-sink diffeomorphism on a 3-sphere*, Math. Notes, 94:6 (2013), 828–845.

[8] V. Grines, O. Pochinka, S. Van Strien, *On 2-diffeomorphisms with one-dimensional basic sets and a finite number of moduli*, Moscow Mathematical Journal, 16:4 (2016), 727–749.

[9] A. Dolgonosova, E. Nozdrinova, O. Pochinka, *On the obstructions to the existence of a simple arc joining the multidimensional Morse-Smale diffeomorphisms*, Динамические системы, 7(35):2, (2017), 103–111.

[10] G. Fleitas, *Replacing tangencies by saddle-nodes*, Bol. Soc. Brasil. Mat., 8:1 (1977), 47–51.

[11] A. B. Katok, B. Hasselblat, it Introduction to the modern theory of dynamical systems / per. from English A. Kononenko with participation of S. Ferleger. - M .: Factorial, (1999), 768.

[12] B. von Kerekjarto, *Uber die periodischen Transformationen der Kreisscheibe und der Kugelflache* Math. Ann, 80:1 (1919), 36–38.

[13] A. G. Mayer, *Rough transformation of a circle to a circle*, Sci. Notes Gorkiy State Univ., 12 (1939) 215–229.

[14] S. Matsumoto, *There are two isotopic Morse-Smale diffeomorphisms which cannot be joined by simple arcs*, Inventiones mathematical, 51 (1979), 1–7.

[15] S. Newhouse, J. Palis, F. Takens, *Bifurcations and stability of families of diffeomorphisms*, Publications mathematiques de l' I.H.E.S, 57 (1983), 5–71.

[16] S. Newhouse, J. Palis, F. Takens, *Stable arcs of diffeomorphisms*, Bull. Amer. Math. Soc., 82:3 (1976), 499–502.

[17] S. Newhouse, M. Peixoto, *There is a simple arc joining any two Morse-Smale fows*, Asterisque, 31 (1976), 15–41.

[18] J. Nielsen, *Die struktur periodischer transformationen von flachen*, Math.-fys. Medd. Danske Vid. Selsk. 15 (1937).

[19] E. Nozdrinova, *Existence of a connected characteristic space for gradient-like diffeomorphisms of surfaces*, Middle Volga Mathematical Society Journal, 19:2 (2017), 91–97.

[20] E. Nozdrinova, *On a stable arc connecting Palis diffeomorphisms on a surface*, Динамические системы, 10(38):2 (2020), 139–148.

[21] E. Nozdrinova, *Rotation number as a complete topological invariant of a simple isotopic class of rough transformations of a circle*, Russian Journal of Nonlinear Dynamics, 14:4 (2018), 543–551.

[22] E. V. Nozdrinova, O. V. Pochinka, *On the solution of the 33rd Palis–Pugh problem for gradient-like diffeomorphisms of a 2-sphere*, Uspekhi Mat. Nauk, 75:2(452) (2020), 195–196; Russian Math. Surveys, 75:2 (2020), 383–385.

[23] E. Nozdrinova, O. Pochinka, *Solution of the 33rd Palis-Pugh problem for gradient-like diffeomorphisms of a two-dimensional sphere*, Discrete and Continuous Dynamical Systems, 41:3 (2021), 1101–1131.

[24] E. Nozdrinova, O. Pochinka, *Stable arcs connecting polar cascades on a torus*, Russian Journal of Non-linear Dynamics, (2021).

[25] J. Palis,*On Morse-Smale dynamical systems*, Topology, 8:4 (1969), 385–404.

[26] J. Palis, C. Pugh, *Fifty problems in dynamical systems*, Lecture Notes in Math., 468 (1975), 345–353.

[27] D. Pixton, *Wild unstable manifolds*, Topology, 16:2 (1977), 167–172.