Lectures on relativistic gravity and cosmology.

Lectures 11-12

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Light propagation in the FLRW universe

Present matter content of the Universe

Dark matter

Dark energy

Three fundamental cosmological constants

The standard LambdaCDM cosmological model

Light propagation and redshift

$$ds = 0 \to r = \pm\eta\,.$$

$$g^{ik}\frac{\partial\psi}{\partial x^i}\frac{\partial\psi}{\partial x^k} = 0 \to \psi = k(x-\eta),\ \omega = \frac{k}{a(t)},\ k = const\,.$$

For massive particles: $\mathbf{p}a(t) = const,\ \gamma v a(t) = const$ where p is the physical relativistic momentum.

The redshift $z(t)$ for light emitted in the past and observed at present $(t = t_0, a(t_0) = a_0)$ is defined as $z = \frac{\omega(t)-\omega(t_0)}{\omega(t)}$. Thus,

$$z = \frac{a_0}{a(t)} - 1,\ \eta_0 - \eta = a_0^{-1}\int_0^z \frac{dz}{H(z)}\,,\ \eta_0 = \eta(t_0)\,.$$

The luminosity distance

The luminosity distance D_L is defined through $I = L/D_L^2$. On the other hand, taking time dilation $d\eta/dt$ and light redshift into account (each of this produces the $\frac{a(\eta_0-\eta)}{a(\eta_0)}$ multiplier), we get

$$I = L\,\frac{a^2(\eta_0-\eta)}{a^4(\eta_0)r^2}, \;\; r = \eta_0 - \eta\,.$$

$$D_L = a_0(\eta_0-\eta)(1+z) = (1+z)\int_0^z \frac{dz}{H(z)}\,.$$

For $z \ll 1$ and restoring c, $D_L(z) = cz/H_0$.
Inversion of the formula gives the model-independent determination of $H(z)$ from $D_L(z)$:

$$H(z) = c\left[\frac{d}{dz}\left(\frac{D_L(z)}{1+z}\right)\right]^{-1}\,.$$

However, the practical application of this formula requires smoothing over some interval of redshift.

Determining partial terms in the Friedmann equation

$$\sum_i \Omega_i + \Omega_{curv} + \Omega_\Lambda = 1,$$

$$\Omega_i = \frac{\rho_i}{\rho_c} = \frac{8\pi G \rho_i}{3H_0^2}, \ \Omega_{curv} = -\frac{\mathcal{K}}{a^2(t_0)H_0^2}, \ \Omega_\Lambda = \frac{\Lambda}{3H_0^2} .$$

1. Spatial curvature:

$$\Omega_{curv} = 0.001 \pm 0.002$$

- arXiv:1807.06209 (Planck 2018 results) - from CMB angular temperature and polarization fluctuations (mostly from the position of the first acoustic peak) + BAO (baryon acoustic oscillations) - to be derived later.

2. Visible matter: stars, galaxies (optical data), hot gas in rich galaxy clusters (X-ray data): $\Omega_{vis} \lesssim 0.01$.

3. Total amount of baryons (p,n) and electrons (e^-):

$$\Omega_b = (0.0457 \pm 0.0002) \left(\frac{70}{H_0}\right)^2$$

- arXiv:1807.06209 (Planck 2018 results).

From two independent sources leading to the same result:
1) CMB angular temperature and polarization fluctuations (mostly from the distance between acoustic peaks and the height of the third acoustic peak) - to be derived later;
2) abundance of light chemical elements (He^4, D, He^3, Li^7), which were produced during the Big Bang nucleosynthesis (BBN) in the early Universe - to be derived later.

No primordial antimatter.
The most part of baryonic matter ('dark baryons') is in the intergalactic medium (IGM).

4. Kinetic energy of baryons - very small compared to their rest-mass energy. Typical peculiar velocities of galaxies with respect to the FLRW background are ~ 500 km/s. The velocity of the Solar system with respect to the reference frame where CMB is at rest on average is ≈ 370 km/s. It is determined from the dipole anisotropy of the CMB temperature. In turn, CMB on average is practically at rest with respect to the FLRW background with $\sim 10^{-5}$ accuracy. The velocity of the Milky Way with respect to CMB is 627 ± 22 km/s.

5. CMB photons (restoring c):

$$\rho_\gamma c^2 = \frac{\pi^2 (k_B T_\gamma)^4}{15 \hbar^3 c^3},$$

$$\Omega_\gamma = 5.045 \cdot 10^{-5} \left(\frac{T_\gamma}{2.725}\right) \left(\frac{70}{H_0}\right)^2$$

6. 3 families of neutrinos $(\nu_e,\ \nu_\mu,\ \nu_\tau)$ from which at least two are massive.

$$\Delta m_{31}^2 \approx \Delta m_{32}^2 \approx 2.5 \cdot 10^{-3}\,\mathrm{eV}^2,\ \Delta \mathrm{m}_{21}^2 \approx 0.75 \cdot 10^{-4}\,\mathrm{eV}^2.$$

The BBN theory predicts that $T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$ for $T_\nu \gg m_\nu$ - to be derived later. Then

$$\sum_{i=1}^{3} m_{\nu i} = 46\Omega_\nu \left(\frac{H_0}{70}\right)^2\ \mathrm{eV}.$$

Observations (arXiv:1807.06209 - Planck 2018 results, model dependent however):

$$\sum_{i=1}^{3} m_{\nu i} < 0.12\,\mathrm{eV} \rightarrow \Omega_\nu < 0.003.$$

Thus, about 95% of the total present energy density of the Universe is due to 'dark entities' not known from terrestrial experiments. We divide these dark entities into dark matter and dark energy.

