On the Capacity Estimation of a Slotted Multiuser Communication Channel

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***Abstract*—We consider a vector-disjunctive channel where users transmit some vector of bits of length L. We estimate the capacity of this channel and derive the lower bound on this value. We present some numerical results for the bound we obtained in the case of different number of active users and different parameters of channel.**

I. Introduction

Due to the recent development in Internet of Things technology the need for wireless networks with high mutual interference is rising. At the same high data rate and low packet loss are still required in these scenarios.

These systems are usually described uncoordinated multiple access channels. While the number of users might be huge the number of active users is usually not that large. The length of the data transmitted by each user is relatively small. This fact and the uncoordinated nature of the multiple access channel is what differs the channel model considered for the ones described in most papers on multiple access. Due to these differences we have to derive new bounds on capacity of such channels.

Almost all well-known low-complexity coding solutions for the traditional MAC channel such as code-division multiple access (CDMA), rate-splitting [1], and interleave-division multiple access [2], implicitly assume some form of coordination between the users and that some parameters of the coding scheme such as the spreading sequence, code rates, time sharing parameters, Tanner graph of the code, etc., are user dependent.

In this paper we focus on a special class of uncoordinated MAC, namely vector-disjunctive channel (logical OR or Z-channel) that was introduced by Cohen, Heller and Viterbi in [3], also see [4], [5]. Papers [6]–[9] are devoted to the capacity of this channel under some additional assumptions.

More precisely, we consider on a special class of vector-disjunctive channel where each active user transmit some binary vector of length $L \geq 1$. We estimate the capacity of this channel and derive the lower bound on this value.

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II. Channel Model

Let us first describe precisely the channel model we use.

A. Channel Description

A disjunctive channel has Q distinct sub-channels. Each user can transmit a unit symbol over one (and only one) of these sub-channels. Only U users are active at the same time. In a vector-disjunctive channel each user can transmit any vector of length L in one of S slots (also known as L-channels). The slot is selected randomly, but the vector is selected from a codebook. The net number of sub-channels is therefore $Q = LS$. The channel computes a disjunction of all vectors transmitted in the same slot and the receiver measures these disjunctions.

If all users transmit in different slots all transmitted vectors are received without any distortions. But we will often consider $U > S$ so that some collisions will always happen and some vectors will likely be distorted.

Let us consider an example of such collision: users i and k have transmitted vectors $(0, 1, 0)$ and $(0, 1, 1)$ respectively in the slot j. The receiver would get $(0, 1, 0) \vee (0, 1, 1) = (0, 1, 1)$ in j-th slot. The vector transmitted by user k is received without any distortions but the vector transmitted by user i has the last bit flipped.

B. Collision Model

Let us define a collision as the event when multiple users transmit in the same slot. Let us define collision order (denoted by κ) as the number of users transmitting in a single slot minus one. Therefore no collision corresponds to a collision of zero order. The probability of no collision is:

$$P_s = 1 - \left(1 - \frac{1}{S}\right)^{U-1}.$$

It is easy to calculate the probability that in a selected slot where a selected user transmits the collision order $\kappa = t$:

$$P(\kappa = t) = \binom{U-1}{t} \left(\frac{1}{S}\right)^t \left(1 - \frac{1}{S}\right)^{U-t-1}.$$

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In this paper we consider noiseless channel and uncoordinated transmission scenario. Thus for a given user we can assume transmissions of all other $U-1$ users as noise (cause of collisions).

In some cases the transmission of other users does not result in incorrect transmission for a given user. For instance if the transmitted information in slot for considering user is $(1, 1, \dots, 1)$ then it will be received correctly regardless of what other users transmit in the same slot.

Let us in consider two users i and k that use common L-channel for transmission in j-th time moment.

Let us assume that users i and k transmit L binary symbols $\mathbf{X}_{ij} = (x_{ij}^{(1)}, x_{ij}^{(2)}, \dots, x_{ij}^{(L)})$ and $\mathbf{X}_{kj} = (x_{kj}^{(1)}, x_{kj}^{(2)}, \dots, x_{kj}^{(L)})$ respectively in L-channel. Since we suppose that our channel is vector-disjunctive then in common L-channel we obtain a disjunction of transmitted binary vectors: $\mathbf{X}_{ij} \vee \mathbf{X}_{kj}$. Now let us focuse on i-th user. Since we suppose that vector-disjunctive channel is noiseless then we can consider transmission k-th user as a noise for i-th user. Taking this assumption into account one can conclude that collision for i-th user takes place if and only if $\mathbf{X}_{ij}$ does not cover $\mathbf{X}_{kj}$.

