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AND OSCILLATIONS
OF NONLINEAR CONTROL
SYSTEMS (PYATNITSKIY'S
CONFERENCE) (STAB)**

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**2022 16th International Conference on Stability and Oscillations
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- General problems of stability and stabilization
- Nonlinear oscillations: general problems and methods
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The conference is biannual. Before 2004 it was called International Workshop.

The first conference was held in Tallinn in 1987. In 1992, 1996 – 2012, 2016, 2018, 2020 the conference was held in Moscow, in 1994 in Samara.

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Optimal control of car active suspension control under delays

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Abstract—In this paper algorithm of control synthesis for car active suspension with delays is examined. The problem is formulated in terms of differential games. External perturbation such as road surface unevenness is considered as actions of some opponent. The commonly known model of quarter car active suspension system will be expanded with delays under 0.5s in control. Proposed algorithm depends on "worst" delay value and calculate robust control for the system. Comparison with LQR regulator by mathematical modeling will be made.

Index Terms—Optimal control, robust control, delayed systems, linear quadratic regulator, delay differential equations, active suspension, Riccati equation.

I. INTRODUCTION

This article considers an actual problem in control theory related to the synthesis of control algorithm that can work with models under delays. Such models can describe behaviour of real world systems more accurately comparing to conventional linear systems. However, the problem of synthesizing optimal control for systems with delays still does not have a general solution [Fridman(2014)]. One of naive methods is to consider system without delays and use well known methods for control task optimal solution such as LQR. LQR control is an optimal control method with quadratic performance indexes. It is rather simple and can achieve closed loop optimal control with linear state feedback or output feedback. Unfortunately, there will be delays in control system caused by various factors as long computation times and actuator inertia. Thus classical approach of LQR regulator synthesis doesn't consider delays in the system and can give the researcher non optimal coefficients [Ghiggi(2008)]. Therefore, in this paper, we propose a method for synthesizing control algorithm that takes into account the "worst" delay time arising in the system. Worst delay value in system is selected from the predetermined interval which can be defined by system physical restrictions and common sense. Algorithm of "worst" delay value selection is separate task and will be considered in this work. A system with control which is synthesised for the "worst" scenario case can be classified as a robust system. Such approach can be useful in problem of control synthesis for active suspension of car. Passive suspension systems have been widely applied to manned vehicles from ancient carriages with flexible leaf springs to modern automobiles with pneumatic and hydraulic

systems. The main purpose of implementing the suspension system is to increase comfort the passengers by isolating them from vibrations due to road unevenness. A common passive suspension system consists of conventional springs and dampers with a fixed spring rate and damping parameters. Selection of such parameters depends on balance between requirements of ride comfort and vehicle handling. As passive suspension is not very complex idea as it has no feedback and actuators. However, modern generation of suspensions implements passive elements of classic mechanics and add controlled system with actuators that can apply external force to the body of the car [Guglielmino(2008)]. In this paper we will consider model of one of the vehicle wheels suspension with active actuator installed between wheel base and car body. The main objective of the control system that drives the actuator will be to deliver riding comfort and minimise wheel travel to reduce possibility of loss of grip with road. Additionally, the disturbance caused by road unevenness will be considered as actions of another player (opponent).

II. PROBLEM FORMULATION

A. Quarter car model

The model of suspension considered in this paper is presented on fig. 1. This model is widely used in the active

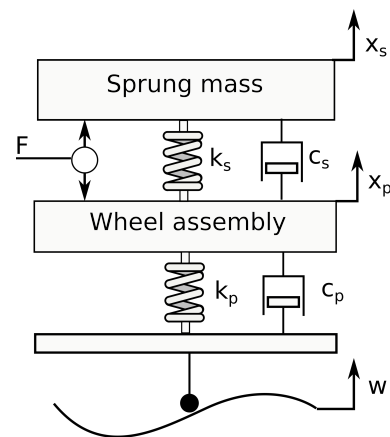


Fig. 1. Graphical representation of quarter car model

vehicle suspension studies [Guglielmino(2008)] and captures

major characteristics of a real suspension system. It consists of car body or sprung mass where passengers are located. This part is connected to wheel assembly with conventional spring and shock-absorber. Tire also provides some dumping behaviour to the system.

