The use of the system dynamics model to determine the probability of company default

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Abstract. This research demonstrates the approach of using the system dynamics model to assess the probability of company default that is a relevant problem in credit risk analysis. System dynamics offers models in which the reality is simulated structurally. According to the principles of system dynamics, the company is represented in the form of continuously interacting elements and external factors. Enterprise dynamics and the enterprise's resistance to various macroeconomic environments are determined by functional dependencies and differential equations that describe the links between the elements of the model. The behavior of random macroeconomic variables is described with a multivariate ARIMA-GARCH model, which is used in econometrics to predict non-stationary time series. The probability of company default is determined as a result of experiments with the obtained system dynamics model using the Monte Carlo simulation. The estimation of a default probability is the overall share of macroeconomic scenarios leading to the ruin of the enterprise. A comparative analysis of the obtained results and data from Moody's and Fitch demonstrates the closeness of the probability of company defaults obtained by simulation and corresponding estimates of rating agencies, which makes it possible to conclude that the considered approach is acceptable for estimating the probability of default of a borrower.

1. Introduction

In this article, the authors consider the problem of assessing the probability of company default, which is relevant in the framework of the credit risk analysis.

Currently, a significant number of mathematical methods for estimating the probability of a borrower's default have been developed, based on the analysis of the values of various quantitative and qualitative indicators of an enterprise [1]. Most of the methods do not take into account the structure of the company, or its dynamics in the context of changing external factors, and these methods imply the presence of a large sample of data about similar companies.

This work demonstrates another approach to assess the probability of a company's bankruptcy, which is based on system dynamics model [2, 3], and eliminates the aforementioned drawbacks. In the system dynamics paradigm, the enterprise under study is represented as continuously interacting elements and external factors. Relations between elements are described by functional dependencies and differential equations that determine the company's dynamics, and the degree of its stability in relation to various macroeconomic scenarios. The behavior of macroeconomic variables which are external factors for the system dynamics model is described by a multivariate ARIMA-GARCH model [4], used in econometrics to predict non-stationary time series. The probability of a company's default is determined

from the results of experiments conducted by the Monte-Carlo method with the system dynamics model. It is the share of macroeconomic scenarios leading to the ruin of an enterprise.

The key difference of the proposed approach to solving problem of assessing the probability of company default is modeling with consideration not only individual financial indicators, but also the features of the company structure and its dynamics with changing external factors. System dynamics tools are used for this purpose.

This article includes four parts. The second part is devoted to methodology of system dynamics and ARIMA-GARCH model. The part 2.1 describes the key properties of the system dynamics model of an oil producing and refining enterprise. The section 2.2 is devoted to the concept of modeling macro parameters with the ARIMA-GARCH model. These parameters are external for an oil producing and refining enterprise, which system dynamic model is represented in the section 2.1. The numerical results of assessing the probability of default are shown in Section 3. This part also presents a comparative analysis of the simulation results with data on the default probability from the Moody's and Fitch rating agencies. The main conclusions about the effectiveness of the considered approach are summarized in Section 4.

2. Methodology

The authors of this paper discuss the possibility of applying the principles of system dynamics to assessing the probability of enterprise default by reproducing its structure and changes under the influence of external economic conditions. For this, the authors consider the system dynamics model of the oil producing and refining enterprise, described in detail in [3], reproduce external economic factors using ARIMA-GARCH models and present the result as input parameters to the system dynamics model. Then the fact of default or non-default of the model is recorded. This procedure is repeated and the default frequency of the enterprise is calculated. Thus, according to the Monte Carlo Method, the probability of company bankruptcy is estimated.

2.1. System dynamics methodology

In this study, the macroeconomic parameters described by the ARIMA-GARCH models are the input parameters for the system dynamics model of an oil producing and refining. The simulation of the system dynamics model with macro parameters on its inputs allows estimating the probability of company default.

System dynamics is a simulation modelling approach developed to describe the structure and dynamics of complex systems based on the concept of flows, stocks and feedbacks, which are translated to differential equation systems as a formal representation. The classic fundamental literature on system dynamics includes [5].

The development of a system dynamics model includes describing the object or phenomenon's structure in a form of stock and flow diagrams, as well as determining the interaction between its parts and elements. The model presented in this form is elaborated with computer-aided simulation, by testing various hypotheses about its behavior, and by checking the constructed interconnections on test data.