Let us describe this event more precisely. Let us denote by $supp(\mathbf{X})$ a support of vector $\mathbf{X}$, i. e.

$$supp(\mathbf{X}) = \{i : x_i \neq 0\}.$$

Thus if i-th and k-th users use the same L-channel then collision for user i does not take place if

$$supp(\mathbf{X}_{kj}) \subset supp(\mathbf{X}_{ij}).$$

III. Channel Analysis

Let us first consider the simpliest case of transmission through the channel we described above. For this purpose let us fix $L = 1$ and collision order $t = 1$. We denote this channel is $\mathcal{C}_1^1$. If p is a probability to transmit 1 then $\mathcal{C}_1^1$ is just simple Z-channel:

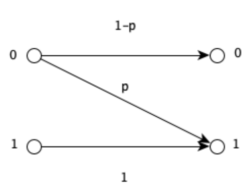

Fig. 1. Z-channel for $L = 1$ and $t = 1$.

Transition probability matrix of $\mathcal{C}_1^1$ has the following form:

$$\mathbf{P}_1^1 = \begin{pmatrix} 1-p & p \\ 0 & 1 \end{pmatrix}.$$

Since $\mathcal{C}_1^1 : \{0,1\} \mapsto \{0,1\}$ and input probabilities $p_{in}(x)$ as follows: $p_{in}(0) = 1-p$, $p_{in}(1) = p$ then it is easy to calculate output probabilities $p_{out}(y)$: $p_{out}(0) = (1-p)^2$, $p_{out}(1) = p(2-p)$.

Now we can estimate the capacity C of this channel:

$$C(\mathcal{C}_1^1) \geq \sum_{x \in \{0,1\}} p_{in}(x) \sum_{y \in \{0,1\}} p(y|x) \log_2 \frac{p(y|x)}{p_{out}(y)} =$$
$$= (1-p)^2 \log_2 \frac{1}{1-p} + p(2-p) \log_2 \frac{1}{2-p} + p \log_2 \frac{1}{p}.$$

Also we will assume that if $t = 0$ then $C(\mathcal{C}_1^0) = 1$.

Now let us consider the case when $L = 2$ and $t = 1$. We denote the channel of this case as $\mathcal{C}_2^1$. The graphical representation of $\mathcal{C}_2^1$ is as follows:

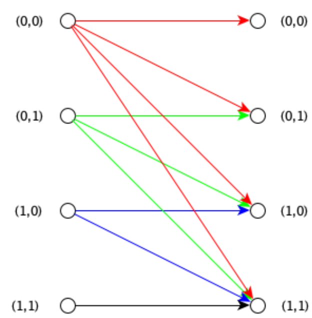

Fig. 2. Channel for $L = 2$ and $t = 1$.

If $p_{in}(0) = 1-p$, $p_{in}(1) = p$ then it is easy to calculate input probabilites of $(0,0)$, ..., $(1,1)$:

$$p_{in}((0,0)) = (1-p)^2$$
$$p_{in}((0,1)) = p_{in}((1,0)) = p(1-p)$$
$$p_{in}((1,1)) = p^2.$$

Transition probability matrix of $\mathcal{C}_2^1$ has the following form:

$$\mathbf{P}_2^1 = \begin{pmatrix} (1-p)^2 & p(1-p) & p(1-p) & p^2 \\ 0 & 1-p & 0 & p \\ 0 & 0 & 1-p & p \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

And it is not difficult to notice that

$$\mathbf{P}_2^1 = \begin{pmatrix} 1-p & p \\ 0 & 1 \end{pmatrix}^{\otimes 2} = \left(\mathbf{P}_1^1\right)^{\otimes 2}, \quad (1)$$

where sign $\mathbf{A}^{\otimes k}$ means k-th Kronecker (tensor) product of matrix $\mathbf{A}$.