Actuator installed between car body and wheel assembly. It can apply force to either increase or reduce distance between this parts. We will consider that force is sufficient for such task.

The mathematical representation is provided by multiple articles:

$$\begin{cases} \ddot{x}_s(t) = \frac{k_s}{m_s}x_p(t) - \frac{k_s}{m_s}x_s(t) + \frac{c_a}{m_s}\dot{x}_p(t) - \\ \quad - \frac{c_a}{m_s}\dot{x}_s(t) + \frac{1}{m_s}F(t), \\ \ddot{x}_p(t) = -\frac{k_s}{m_p}x_p(t) - \frac{k_t}{m_p}x_p(t) + \frac{k_s}{m_p}x_s(t) - \\ \quad - \frac{c_a}{m_p}\dot{x}_p(t) + \frac{c_a}{m_p}\dot{x}_s(t) + \\ \quad + \frac{k_t}{m_p}w(t) - \frac{1}{m_p}F(t). \end{cases} \quad (1)$$

Where k_s is suspension stiffness, c_a - suspension damping rate, k_t -tire stiffness, m_s - mass of car body, m_p - mass of wheel assembly. The coordinates of system are $w(t)$ - road profile height under the tire, $x_p(t)$ - change of position of wheel assembly, $x_s(t)$ - sprung mass position and $F(t)$ - force applied to sprung mass by actuator.

The delay is introduced by argument of control: $F(t) = u(t - \tau)$, where $\tau \in \Upsilon$ is delay period. The dynamic equations (1) can be rewritten:

$$\begin{cases} \dot{x}_s(t) = v_s(t), \\ \dot{v}_s(t) = -\frac{k_s}{m_s}x_s(t) - \frac{c_a}{m_s}v_s(t) + \frac{k_s}{m_s}x_p(t) + \\ \quad + \frac{c_a}{m_s}v_p(t) + \frac{1}{m_s}u(t - \tau), \\ \dot{x}_p(t) = v_p(t), \\ \dot{v}_p(t) = \frac{k_s}{m_p}x_s(t) + \frac{c_a}{m_p}v_s(t) - \frac{c_a}{m_p}v_p(t) - \\ \quad - \left[\frac{k_s}{m_p} + \frac{k_t}{m_p} \right] x_p(t) + \frac{k_t}{m_p}w(t) - \frac{1}{m_p}u(t - \tau). \end{cases} \quad (2)$$

The system (2) can be represented in the form of state space as

$$\dot{x}(t) = A \cdot x(t) + D \cdot w(t) + B \cdot u(t - \tau). \quad (3)$$

The states of the model are defined as

$$x(t) = [x_s(t) \quad v_s(t) \quad x_p(t) \quad v_p(t)]^T$$

and

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_s}{m_s} & -\frac{c_a}{m_s} & \frac{k_s}{m_s} & \frac{c_a}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_p} & \frac{c_a}{m_p} & \left[-\frac{k_s}{m_p} - \frac{k_t}{m_p} \right] & -\frac{c_a}{m_p} \end{bmatrix}, \quad (4)$$

$$D = \begin{bmatrix} 0 & 0 & 0 & \frac{k_t}{m_p} \end{bmatrix}^T, \quad (5)$$

$$B = \begin{bmatrix} 0 & \frac{1}{m_s} & 0 & -\frac{1}{m_p} \end{bmatrix}^T. \quad (6)$$

The resulting system (3) can be considered as a system, where control is implemented with a time delay, this is reflected in the summand $u(t - \tau)$. And (3) also contains an uncontrolled disturbing effect in the form of a changing road profile $w(t)$, which can be considered as an enemy acting against us, but having reasonable restrictions.

Note that it is not always possible to measure all state coordinates directly or sensors can give only noisy data [Akbari(2008)]. An observer should be used in real-world applications, but the synthesis of such filter lays out of the scope of this work and we consider state vector directly observable without any noises.