The purpose of this work is to study the possibility of using the system dynamics' principles to assess the default probability of enterprise. Based on the financial statements for 2007-2015 and information from other open sources, we built a system dynamics model of the Bashneft company, which has been producing oil since 1932 and has developed about 170 fields in different regions of Russia. For the purposes of this article, Bashneft was selected as it met the following critical conditions:

- Open sources have a large amount of information about the structure and financial activities of the company for at least 7 years. At the same time, the structure of the company changed during this time not significantly.
- The company is rated by the largest rating agencies (for example, Moody's, Fitch).
- The company is a major representative of one of the key sectors of the economy.

The Bashneft model was built on the basis of data from the period 2007-2014, since in 2015-2016 the company was acquired by Rosneft, which led to significant changes in both the production and financial activities of the company. For the purposes of the study, it is sufficient to consider information from the period 2007-2014.

A complete description of the considered system dynamics model is contained in [3]. In this paper, we describe it only in general terms. This model consists of three main parts which continuously interact with each other: production, financial, and core. The production part demonstrates the production process of the company, which results in products for sale. The main stocks of the production part aggregate material factors. In the considered case, they are oil and petroleum products. Flows provide the connection between these stocks, and represent the production process this way. The production part of the system dynamic model is presented in figure 1, under the number (1).

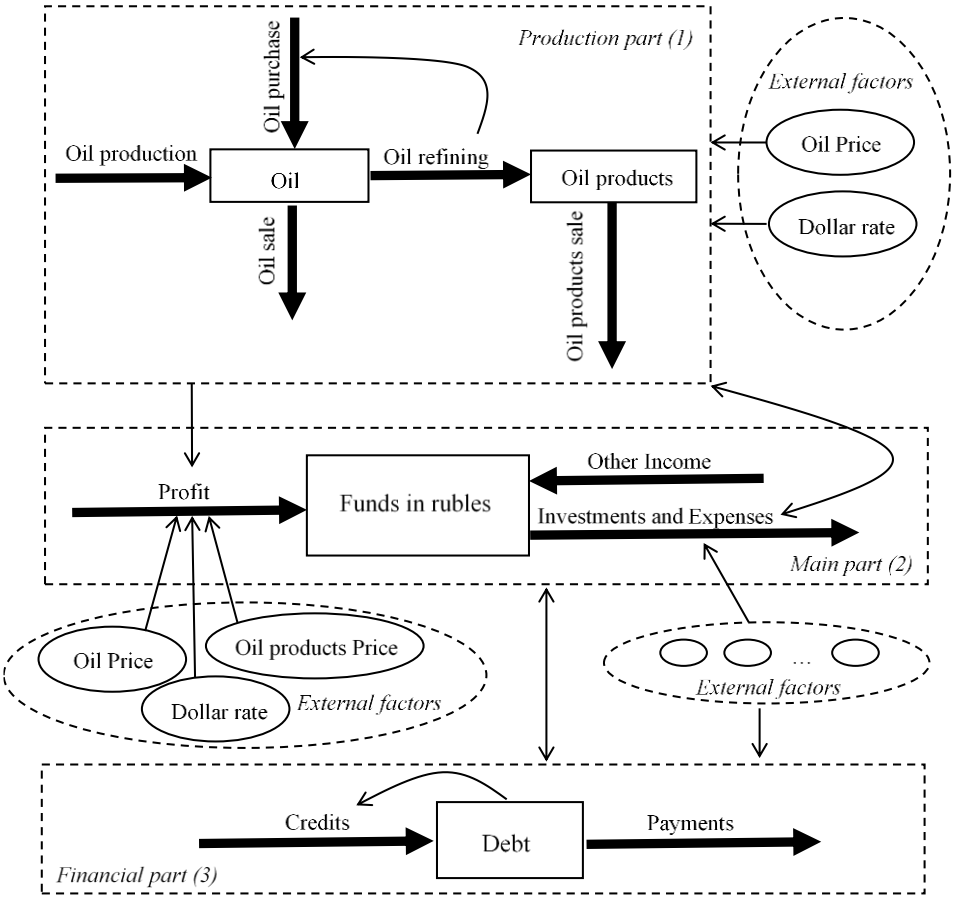

Figure 1. Stock and flow diagram of the system dynamic model of an oil producing and refining enterprise.