Before we use this observation let us calculate the capacity of $\mathcal{C}_2^1$. In order to do this one need to calculate output probabilities of $(0,0)$, $(0,1)$, $(1,0)$, $(1,1)$:

$$p_{out}((0,0)) = (1-p)^4$$
$$p_{out}((0,1)) = p_{out}((1,0)) = p(2-p)(1-p)^2$$
$$p_{out}((1,1)) = p^2(2-p)^2.$$

Thus, applying the same expression as for $C\left(\mathcal{C}_1^1\right)$ for input/output alphabets $\mathcal{X}=\mathcal{Y}=\{(0,0),\,(0,1),\,(1,0),\,(1,1)\}$ we have:

$$C\left(\mathcal{C}_1^2\right) \geq (1-p)^4\log_2\frac{1}{(1-p)^2} + p(1-p)^3\log_2\frac{1}{(1-p)(2-p)} + $$
$$+p(1-p)^2\log_2\frac{1}{p(1-p)(2-p)} + p(1-p)^3\log_2\frac{1}{(1-p)(2-p)} + $$
$$+p^2(1-p)^2\log_2\frac{1}{(2-p)^2} + p^2(1-p)\log_2\frac{1}{p(2-p)^2} + $$
$$+p(1-p)^2\log_2\frac{1}{p(1-p)(2-p)} + p^2(1-p)\log_2\frac{1}{p(2-p)^2} + $$
$$+p^2\log_2\frac{1}{p^2(2-p)^2}.$$

After long but rather straightforward simplifications of the latest expression we can find that

$$C\left(\mathcal{C}_1^2\right) = 2C\left(\mathcal{C}_1^1\right). \tag{2}$$

Previously we showed that $\mathbf{P}_2^1 = \left(\mathbf{P}_1^1\right)^{\otimes 2}$. And now we show that equation (2) goes directly from (1) not only for channel we described above but for any discrete memoryless one.

But first of all let us prove generalization of equation (1). For this case let us consider communication channel $\mathcal{C}_L^1$ for $t=1$ and arbitrary $L \geq 1$. Thus this channel has 2^L inputs and 2^L outputs: $\mathcal{X}=\mathcal{Y}=\{(0,\ldots,0),\ldots,(1,\ldots,1)\}$. In order to simplify further expressions we will assume that $\mathcal{X}=\mathcal{Y}=\{\mathbf{0},\mathbf{1},\ldots,\mathbf{2^L-1}\}$, where under $\mathbf{i}$ we assume binary, length L representation of integer i, $0 \leq i \leq 2^L-1$.

If p is a probability of transmitting 1 for each coordinate in length L binary vector, then

$$p_{in}(\mathbf{i}) = p^{wt(\mathbf{i})}(1-p)^{L-wt(\mathbf{i})},$$

where $wt(\mathbf{x})$ is a Hamming weight of vector $\mathbf{x}$.

The transition probability matrix of $\mathcal{C}_L^1$ has size $2^L \times 2^L$. We will denote it by

$$\mathbf{P}_L^1 = \left[p_{ij}\right]_{i,j=0,0}^{2^L-1,2^L-1},$$

where $p_{ij} = p(\mathbf{j}|\mathbf{i})$ – are transition probabilities to obtain vector $\mathbf{j}$ for input vector $\mathbf{i}$.

It is obvious that $p_{ij}=0$ if $supp(\mathbf{i}) \not\subset supp(\mathbf{j})$. Let us calculate p_{ij} for the case when $supp(\mathbf{i}) \subset supp(\mathbf{j})$. Without loss of generality one can assume that $supp(\mathbf{i})=[k_1]$ and $supp(\mathbf{j})=[k_2]$, where $0 \leq k_1 \leq k_2 \leq L$, then

$$p_{ij} = \sum_{s=0}^{k_1}\binom{k_1}{s}p^{s+k_2-k_1}(1-p)^{L-s-k_2+k_1}.$$

Then we can write these transition probabilities in more general way:

$$p_{ij} = \sum_{s=0}^{wt(\mathbf{i})}\binom{wt(\mathbf{i})}{s}p^{s+wt(\mathbf{j})-wt(\mathbf{i})}(1-p)^{L-s-wt(\mathbf{j})+wt(\mathbf{i})} \times$$
$$\times I_{supp(\mathbf{i})\subset supp(\mathbf{j})},$$

where I - indicator function, which is 1 if corresponding condition is fulfilled and 0 otherwise.