B. Problem Statement

The construction of control actions will be carried out in two stages. First, the problem of synthesizing control actions without delay will be solved in the case when there is an "enemy" in the system. Next, the resulting control will be rebuilt to compensate for the worst time delay that occurs in real systems.

The algorithm for constructing the controller is based on the ideas of differential zero-sum games [Afanasiev(2014)]. Hence let us introduce the cost function:

$$J(x(\cdot), u(\cdot), w(\cdot)) = \lim_{t_f \rightarrow \infty} \int_{t_0}^{t_f} \left\{ \|x(t)\|_Q + \|u(t - \tau)\|_R - \|w(t)\|_P \right\} dt. \quad (7)$$

The goal of our control $u(t - \tau)$ is to minimise the cost function (7), on the other hand enemy control $w(t)$ is trying to maximize it. We should also compare performance of active and passive suspensions by measuring RMS of sprung mass acceleration (\dot{v}_s). The recommended by ISO 2631 (ISO, 1997) RMS sprung mass acceleration must be below $0.315m/s^2$ to passengers feel highly comfortable [Ahlin(2001)].

III. CONTROL DEVELOPMENT

We will split the process of regulator development by two stages.

Initially we will assume that there is no delay of actuator and synthesize control for both players [Nagarkar(2016)]. As it stated in earlier studies the controls will be

$$u(t) = -K_u \cdot x(t) = -R^{-1}B^T S \cdot x(t), \quad (8)$$

$$w(t) = K_w \cdot x(t) = P^{-1}D^T S \cdot x(t), \quad (9)$$

where S is solution of Riccati equation with matrices (4)-(6):

$$SA + A^T S - S[BR^{-1}B^T - DP^{-1}D^T]S + Q = 0. \quad (10)$$

Then we will use information from previous stage to specify regulator coefficients for a case of presenting delay.

Note that $x(t - \tau) = x(t) - \tau \cdot \dot{x}(t)$ near $x(t) = 0$ then control law can be rewritten as

$$u(t - \tau) = -K_u \cdot x(t - \tau) = -K_u \cdot x(t) + \tau K_u \dot{x}(t). \quad (11)$$

Substituting approximation (11) into state space representation (3):

$$\dot{x}(t) = A \cdot x(t) + DK_w \cdot x(t) - B \cdot K_u \cdot x(t) + \tau BK_u \dot{x}(t) \quad (12)$$

or finally

$$\dot{x}(t) = \left\{ [1 - \tau BK_u]^{-1} [A - BK_u + DK_w] \right\} \cdot x(t). \quad (13)$$

Since the exact value of τ is not specified for the system, moreover, it can be different depending on the situation, so we will build the control based on the worst value of τ from the possible interval. It should be noted that the system (13) can be both stable and unstable. In the case of a stable system, we will call the delay value τ^* the worst case scenario, in which the system comes to a stable state for as long as possible. In the case of an unstable system, τ^* will be considered the worst, at which the system gains maximum speed when deviating from equilibrium. Fig. 2 shows abstract graphs of the behavior of the system in the case a), when the system is stable, and in the case b), when the system is unstable. In both cases, the dotted graphs correspond to the worst delay values τ^* .

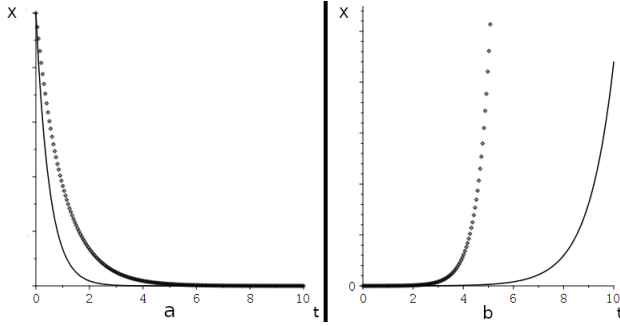


Fig. 2. Trajectories of stable a) and unstable b) systems

To find exact value τ^* we can find roots $\lambda(\tau)$ of characteristic equation for matrix

$$A_s = \left\{ [1 - \tau BK_u]^{-1} [A - BK_u + DK_w] \right\} \quad (14)$$

and maximize its real part: $\max_{\tau \in \Upsilon} \lambda(\tau)$. The other way to find τ^* is to consider norm:

$$M = \|x(t, \tau)\|^2 = \frac{1}{2} x^T(t, \tau) x(t, \tau), \quad (15)$$

one can notice that τ^* meets the greatest derivative of (15): $\frac{d}{d\tau} M(\tau^*) > \frac{d}{d\tau} M(\tau), \forall \tau \neq \tau^*$.

