On-line «Master in Computer Vision» educational program.
Entrance examination test (each problem is evaluated by 10 points).
Variant 4

1. Find all solutions in integer numbers $x, y \in \{0,1,2, \ldots ,2020,2021\}$ of the equation

$$\frac{e^x}{e^x + e^y} = \frac{e^{2019}}{e^{2019} + 1}$$

2. There are given three row vectors of dimension 3:

$$u = (0, a, -1), \quad v = (1, 1, a), \quad z = (3, 5, 1)$$

Find all values of $a \in R$, such that the vector z is a linear combination of the vectors u and v.

3. Independent identically distributed discrete random variables X and Y have probability mass function (probability distribution) defined by the table

Values of X (or Y)	-2	1	2
Probability	1/5	2/5	2/5

Random variables U and V are defined by

$$U = 2X + Y, V = 2X - Y$$

Find expectation (expected value, mean) of the random variable $Z = U \cdot V$

4. In the following problem it is necessary to suggest the most efficient algorithms. Full points are given for the most efficient algorithm having the lowest computational complexity. The lower the efficiency of the suggested solution, the lower are the points.

For a given array of real numbers $a[i]$, $i=0,1,2,...,n-1$ and a given threshold $t \in R$, a t-chain in the array is a sequence of elements

$$a[k], a[k+1], a[k+2],..., a[k+l-1]$$

such that $a[k+j] \geq t$ for $j=0,1,...,l-1$.
Write a pseudo-code (or code on any programming language) of an algorithm, which takes as input an array of real numbers and a threshold t and gives as output the maximal length of t-chain in the array. Discuss the computational complexity of your algorithm.