Now let us prove the following theorem:

Theorem 1. $\mathbf{P}_L^1 = \left(\mathbf{P}_1^1\right)^{\otimes L}$

Proof. We will prove this theorem by induction on L.

- Base of induction: $L=2$. For this case it was shown that $\mathbf{P}_2^1 = \left(\mathbf{P}_1^1\right)^{\otimes 2}$.
- Induction hypothesis: $\mathbf{P}_{L-1}^1 = \left(\mathbf{P}_1^1\right)^{\otimes (L-1)}$.
- Induction step: $L-1 \mapsto L$. In order to prove the theorem we must show that $\mathbf{P}_L^1 = \mathbf{P}_{L-1}^1 \otimes \mathbf{P}_1^1$. Let us consider arbitrary i,j: $0 \leq i,j \leq 2^{L-1}-1$ and corresponding vectors $\mathbf{i}$ and $\mathbf{j}$.

  There are two possible cases:

  1) $supp(\mathbf{i}) \not\subset supp(\mathbf{j})$,
  2) $supp(\mathbf{i}) \subset supp(\mathbf{j})$.

  It is evident that $p_{ij}=0$ for the first case.

  When we substitute $L-1$ by L a number of channel inputs and outputs increase twice. It means that each vector $\mathbf{i}$ splits into $(\mathbf{i},\,0)$ and $(\mathbf{i},\,1)$. If $supp(\mathbf{i}) \not\subset supp(\mathbf{j})$ then $supp((\mathbf{i},0)) \not\subset supp((\mathbf{j},0))$, $supp((\mathbf{i},1)) \not\subset supp((\mathbf{j},0))$, $supp((\mathbf{i},0)) \not\subset supp((\mathbf{j},1))$ and $supp((\mathbf{i},1)) \not\subset supp((\mathbf{j},1))$. It means that:

$$p((\mathbf{j},0)|(\mathbf{i},0)) = p((\mathbf{j},1)|(\mathbf{i},0)) = p((\mathbf{j},0)|(\mathbf{i},1)) =$$
$$= p((\mathbf{j},1)|(\mathbf{i},1)) = 0.$$

Which indicates that 0 is subsitited by 2×2 all-zeros matrix when $L-1 \mapsto L$ and we proved the theorem for the first case.

Now let us consider the second case when $supp(\mathbf{i}) \subset supp(\mathbf{j})$ and $p_{ij}>0$. Let us substitute $L-1$ by L. If $\mathbf{i}$ and $\mathbf{j}$ have length $L-1$ then $\mathbf{i} \mapsto \mathbf{i}'$, $\mathbf{j} \mapsto \mathbf{j}'$ where $\mathbf{i}'=(\mathbf{i},0)$ or $\mathbf{i}'=(\mathbf{i},1)$ or $\mathbf{j}'=(\mathbf{j},0)$ or $\mathbf{j}'=(\mathbf{j},1)$. There are four possible cases for $p(\mathbf{j}'|\mathbf{i}')$.

1) $\mathbf{i}'=(\mathbf{i},0)$ and $\mathbf{j}'=(\mathbf{j},0)$ in this case we can obtain $\mathbf{j}'$ from $\mathbf{i}'$ only when $\mathbf{i}'(L)=0$. The probability of this event is $1-p$ and thus $p(\mathbf{j}'|\mathbf{i}')=p_{ij}(1-p)$.
2) For the case when $\mathbf{i}'=(\mathbf{i},0)$ and $\mathbf{j}'=(\mathbf{j},1)$, $p(\mathbf{j}'|\mathbf{i}')=p_{ij}\cdot p$
3) For the case when $\mathbf{i}'=(\mathbf{i},1)$ and $\mathbf{j}'=(\mathbf{j},1)$, $p(\mathbf{j}'|\mathbf{i}')=p_{ij}$
4) For the case when $\mathbf{i}'=(\mathbf{i},1)$ and $\mathbf{j}'=(\mathbf{j},0)$, $supp(\mathbf{i}') \not\subset supp(\mathbf{j}')$ and $p(\mathbf{j}'|\mathbf{i}')=0$