Substituting value $\tau = \tau^*$ in (13), we gain majoring system for the original system

$$\dot{z}(t) = \left\{ [1 - \tau^* BK_u]^{-1} [A - BK_u + DK_w] \right\} \cdot z(t), \quad (16)$$

or in other form

$$\begin{aligned} \dot{z}(t) = & [1 - \tau^* BK_u]^{-1} A \cdot z(t) + \\ & + [1 - \tau^* BK_u]^{-1} B u_z(t) + [1 - \tau^* BK_u]^{-1} D w_z(t), \end{aligned} \quad (17)$$

here $u_z(t) = -K_u \cdot z(t)$ and $w_z(t) = K_w \cdot z(t)$. Let's rewrite (17) in general form

$$\dot{z}(t) = A_z \cdot z(t) + B_z \cdot u_z(t) + D_z \cdot w_z(t), \quad (18)$$

where matrices A_z, B_z, D_z are obtained from (17) and have the following form:

$$\begin{aligned} A_z &= [1 - \tau^* BK_u]^{-1} \cdot A, \\ B_z &= [1 - \tau^* BK_u]^{-1} \cdot B, \\ D_z &= [1 - \tau^* BK_u]^{-1} \cdot D. \end{aligned}$$

This representation of the system in the form (18) makes it possible to calculate $K_z = R^{-1} B_z^T S_z$, where S_z is solution of Riccati equation:

$$S_z A_z + A_z^T S_z - S_z [B_z R^{-1} B_z^T - D_z P^{-1} D_z^T] S_z + Q = 0. \quad (19)$$

Value of K_z should be substituted as regulator coefficients. Let us apply suggested algorithm to the system (1).

A. Nondelayed case

On this stage we will assume that $\tau = 0$. This case leads us to simple LQR problem, where matrices (4), (5) and (6) are described in previous section. Solving the Riccati equation (10) with parameters provided in table 1. we receive

$$K_u = [3.677 \cdot 10^3, 2.398 \cdot 10^3, -4.859 \cdot 10^3, 1.094 \cdot 10^{-4}]$$

B. Delayed case

As we obtained coefficient K_u for non delayed case we can now calculate regulator coefficient for delayed case.

For τ^* selection we will use norm (15).

As shown in fig. 3 the largest delay value is not always worst scenario for the control task. For this particular object $\tau^* = 0.0152$.

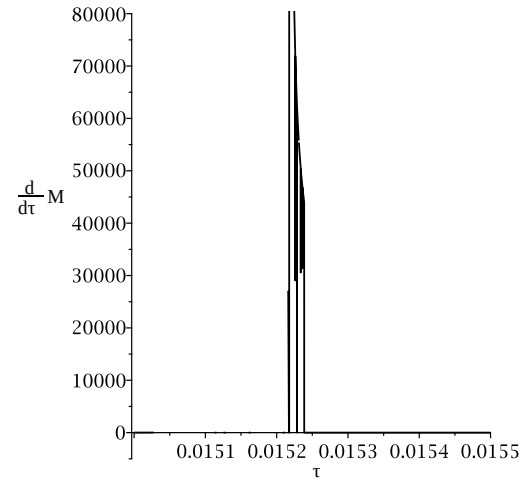


Fig. 3. Plot of $\frac{d}{d\tau} M$ function used for τ^* determination

After obtaining K_u and τ^* we can now solve control task for (18)-(19) and use K_z as regulator gain

$$K_z = [3.37853 \cdot 10^4, 1.00977 \cdot 10^4, -3.70699 \cdot 10^4, 0.00565]$$

Model parameters	symbol	values	unit
suspension stiffness	k_s	16812	N/m
suspension damping rate	c_a	1000	$N/(m/s)$
tire stiffness	k_t	190000	N/m
mass of car body	m_s	250	kg
mass of wheel assembly	m_p	50	kg
delay	τ	0.01	s

TABLE I
SIMULATION PARAMETERS

IV. SIMULATION RESULTS

The simulation was made in MATLAB Simulink with parameters provided in table 1.

