Comparative Analysis of MADM Approaches: ELECTRE, TOPSIS and Multi-level LDM Methodology

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***Abstract*— There are multiple Multi-Attribute Decision Making methods elaborated for the past years. Those methods are targeted at aggregating assessments provided by the stakeholders of the problematic situation in order to choose the best alternative from the set of given ones. This paper considers ELECTRE, TOPSIS and the Multi-Level Linguistic Decision Making Methodology. In this paper we try to challenge first two methods by comparing it to latter method. One of the biggest challenges of modern Decision Making methods is flexibility to accept not only quantitative assessments but also hybrid ones: qualitative, interval, mixed etc. This brings the necessity for fuzzy computations. Decision Making methods analysis is performed through deep dive in constitution of each method and comparison across the set of elaborated criteria. Key criteria for the Decision Making methods assessment were identified and the comparative analysis on the base of two scenarios of different complexity was elaborated. The conclusion is made that ELECTRE and TOPSIS are well suited for small problems containing only several (less than a dozen) alternatives and criteria while being hardly generalized for the case of poorly structured problems (pollution, hunger, poverty). At the same time, Multi-Level Linguistic Decision Making Methodology excels at analyzing the problem from multiple aspects and considering any number of experts with arbitrary expertise that is beneficial in complex decision making cases.**

Keywords*— *multi-attribute decisionmaking; linguistic decision making; fuzzy logic

I. INTRODUCTION

The decision making process is usually connected to the huge list of alternative solutions. The main difficulty in selecting one of them is the fact that criteria that are used to compare the alternatives are often conflicting and it is almost impossible to find such an alternative that would be a head above all other through all the criteria.

More importantly, criteria or as they are traditionally referred to as attributes can be both qualitative and quantitative. Therefore, the process of decision making is referred to as Multi-Attribute Decision Making (MADM) or Multi-Criteria Decision Making (MCDM) [1]. Also, criteria values can have different nature resulting in different measurement units. Finally, it is also difficult to define criteria importance. Even assigning a-priori importance weights to each criterion can be a failure strategy if, for example, you are not well informed about the topic and as you dive into and better understand the context your views on importance of criteria might change.

There are two classical methods in the MADM field: ELECTRE [2, 3] and TOPSIS [4]. They are often used in solving real-life cases [5, 6] and as a basis for the development of more sophisticated approaches [7, 8]. At the same time those methods obtain several considerable gaps that leave the room for further improvement of MADM methods [9]. Therefore, the goal of this article is to analyze if those gaps be covered by the newly introduced methodology [10] as well defining recommendations for Decision Makers on choosing the appropriate method depending on the complexity of their use case.

This paper is organized as follows. Section II contains motivation of decision making methods selection and draws attention to important details about ELECTRE, TOPSIS and ML-LDM. Comparison results are covered in Section III with the concluding remarks in Section IV.

II. DECISION MAKING METHODS ARCHITECTURE

The decision making process usually consists of 8 consecutive steps [9]:

1. defining the problem;
2. determining the requirements;
3. establishing the goals;
4. identifying alternatives;
5. developing evaluation criteria;
6. selecting the decision making tool;
7. application of the tool;
8. checking the answer.

When we review the decision making methods, like ELECTRE [2, 3] or TOPSIS [4], we position ourselves on the steps 6 and 7 where both the alternatives and criteria are prepared by someone. In contrary, several novel approaches

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propose a bigger influence on the Decision Making chain. For example, the recently proposed method ML-LDM [4] focuses also on the semantics of criteria trying to build them in the hierarchy and utilize this characteristic when the decision making method is applied to the assessments collected.

Although the topic of comparing ELECTRE and TOPSIS method is not novel and has been covered in [7, 9, 11], we will focus on the most considerable ideas. Firstly, there is a huge interest to both methods in the applied field: environmental issues [12], urban problems [5], different system selection [6] etc. Secondly, there were made numerous attempts of improving those methods in several directions: weighting techniques for criteria [8], fuzzy computations support [6] etc. Thirdly, novel approaches appear with the focus on simulation capabilities, ability to accept multiple experts' assessments [10]. In the next section we will review ELECTRE, TOPSIS and ML-LDM.

A. ELECTRE Methods

The ELECTRE (ELimination and Choice Expressing REality) methods family was originally proposed and later improved in [2], [3]. In general, those approaches provide the ability to evaluate dominance of alternative solutions compared to each other using the concordance analysis. The decision process starts with collecting the estimations of each alternative for each criterion. This results in the decision matrix $A = (x_{ij})$, that is of the form where x_{ij}denotes theassessment given for the i_{th} alternative for the j_{th}criterion:

The original method contained 9 successive steps [1]:

1. calculating the normalized decision matrix;
2. calculating the weighted normalized decision matrix;
3. building the concordance and discordance sets;
4. calculating the concordance matrix;
5. calculating the discordance matrix;
6. building the concordance dominance matrix;
7. building the discordance dominance matrix;
8. buildingtheaggregatedominancematrix;
9. eliminating the less favorable alternatives.