These four cases indicates that $p_{ij}>0$ is subsitited by 2×2 matrix with the following form:

$$\begin{pmatrix}(1-p)p_{ij} & p\cdot p_{ij}\\ 0 & p_{ij}\end{pmatrix} = p_{ij}\mathbf{P}_1^1.$$

Thus finally $\mathbf{P}_L^1 = \mathbf{P}_{L-1}^1 \otimes \mathbf{P}_1^1$ and the theorem is proved. ■

Remark 1. *Let $\mathbf{P}_A$ and $\mathbf{P}_B$ are transition probabilites matrices for arbitraty discrete memoryless channels $\mathcal{C}_A$ and $\mathcal{C}_B$ with input alphabets $\mathcal{X}_A$, $\mathcal{X}_B$ and output alphabets $\mathcal{Y}_A$, $\mathcal{Y}_B$ such that $|\mathcal{X}_A|=n_A$, $|\mathcal{X}_B|=n_B$, $|\mathcal{Y}_A|=m_A$, $|\mathcal{Y}_B|=m_B$, $\mathcal{X}_A=\{x_0^A,x_1^A,\ldots,x_{n_A-1}^A\}$, $\mathcal{X}_B=\{x_0^B,x_1^B,\ldots,x_{n_B-1}^B\}$, $\mathcal{Y}_A=\{y_0^A,y_1^A,\ldots,y_{m_A-1}^A\}$, $\mathcal{Y}_B=\{y_0^B,y_1^B,\ldots,y_{m_B-1}^B\}$. If we consider matrix $\mathbf{P}_C=\mathbf{P}_A\otimes\mathbf{P}_B$ then*

$$\mathbf{P}_C(i_1+i_2n_A, j_1+j_2m_A) = \mathbf{P}_A(i_1,j_1)\mathbf{P}_B(i_2,j_2),$$

where $0 \le i_1 < n_A$, $0 \le i_2 < n_B$, $0 \le j_1 < m_A$, $0 \le j_2 < m_B$ *. In the other hand*

$$\mathbf{P}_C(i_1 + i_2 n_A, j_1 + j_2 m_A) = p(y^A_{i_1}, y^B_{i_2} | x^A_{j_1}, x^B_{j_2}),$$
$$\mathbf{P}_A(i_1, j_1) = p(y^A_{i_1} | x^A_{j_1}),$$
$$\mathbf{P}_B(i_2, j_2) = p(y^B_{i_2} | x^B_{j_2}).$$

It means that

$$p(y^A_{i_1}, y^B_{i_2} | x^A_{j_1}, x^B_{j_2}) = p(y^A_{i_1} | x^A_{j_1}) p(y^B_{i_2} | x^B_{j_2})$$

and transmission over channel $\mathcal{C}_C$ *with transition probabilities matrix* $\mathbf{P}_C$ *is equvalent of transmission over parallel and independent channels* $\mathcal{C}_A$ *and* $\mathcal{C}_B$*.*

From Theorem 1 and Remark 1 the following Theorem is followed:

Theorem 2. *Let* $\mathbf{P}$ *is a transition probability matrix for some discrete memoryless channel* $\mathcal{C}$ *with input alphabet* $\mathcal{X}$ *and output alphabet* $\mathcal{Y}$ *and capacity* C_1*. Consider* $l \in \mathbb{N}$ *and channel* $\mathcal{C}_l : \mathcal{X}^l \mapsto \mathcal{Y}^l$ *with transition probability matrix* $P^{\otimes l}$*. Capacity* C_l *of* $\mathcal{C}_l$ *: can be calculated as* $C_l = lC_1$*.*