The financial part reflects the features of the company credit policy. One of the main elements of the financial part is the stock named ‘debt’. The flows correspond to attracting new loans and loan repayments. The financial part is presented in figure 1 under number (3).

The core part of the model (figure 1, number (2)) consists of the ‘funds in rubles’ stock, which accumulates the results of production and financial activities. This stock indicates the amount of funds available to the company. The zero value of this stock means the default of the enterprise.

The links between different parts of the model indicate their influence on each other. For example, investments affect the volume of oil production, while the volume of production determines the

corresponding expenses. In addition, the credits increase the value of the 'funds in rubles' stock, and the loan repayments reduce it.

The macroparameters affecting the state of the model are the prices for oil and petroleum products traded by the company, the US dollar to ruble rate, the rate of attracted loans, the basic mineral tax rate, the cost of unit production, processing, and general business expenses (table 1). The paper considers the factors presented in table 1, because they determine dynamics of the enterprise under study. It should be noted that the events of 2016, related to the purchase of this company by Rosneft, were not taken into account when the model was building.

2.2. ARIMA-GARCH methodology

In the framework of this study, the macro parameters are described using a multivariate ARIMA-GARCH model, which is used in econometrics for analyzing and predicting non-stationary time series [4], [6], 7]. In the hybrid ARIMA-GARCH model, two stages can be arbitrarily distinguished. In the first stage, we use ARIMA model to move from non-stationary process to stationary one by taking the differences of some order from the initial non-stationary process. In the second stage, we use the GARCH model in order to contain non-linear residuals patterns. The estimation procedure of ARIMA-GARCH model are based on maximum likelihood method.

The basic concept of ARIMA models is to determine the linear dependence of the current value of a time series on its previous values and exogenous factors, taking into account random errors. In this case, the GARCH model describes variance of the current value of these random errors as a function of its previous terms. Thus, the classic ARIMA-GARCH model consists of two parts.

Integrated autoregressive moving average models (ARIMA) are a generalization of ARMA models for non-stationary time series. The basic idea is to move from a non-stationary process to a stationary one by taking the differences of some order. In fact, the ARIMA model (p, d, q) means that the differences of the time series of order d obey the ARMA model (p, q):

$$\Delta^d Y_t = c + \sum_{i=1}^{p} a_i \cdot \Delta^d Y_{t-i} + \sum_{i=1}^{q} b_i \cdot e_{t-i} + e_t, \tag{1}$$

where Δ^d is the difference operator of a time series with order d, which means taking the differences of the first order d times consecutively. Using the lag operator $L : LY_t = Y_{t-1}$ formula (1) can be written in the following form:

$$(1-L)^d Y_t = c + (\sum_{i=1}^{p} a_i \cdot L^i)(1-L)^d Y_t + (1 + \sum_{i=1}^{q} b_i \cdot L^i) e_t.$$

Generalized models of autoregressive conditional heteroscedasticity (GARCH) are used to analyze the time series whose conditional variance is changed and depends on its previous values and past values of the series. In the context of ARIMA models, they are applied to the e_t tationary process:

$$e_t = \sigma_t \cdot z_t,$$

$$\sigma_t^2 = c_0 + \sum_{i=1}^{s} \gamma_i \cdot \sigma_{t-i}^2 + \sum_{i=1}^{r} \beta_i \cdot e_{t-i}^2,$$

where c_0 is a constant; γ_i nd β_i re the model's coefficients; $\{z_t\}$ s a random process of independent identically distributed random variables. In this study, $\{z_t\}$ beys the standard normal distribution. Thus, the set of equations describing the ARIMA (*p*,*d*,*q*)-GARCH (*r*,*s*) model has the following form:

$$(1-L)^d Y_t = c + (\sum_{i=1}^{p} a_i \cdot L^i)(1-L)^d Y_t + (1+\sum_{i=1}^{q} b_i \cdot L^i)e_t,$$

$$e_t = \sigma_t \cdot z_t,$$

$$\sigma_t^2 = c_0 + \sum_{i=1}^{s} \gamma_i \cdot \sigma_{t-i}^2 + \sum_{i=1}^{r} \beta_i \cdot e_{t-i}^2.$$