B. TOPSIS Methods

The TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method was proposed by Yoon and Hwang [1]. The idea behind this technique is quite simple: after defining the "ideal" and "negative-ideal" solutions we try to figure out what alternative is simultaneously the closest to the "ideal" solution and the furthest to the "negative-ideal" one. The decision process starts with collecting the estimations of each alternative for each criterion. This results in the decision matrix A. The original method contained 6 successive steps [1]:

1. calculating the normalized decision matrix;
2. calculating the weighted normalized decision matrix;
3. defining "ideal" and "negative-ideal" solutions;
4. calculating the separation measure;
5. calculating the relative closeness to the ideal solution;
6. ranking the preference order.

C. Multi-Hierarchical Linguistic Decision Making Methodology

The Multi-Hierarchical Linguistic Decision Making Methodology was recently proposed in [10]. The core idea behind the methodology is utilizing two important features of any poorly structured problem: multiple aspects of the problematic situation (political, economic, ecological etc.) and huge list of stakeholders as well as experts who are usually invited to the Working Group to elaborate the best possible solution. Both ELECTRE and TOPSIS approaches are focusing mostly on the aggregation of the assessments and finding out the final aggregate characteristic that can be used to compare alternatives and choose the best one. It is important to note that ML-LDM inherently works with assessments of various types, including qualitative and interval ones based on fuzzy computations over 2-tuple model [13] and several add-ons [14]. The 2-tuple model is based on the concept of symbolic translation [4].

Definition 1. A 2-tuple structure includes a pair (s_i, α) where $s_i \in S = \{s_0, \dots, s_g\}$ – is a linguistic term (concept), α – a numeric value or a symbolic translation that shows a result of execution of membership function. It shows the distance to the closest concept $s_i \in S = \{s_0, \dots, s_g\}$ if a membership function does not result in an exact value (s_i).

Definition 2. Translation rule. Let $S = \{s_0, \dots, s_g\}$ be a linguistic scale, where $g = \tau + 1$ denotes granularity level of S. If $\beta \in [0,1]$ is a result of symbolic aggregation, then there is a way to recover a corresponding 2-tuple element:

$$\Delta_g = [0,1] \to S \times [-0.5, 0.5)$$
$$\Delta_g(\beta) = (s_i, \alpha) = \begin{cases} s_i, & i = round(\beta\tau) \\ \alpha = \beta\tau - i, & \alpha \in [-0.5, 0.5) \end{cases} \quad (1)$$

Definition 3. Reverse translation rule. Let $S = \{s_0, \dots, s_g\}$ be a linguistic scale, where $g = \tau + 1$ denotes granularity level of S. Let (s_i, α) be a 2-tuple element on a linguistic scale S, where $\alpha \in [-0.5, 0.5)$. Then there is a way to transform 2-tuple element to a numeric form of $\beta \in [0,1]$:

$$\Delta_g^{-1} = S \times [-0.5, 0.5) \to [0,1]$$
$$\Delta_g^{-1}(s_i, \alpha) = \frac{i + \alpha}{\tau} \quad (2)$$

The aggregation step of the methodology consists of 6 steps [10]:

1. formulating matrices of interval estimations;
2. aggregating estimations by criteria;
3. translating estimations to abstraction levels;
4. aggregating estimations by experts;

5. aggregating estimations by levels of abstraction;
6. eliminating the less favorable alternatives.

III. Methods Comparison

After we have made the overview of three approaches to aggregating assessments, it is important to analyze them and compare on several aspects that were elaborated:

1. Simplicity. Any method should be easy to use for the Decision Maker.

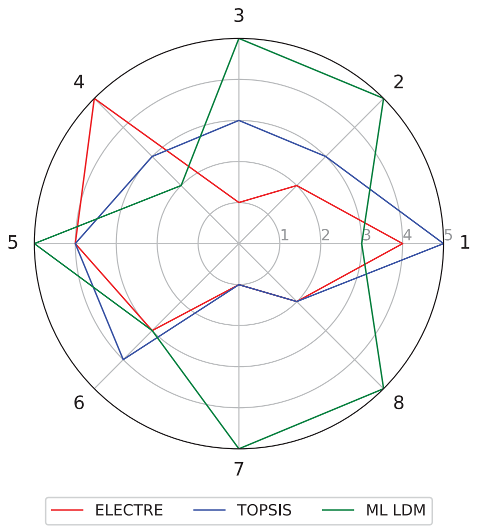

Fig. 1. A polar graph with estimations for three MADM methods

2. Flexibility to accept hybrid values. In a complex case there are multiple qualitative criteria. Any attempt to translate linguistic terms to particular integer value on the scale results in information loss.
3. Extensibility to work with multiple experts. In case of poorly structured problems, it is not enough to collect assessments from a single person as a profound expertise on multiple domain is required.
4. Number of external parameters. Each additional parameter needs to be selected in some manner and the final result can highly depend on the selected value of each parameter. Authors strongly consider that the lesser external parameters the method uses and the more it extracts from the data, the better and more robust the overall solution is.
5. Clarity to the decision maker. It is a known problem of almost every DM method that it results in the single floating-point value per the alternative that brings no information to the Decision Maker about the origins of this mark and produces low trust especially for people who use the DM method as a black box.
6. Computational complexity. The DM method should not be computationally expensive that would create obstacles for its widespread adoption.
7. Criteria semantics considered. Criteria can be organized in some meaningful manner that itself is the useful artifact for the Decision Maker that can be incorporated in the method or used externally for the criteria weights distribution.
8. Simulation capabilities. The bigger the cases are that we are trying to solve the more complex and unintuitive is the DM process. We already face the need in simulating the DM process in order to understand the internal nature of agents' interaction, for example to track trust level dynamics among them.

As the basis for the methods assessments we used two scenarios from the literature that from our perspective reflect the simple and the complex cases respectively. The first case is taken from [1, p. 62]. It is devoted to the selection of a Fighter Aircraft. There are 6 main criteria: maximum speed (X1), ferry range (X2), maximum payload (X3), purchasing cost (X4), reliability (X5), maneuverability (X6). Also, there are 4 alternative unnamed jets $A_i, i = 1, 2, 3, 4$. The second case that we considered as a complex one is taken from [15]. Under the focus of this case is the problem with decreasing levels of rice production in the state Chhattisgarh (India). There are 26 alternative solutions, for example, increase of crop via irrigation system implementation (A.ELA.7) or decrease fertilizers usage (A.SLA.3). Also, there are 28 criteria elaborated by all the stakeholders, for example, water pollution level (C.SLA.2) or per unit cost (C.ELA.5). It is important to note that both alternatives and criteria were defined for each abstraction level. Full decision matrix is dropped because of current work limits, it can be found in the original paper [10]).

TABLE I. Decision Making methods compared

Criteria	**Decision Making Methods**		
	ELECTRE	***TOPSIS***	***ML-LDM***
Simplicity	4	5	3
Flexibility to accept hybrid values	2	3	5
Extensibility to work with multiple experts	1	3	5
Number of external parameters	5	3	2
Clarity to the decision maker	4	4	5
Computational complexity	3	4	3
Criteria semantics considered	1	1	5
Simulation capabilities	2	2	5

After solving both cases using three alternative DM methods, a single expert whose profound knowledge in the field allows to get objective estimations has estimated each method. Results are present in Table 1 and polar graph (Fig. 1).The scale 1–5 was used for the assessment, each value of a scale has a corresponding transcription in linguistic terms. The scale is presented in Table 2. 6 out of 8 criteria are benefit-like, that we want to maximize, while two others (Number of external parameters and Computational complexity) are cost-like. Nevertheless, when estimating the alternative DM methods, it is incorporated in the mark given by the expert. For instance, small number of parameters is assessed as "excellent" in terms of scale.

TABLE II. DECISION MAKING METHOD ESTIMATIONS MEANING

Estimation	***Meaning***
1	Very poor
2	Poor
3	Satisfactory
4	Good
5	Excellent

In Fig. 1 we can see that ELECTRE and TOPSIS are better for simple or mid-level complexity cases than the ML-LDM because of higher scores on corresponding criteria: Simplicity, Computational complexity, Number of external parameters. At the same time, the ML-LDM methodology is better at the following criteria: Criteria semantics considered, Clarity to the decision maker. When speaking about advanced features, ML-LDM outperforms canonical methods (ELECTRE, TOPSIS) on three criteria: Flexibility to accept hybrid values, Extensibility to work with multiple experts, Simulation capabilities. This brings us to the conclusion that ML-LDM is a method of choice for complex DM cases.

IV. CONCLUSIONS

Both ELECTRE and TOPSIS methods are canonical ones and it is recommended to take them as the baseline for analyzing new methods. New approaches are likely to be extremely focused on the use case they were created for while losing generality and universality of original methods. Moreover, ELECTRE and TOPSIS are already implemented in several software products and will be dominating for the long time in the unchanged form. At the same time, it is important to continue investigation of important properties of those methods and extend the comparison in future to include their modern derivatives especially in the field of fuzzy computations.

To sum up, each approach should be selected for the particular problem which lies itself in a MADM field. This paper highlights main criteria to be used when selection process happens and helps to better understand internal constitution of those methods.

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