Proof. See Chapter 7: Channel Capacity" in [10]. ∎

From this theorem we can conclude that

$$C(\mathcal{C}^L_1) = L \cdot C(\mathcal{C}^1_1). \tag{3}$$

Now let us consider a case when $L = 1$ and collision order $t \in \mathbb{N}$ is an arbutrary number. We will denote the corresponding channel as $\mathcal{C}^t_1$. The input/output alphabets for this channel are the same as for $\mathcal{C}^1_1$. If p is a probability to each of t users send 1, then transition probability to obtain 0 for input 0 is $p_{00} = p(0|0,t) = (1-p)^t$. In the same manner we can calculate $p_{01} = p(1|0,t) = 1-(1-p)^t$. Since disjunction of t random binary variables equals 1 if at least one of these variables equals 1 then, $p_{10} = p(0|1,t) = 0$ and $p_{11} = p(1|1,t) = 1$. Thus transition probability matrix $\mathbf{P}^t_1$ of $\mathcal{C}^t_1$ is:

$$\mathbf{P}^t_1 = \begin{pmatrix} (1-p)^t & 1-(1-p)^t \\ 0 & 1 \end{pmatrix} = \left(\mathbf{P}^1_1\right)^t. \tag{4}$$

It means that channel $\mathcal{C}^t_1$ is just serial connection of t $\mathcal{C}^1_1$ channels: Thus we can calculate a capacity of channel $\mathcal{C}^t_1$

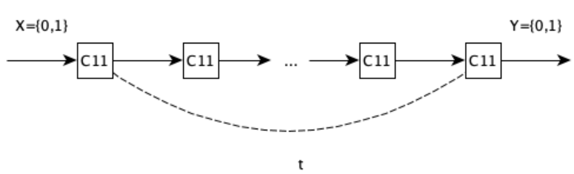

Fig. 3. Graphical representation of $\mathcal{C}^t_1$

$(t > 0)$ as follows:

$$C(\mathcal{C}^t_1) = (1-p)^{t+1} \log_2 \frac{1}{1-p} +$$
$$+(1-p)\left(1-(1-p)^t\right) \log_2 \left(\frac{1-(1-p)^t}{1-(1-p)^{t+1}}\right) +$$
$$+p \log_2 \left(\frac{1}{1-(1-p)^{t+1}}\right).$$

As it was mentioned above, if $t = 0$ then $\mathcal{C}^0_1 = 1$.
And finally we can calculate capacity of $\mathcal{C}^t_L$ as follows:

$$C(\mathcal{C}^t_L) = \begin{cases} L \cdot C(\mathcal{C}^t_1), & t > 0 \\ L, & t = 0 \end{cases}.$$

In Figure 4 the dependence between $C(\mathcal{C}^t_L)$ and p for $L = 8$ and $t = 1, 2, 3, 4, 5, 10$ is presented.

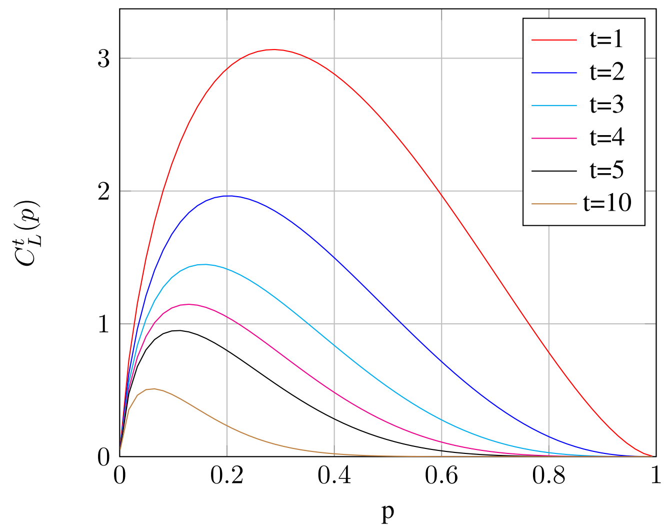

Fig. 4. Dependence between $C(\mathcal{C}^t_L)$ and p for $L = 8$ and $t = 1, 2, 3, 4, 5, 10$

It is obvious that the larger t the smaller capacity $C(\mathcal{C}^t_L)$. Moreover, for each t there is $p = p_{opt}$ which is the optimal probability that maximizes $C(\mathcal{C}^t_L)$, and the larger t the smaller p_{opt}.

Now let us estimate a capacity $C = C(U, Q, L, p)$ of slotted vector-disjunctive channel for a given Q, L, probability p and fixed number of users U.