In this work, macroeconomic variables are considered as risk factors (table 1), whose dynamics are represented using the multivariate ARIMA-GARCH model:

$$(1-L)^d Y_t = c + \left(\sum_{i=1}^{p} a_i \cdot L^i \right)(1-L)^d Y_t + \left(1+\sum_{i=1}^{q} b_i \cdot L^i \right)\varepsilon_t,$$

$$\varepsilon_t = H_t^{1/2} \cdot e_t.$$

Here $X_t = \left(\ln\left(x_t^1\right),\ldots,\ln\left(x_t^N\right)\right)'$is the vector of logarithms of the macroeconomic parameters at the moment of time *t*, $L : LX_t = X_{t-1}$is the lag operator; $a_1,\ldots,a_p$; d $b_1,\ldots,b_q$ are the matrices of real numbers, which are autoregressive coefficients and moving average coefficients; *p, q* и *d* are the natural numbers that determine the order of the model; $\{e_t\}$ s the vector of random processes of independent random variables which are identically normally distributed, *c* is the constant; H_t the positive definite covariance matrix, representable as follows [7]:

$$H_t = B'B + \sum_{k=1}^{r} U_k' \varepsilon_{t-1} \left(\varepsilon_{t-1}\right)' U_k + \sum_{k=1}^{r} J_k' H_{t-1} J_k,$$

where *B*, J_k ,U_k, are the matrices of parameters that have dimension $N \times N$ while *B* is the upper triangular matrix.

Within this section, the relevant macro parameters (table 1) are described using the multivariate ARIMA-GARCH model, based on the quarterly data for the period 2004-2014. The model was realized in a computing environment using the MATLAB programming language.

The order of the multivariate ARIMA-GARCH model is selected based on the Bayesian Information Criterion (*BIC*). To determine the optimal values of the quantities $p \in \{1,2,3,4,5\}, d \in \{1,2,3,4,5\}, q \in \{1,2,3,4,5\}, r \in \{1,2,3,4,5\}$ the *BIC* was calculated for each combination of these parameters. The minimum value of *BIC* = 383.74 corresponds to the following set of parameters: *p* = 3, *d* = 2, *q* = 2, *r* = 1.

The statistical significance and quality of all the constructed models are also estimated on the basis of the Fisher asymptotic test, and the analysis of the coefficient of determination.

The ARIMA-GARCH model is used to reproduce the dynamics of the parameters presented in table 1, because they determine the changes in the system dynamics model, and, therefore, the stability of the enterprise. The plots shown in figures 2 and 3, illustrate the behavior of the main external factors during the 22 quarters of the 2010-2014 historical period and 50 implementations of the ARIMA-GARCH model for the forecast period 2015-2021. The graphs make it possible to verify the non-stationary of the financial time series under consideration and to see the dynamics of external factors that will be fed to the input of the system dynamics model. The period of consideration of the ARIMA-GARCH model is selected in accordance with the period of construction and modeling of the system dynamics model. At the same time, the aim of the study is not to accurately predict time series. The purpose of the study is to determine the probability of company default using a system dynamics model that determines the

behavior and stability of the enterprise. However, for this purpose it is necessary to reproduce the dynamics of economic indicators as external parameters of the system dynamics model. For this, the ARIMA-GARCH model is used.

Table 1. External parameters that are considered in the system dynamics model of an oil company.

Name of macroeconomic parameter	Description
US dollar to ruble rate	The US dollar exchange rate against the ruble, set by the Central Bank of the Russian Federation.
Oil price (world)	World oil price, brand «Urals» (US dollars per barrel).
Oil price (Russia)	«Urals» oil price in the Russian market (rubles per ton).
Diesel price (world)	World diesel price (US dollars per ton).
Diesel price (Russia)	Diesel price in the Russian market (rubles per ton).
Gasoil price (world)	World gasoil price (US dollars per ton).
Gasoil price (Russia)	Gasoil price in the Russian market (rubles per ton).
Motor oil price (world)	World motor oil price (dollars per ton).
Motor oil price (Russia)	Motor oil price in the Russian market (rubles per ton).
Other petroleum products price (world)	The category ‘other’ includes petroleum products, the quarterly sales volume of which in 2007-2015 was less than 0.08 million tons: low-octane gasoline, etc. In addition, vacuum gas oil was included in the ‘others’ category, the sales volume of which decreased during the period under consideration, and, in 2015, amounted to less than 0.04 million tons. The average price for all petroleum products that are included in the ‘other’ category (dollars per ton) was used in the model.
Other petroleum products price (Russia)	‘Other’ price in the Russian market (rubles per ton).
Basic mineral tax rate	The basic mineral tax rate (or basic mineral extraction tax rate) is established annually by the corresponding Russian law and determines the total mineral tax rate.
Unit cost of general running	Monetary value of the general running cost per one ton of oil. It is related to selling, general, and administrative expenses, and is a major non-production cost presented in an income statement.
Unit cost of processing	Monetary value of the processing cost for one ton of oil.
Unit cost of production	Monetary value of the production cost for one ton of oil.
Mosprime rate (rate of loans)	The Mosprime rate is based on the offer rates of Russian ruble deposits, as quoted by Contributor Banks – the leading participants of the Russian money market - to the first class financial institutions.