Step with size of 0.025 m was chosen to simulate the road $w(t)$.

We will compare performance of passive (dash-dot line) and active suspensions (solid and dashed lines) on the given road section. Dashed line corresponds to behaviour of suspension under lqr regulator synthesized without delay value consideration. Solid line corresponds to control synthesised for found τ^* .

The deflection of suspension space (x_s) is shown in fig. 4. It is good to have less deflection in suspension for better handling of the car as it have better grip to the road. Maximum deflection of system under lqr synthesized for τ^* is lower that other cases.

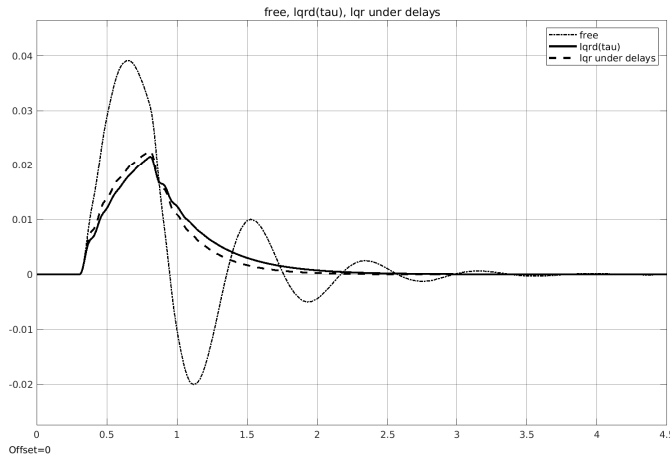


Fig. 4. Sprung mass position ($x_s(t)$) under different controls

The provided in fig. 5 plots show the RMS of car body acceleration that also affects on passengers.

As one can notice, passengers feel more comfortable when lqr is synthesized for worst delay value as RMS of sprung mass acceleration is lower.

V. CONCLUSIONS

This article demonstrates control algorithm development for the system with delayed control which shows capabilities to stabilise active suspension under influence of delays. The control algorithm is based on widely known linear quadratic regulator and is synthesized for the system with the "worst" delay value from possible delays interval. This method was

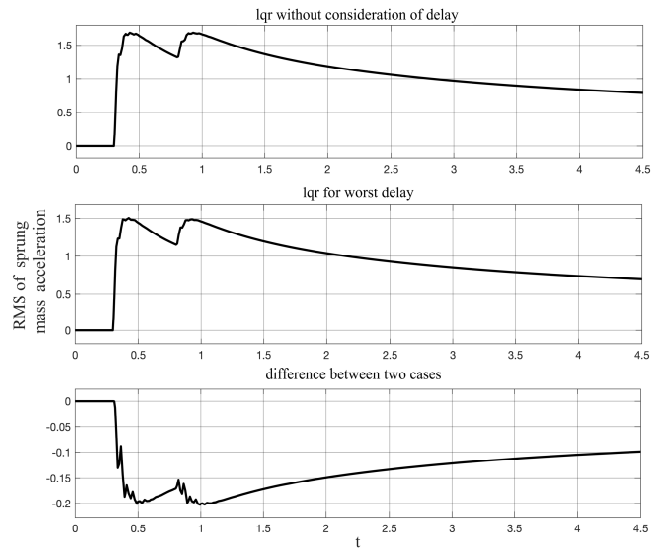


Fig. 5. RMS of car body acceleration

applied to the active suspension system model extended by adding delay to the control loop. The active suspension with provided algorithm improves vehicle handling and comfort of passengers. Offered approach was tested by modelling system in MATLAB Simulink. Provided algorithm shows better results than LQR synthesised for system without taking delay value in consideration.

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