By the definition of capacity:

$$C = \sup_{p_{\text{in}}(\mathcal{X})} I(\mathcal{X}; \mathcal{Y}),$$

where $I(\mathcal{X}; \mathcal{Y})$ is a mutual information between random variables $\mathcal{X}$ and $\mathcal{Y}$. Supremum takes over all possible distributions $p_{\text{in}} \in \mathcal{P}(\mathcal{X}^L)$ of input $\mathcal{X}^L$. Thus

$$C = \sup_{p_{\text{in}} \in \mathcal{P}(\mathcal{X}^L)} \sum_{x \in \mathcal{X}^L} p_{\text{in}}(x) \sum_{y \in \mathcal{Y}^L} p(y|x) \log_2 \frac{p(y|x)}{p_{\text{out}}(y)} \tag{5}$$

Let us define all the probabilities in (5).

$$p_{\text{in}}(\mathbf{x}) = p^{\text{wt}(\mathbf{x})}(1-p)^{L-\text{wt}(\mathbf{x})},$$

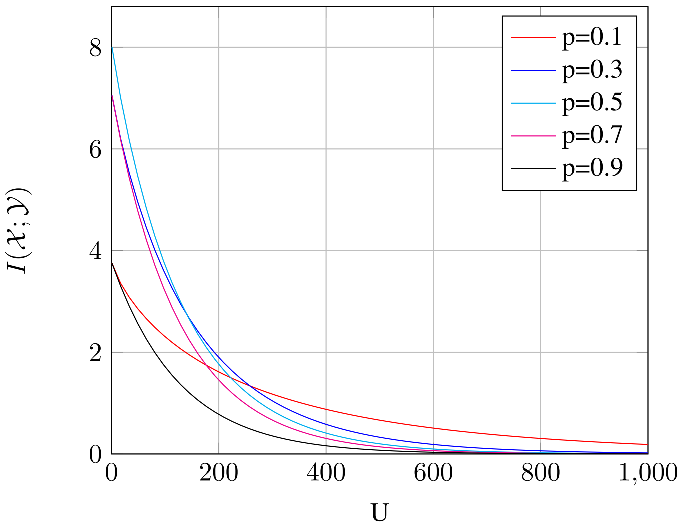

Fig. 5. Mutual information (6) as a function of number of users for $Q = 1024$, L=8.

$$p(\mathbf{y}|\mathbf{x},U) = \begin{cases} \sum_{t=0}^{U-1} \Pr\{\kappa = t\} p^{t(\mathrm{wt}(\mathbf{y})-\mathrm{wt}(\mathbf{x}))} (1-p^t)^{L-\mathrm{wt}(\mathbf{y})}, \\ \qquad\qquad\qquad \mathrm{supp}(\mathbf{x}) \subseteq \mathrm{supp}(\mathbf{y}) \\ 0, \quad \mathrm{supp}(\mathbf{x}) \nsubseteq \mathrm{supp}(\mathbf{y}) \end{cases}$$

To define p_{out} we need to introduce the probabilities as functions of weight. As $p(\mathbf{y}|\mathbf{x},U)$ only depends on the weight of $\mathbf{x}$ and $\mathbf{y}$ and not on their values let us define

$$p(w'|w,U) = p(\mathbf{y}|\mathbf{x},U), \quad \begin{array}{r} \forall \mathbf{x} \in \mathcal{X}^L : \mathrm{wt}(\mathbf{x}) = w \\ \forall \mathbf{y} \in \mathcal{Y}^L : \mathrm{wt}(\mathbf{y}) = w' \\ \mathrm{supp}(\mathbf{x}) \subseteq \mathrm{supp}(\mathbf{y}) \end{array}$$

$$p_{\text{in}}(w) = p_{\text{in}}(\mathbf{x}), \quad \forall \mathbf{x} \in \mathcal{X}^L : \mathrm{wt}(\mathbf{x}) = w$$

$$\begin{aligned} p_{\text{out}}(\mathbf{y}) &= \sum_{\mathbf{x} \in \mathcal{X}^L} p_{\text{in}}(\mathbf{x}) p(\mathbf{y}|\mathbf{x},U) \\ &= \sum_{w=0}^{\mathrm{wt}(y)} \binom{\mathrm{wt}(\mathbf{y})}{w} p_{\text{in}}(w) p(\mathrm{wt}(\mathbf{y})|w,U) \end{aligned}$$