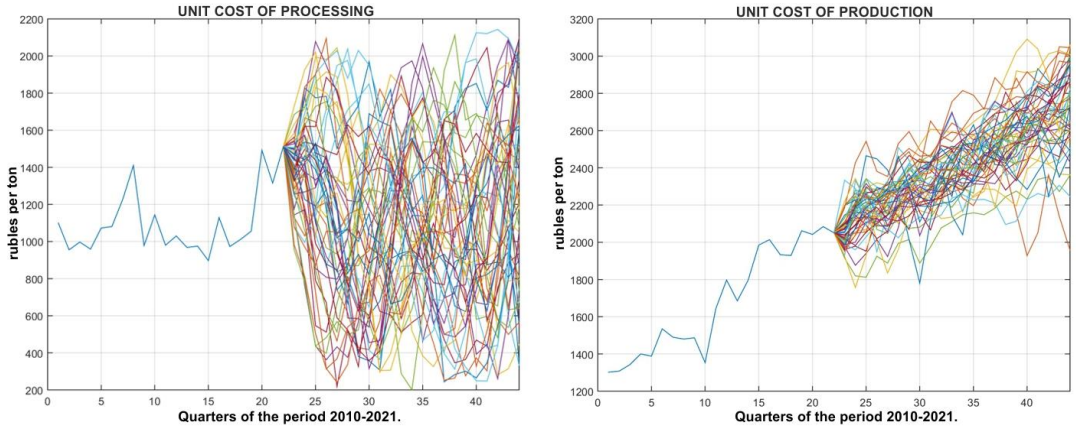

Figure 2. External parameters. Historical curves and realizations ARIMA-GARCH model during the simulated period.

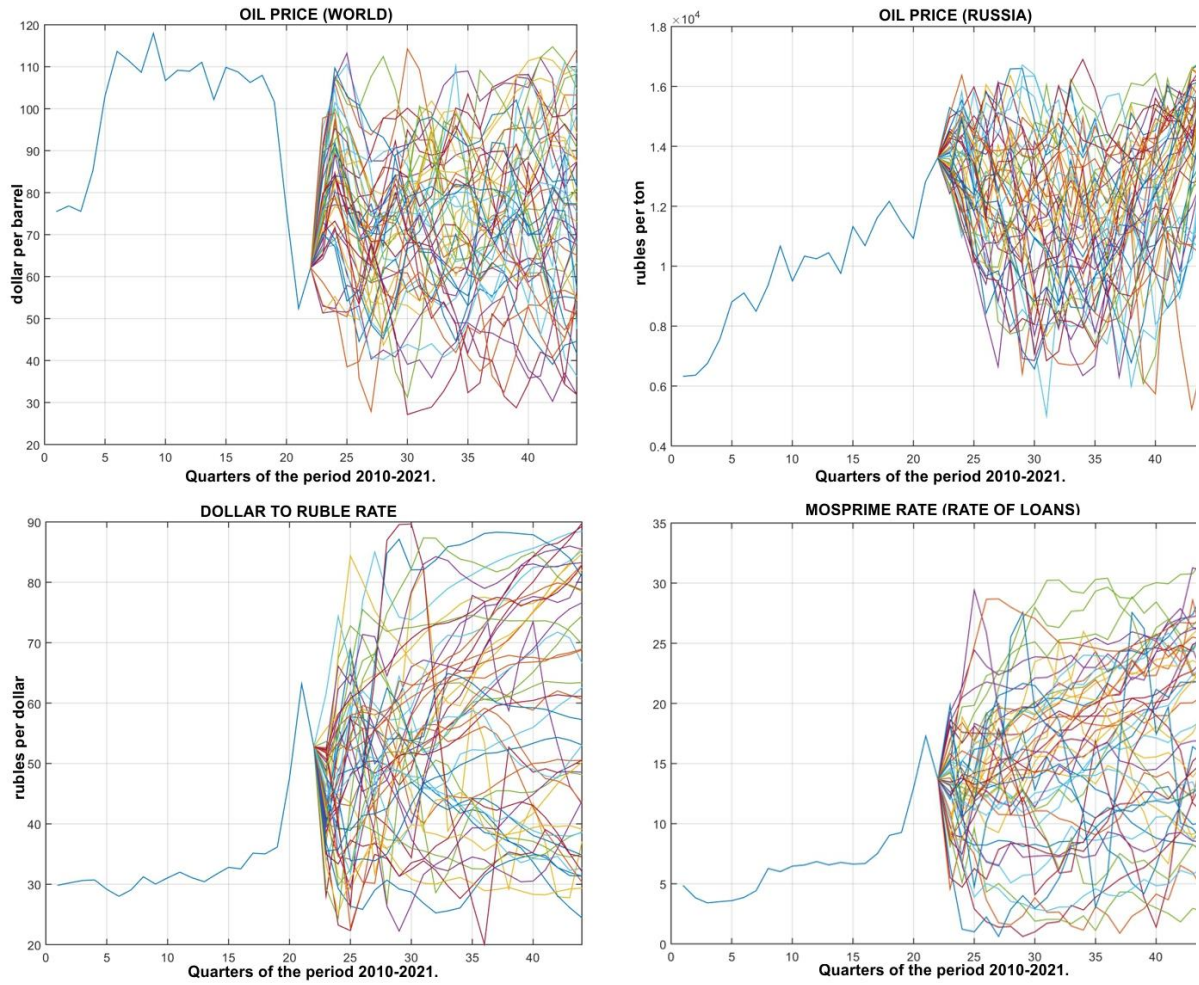

Figure 3. External macroeconomic parameters. Historical curves and realizations of ARIMA-GARCH model during the simulated period.

3. Estimating the probability of company default

In order to assess the default probability of the oil producing and oil refining enterprise, the system dynamic model of the company was run from the second quarter of 2015, taking into account various scenarios of external macroeconomic parameters simulated by the described multivariate ARIMA-GARCH model. The total number of experiments with two outcomes was 10,000. The facts of default were registered during various periods of time: one, two, three, four, five and ten, years. As a result, the probability of default for each time interval was calculated by the formula

$$p = \frac{k}{n} \cdot 100 \,,$$

where k is the number of experiments, in which the company defaulted, and n is the total number of model launches. Then the confidence intervals with trust levels α ()5% and 99%, by the standard method for binomial distribution, was built:

$$p - z_\alpha \times \sqrt{\frac{p \cdot (1-p)}{n}} \le p \le p + z_\alpha \times \sqrt{\frac{p \cdot (1-p)}{n}},$$

where z_α is the $1-\alpha/2$ quantile of a standard normal distribution. The results are presented in table. 3.

Table 2. Estimating the probability of oil enterprise default.

The period of simulation	L_{lower}^{95}	L_{upper}^{95}	L_{lower}^{99}	L_{upper}^{99}	PD (%)	Moody's (1983-2016)	Fitch (1981-2015)	Moody's (2015)
1 year	0.335	1.525	0.147	1.713	0.93	0.47	0.77	0.905
2 years	1.307	3.133	1.018	3.422	2.22	1.54	2.51	-
3 years	2.539	4.881	2.168	5.252	3.71	2.85	4.04	-
4 years	3.667	6.373	3.238	6.802	5.02	4.15	5.58	-
5 years	7.244	10.796	6.683	11.357	9.02	5.47	6.83	-
10 years	9.513	13.467	8.888	14.092	11.49	10.36	9.92	-

L_{lower}^{95} L_{upper}^{95} – lower and upper limits of the confidence interval with the confidence level 95%.