Likewise let us define

$$p_{\text{out}}(w') = p_{\text{out}}(\mathbf{y}), \quad \forall \mathbf{y} \in \mathcal{Y}^L : \mathrm{wt}(\mathbf{y}) = w'$$

Combining these expressions one can estimate C as follows:

$$C \geq \sum_{w=0}^{L} p_{\text{in}}(w) \sum_{w'=w}^{L} \binom{L}{w, w'-w} p(w'|w,U) \log_2 \frac{p(w'|w,U)}{p_{\text{out}}(w')}. \tag{6}$$

In Fig. 5 the dependence between expression (6) and number of active users U for fixed $Q = 1024$, $L = 8$ and different values of p is presented. It is easy to notice that $p = 0.5$ is not optimal for all cases: it maximizes (6) only when the number of active users is relatively small ($U < S$). In other cases smaller values of p result in better capacity.

IV. Conclusion

In this paper we consider a very simple model of multiple access system where users send short bit packets to base station which recovers a disjunction of all transmitted data. We estimate the capacity of this system in terms of maximal achievable throughput per one channel use and obtain lower bound on this value. The numerical results, presented in the paper allow us to make a conclusion that capacity still remains non-zero when the number of active users becomes greater than S — number of orthogonal slots. It means that proposed transmission scenario potentially allows a large number of users transmit simultaneously in uncoordinated fashion even when this number becomes greater than the maximal number of users that can work in orthogonal manner.

Moreover, our proposed scheme has such advantages as simple receiver, uncoordinated transmission, number of active users are not strictly fixed and can be changed dynamically.

References

[1] B. Rimoldi and R. Urbanke, "A rate-splitting approach to the Gaussian multiple-access channel," IEEE Transactions on Information Theory, vol. 42, no. 2, pp. 364–375, 1996.

[2] L. Ping, L. Liu, K. Wu, and W. K. Leung, "Interleave division multiple-access," IEEE Transactions on Wireless Communications, vol. 5, no. 4, pp. 938–947, 2006.

[3] A. R. Cohen, J. A. Heller and A. J. Viterbi, "A new coding technique for asynchronous multiple access communication," IEEE Transactions on Communication Technology, vol. 19, pp. 849–855, October 1971.

[4] D. S. Osipov, A. A. Frolov and V. V. Zyablov, "Multiple access system for a vector disjunctive channel," Problems of information Transmission, vol. 48, no. 3, pp. 243–249, 2012.

[5] A. A. Frolov, V. V. Zyablov, V. R. Sidorenko and R. Fischer, "On a multiple-access in a vector disjunctive channel," Information Theory (ISIT), Proceedings of IEEE International Symposium on Information Theory (ISIT), pp. 211–215, 2013.

[6] L. Wilhelmsson and K. S. Zigangirov, "On the Asymptotic Capacity of a Multiple-Access Channel," Problems of Information Transmission, vol. 33, no. 1, pp. 9–16, 1997.

[7] S. C. Chang and J. Wolf, "On the t-User m-Frequency Noiseless Multiple-Access Channel with and without Intensity Information," IEEE Transactions on Information Theory, vol. 27, no. 1, pp. 41–48, Jan. 1981.

[8] L. A. Bassalygo and M. S. Pinsker, "Evaluation of the Asymptotics of the Summarized Capacity of an m-Frequency t-User Noiseless Multiple-Access Channel," Problems of Information Transmission, vol. 36, no. 2, pp. 91–97, 2000.

[9] A. Han Vinck and K. Keuning, "On the Capacity of the Asynchronous t-User m-Frequency Noiseless Multiple-Access Channel without Intensity Information," IEEE Transactions on Information Theory, vol. 42, no. 6, pp. 2235–2238, Nov. 1996.

[10] Thomas M. Cover and Joy A. Thomas, "Elements of Information Theory" (Second ed.). Wiley-Interscience. pp. 206207, 2006.