L_{lower}^{99} L_{upper}^{99} – lower and upper limits of the confidence interval with the confidence level 99%.

PD (%) – calculated probability of default.
Moody's (1983-2016) – the average percentage of default companies over different periods (1-5 years and 10 years) and rated Ba1.
Fitch (1981-2015) – the average percentage of default companies over different periods (1-5 years and 10 years) and rated BB+.
Moody's (2015) – the percentage of companies that failed in 2015 with a Ba1 rating.

In 2015 Bashneft was rated Ba1 by Moody's [8] and BB + by Fitch [8]. Table 2 shows the average percentage of bankrupt companies with this rating over various periods (1-5 and 10 years), established on the basis of data for 1983-2016 for Moody's and 1981-2015 for Fitch. The last column contains information on the percentage of companies that had a Ba1 rating from Moody's at the beginning of 2015 and defaulted during this year. Note that for 1983-2016 the average percentage of companies that collapsed during the year is lower than during 2015. At the same time, the data of rating agencies differ from each other by 1-1.5 percent. It is standard **Ошибка! Источник ссылки не найден.** for rating agencies to indicate the correspondence between the rating and the probability of default over different time periods of 1-5 and 10 years. In this case, the forecast of the probability of default for 5 and 10 years is for guidance only, and more attention is paid to the forecast for 1, 2, 3 and 4 years.

On the basis of table 2, it can be concluded that data from rating agencies are close to the estimations of the default probability obtained by using the system dynamics model. In most cases, the modeled default probability lies between the corresponding estimates from ratings agencies, and the found confidence intervals cover the average percentage of bankrupt companies from Moody's. The most accurate results were obtained when assessing the probability of default occurring within one, two, three, and four, years. The discrepancies for forecast periods of 5 and 10 years can be explained by the insufficient accuracy of ARIMA-GARCH models for such long periods. These models are unable to take into account the changes that have occurred in the market during that time. In addition, the information used to build the system dynamics model of Bashneft is not exhaustive, because it was based only on the analysis of open sources. Note that banks have the opportunity to receive any data from their borrowers and make their system dynamics models more accurate.

Comparisons of the estimates obtained by rating agencies and the system dynamics model allow us to conclude that the system dynamics modeling is effective in the context of determining the probability of company default.

4. Conclusion

The main result of this paper is concluded in the consideration alternative method for estimating the probability of default for an enterprise, based on the system dynamics model that describes the structure and behavior of the company under study. External parameters affecting the state of the company under consideration are the prices for oil and oil products traded by the company, the dollar rate against the

ruble, the rate of attracted and repaid loans, the basic mineral tax rate, the unit cost of production, refining and general business expenses. The dynamics of external factors during the period 2015-2025 was simulated using the Monte Carlo method, based on ARIMA-GARCH models.

A comparative analysis of the results and data from Moody's and Fitch demonstrates the closeness of the simulated probability of the default of the enterprise, and the corresponding estimations from rating agencies, which makes it possible to conclude that the described approach is acceptable for estimating the probability of default of a borrower.

References

[1] Gurný P and Gurný M 2013 Comparison of credit scoring models on probability of default estimation for us banks *Prague economic papers* **2** 163-81
[2] Forrester J W 1971 *World Dynamics* (Cambridge, MA, US: Wright-Allen Press)
[3] Kurennoy D S and Golembiovskiy D Yu 2017 System dynamics credit risk model of an oil company *Issues of Risk Analysis* **1** 6-22
[4] Box G, Jenkins G and Reinsel G 2008 *Time Series Analysis: Forecasting and Control* 4th edn (New York, US: Wiley)
[5] Forrester J W 1969 *Urban Dynamics* (Waltham, MA, US: Pegasus Communications)
[6] Dritsaki C 2018 The Performance of Hybrid ARIMA-GARCH Modeling and Forecasting Oil Price *International Journal of Energy Economics and Policy* **8(3)** 14-21
[7] Bollerslev T 1986 Generalized autoregressive conditional heteroskedasticity *Journal of Econometrics* **31** 309-28
[8] Moody's Default Research https://www.moodys.com/Pages/GuideToDefaultResearch.aspx
[9] Fitch regulatory affairs https://www.fitchratings.com/site/regulatory
[10] Schwarz G 1978 Estimating the dimension of a model *Annals of Statistics* **2** 461-4