Dark matter

Dark matter and dark energy are seen through gravitational interaction only – we know the structure of their effective energy-momentum tensor.

DM - non-relativistic, gravitationally clustered.
DE - relativistic, unclustered.
Definition of their effective EMT – through equations (chosen conventionally). We choose what follows from GR.

For non-relativistic matter in the longitudinal gauge (valid in the first order in $|\Phi|$ only):

$$ds^2 = (1 + 2\Phi)\, dt^2 - (1 - 2\Phi)\, a^2(t)(dx^2 + dy^2 + dz^2),$$

we get the generalized Poisson equation (to be derived later):

$$\frac{\triangle\Phi}{a^2} = 4\pi G(\rho - \rho_0(t)).$$

Observational determination of Φ

To determine the amount of DM and its properties, we consider scales less than ~ 100 Mpc at which gravitational matter clustering is seen.

$\Phi(\mathbf{r}, t)$ is measured using the motion of 'test particles' in it.

a) Stars in galaxies $\rightarrow$ rotation curves.

b) Galaxies $\rightarrow$ peculiar velocities.
Peculiar velocity is $\mathbf{v} - H_0\mathbf{r}$.

c) Hot gas in galaxies $\rightarrow$ X-ray profiles.

d) Photons $\rightarrow$ gravitational lensing (strong and weak).
Strong lensing - multiple images of a source.
Weak lensing - astigmatism, distortion of the shape of galaxies.

Amount and properties of DM

Observations: DM is non-relativistic, has a dust-like EMT – $p \ll \rho, \quad p > 0$, collisionless in the first approximation – $\sigma/m < 1\ \mathrm{cm}^2/\mathrm{g}$, and has the same spatial distribution as visible matter for scales exceeding a few Mpc. However, its amount is about 5 times more than that of baryonic matter.

$$\Omega_{DM} \approx 0.25, \ \Omega_m = \Omega_{DM} + \Omega_b \approx 0.3$$

(arXiv:1807.06209: $\Omega_m = 0.315 \pm 0.007$, model dependent).

In addition, DM is cold - CDM: relative (e.g., thermal) velocities of particles which constitute it (if exist) are much less than the velocity $v = |-\nabla\Phi|/a(t)$ acquired in the cosmic gravitational field. This follows from the absence of damping of matter perturbations at galaxy and smaller scales (to be discussed later).

At smaller scales, DM is less clustered than baryonic matter.

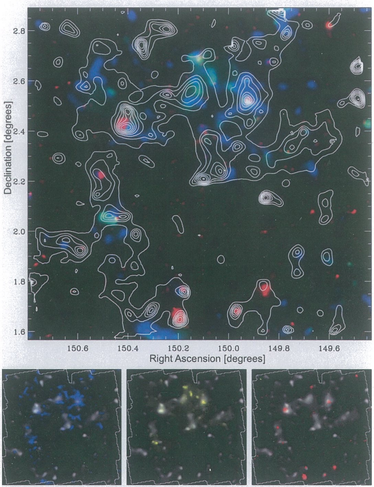

Figure 3 | Comparison of baryonic and non-baryonic large-scale structure. The total projected mass from weak lensing, dominated by dark matter, is shown as contours in panel **a** and as a linear grey scale in panels **b**, **c** and **d**. Independent baryonic tracers comprise (i) stellar mass (blue, colour scale peaks at 2.3×10^{11} $M_{\rm sun}$ deg^{-2} within Δz=0.1), (ii) galaxy number density (yellow, peak at 1.4×10^{5} deg^{-2} within Δz=0.1) seen in optical and near-IR light (adjusted to the redshift sensitivity function of the lensing mass), and (iii) hot gas (red, peak at 2.6×10^{-14} $erg/s/cm^2/arcmin^2$) seen in *x*-rays after removal of point sources.

Possible candidates for DM

Purely hypothetical at present.
1. Weakly interacting massive particles (WIMPs) which were in thermal equilibrium with CMB during BBN or earlier. Appear in extensions of the Standard Model of elementary particles (e.g. in its supersymmetric extensions). Ground experiments: spin-independent cross section $\sigma < 10^{-46}\ \mathrm{cm}^2$ for m_{WIMP} in the range $(10 - 300)$ GeV (arXiv:2207.03764).

2. Light cold particles which were never in thermal equilibrium with CMB and form a Bose-Einstein condensate. A popular example: an axion with the mass in the range $(10^{-6} - 10^{-3})$ eV.

3. Primordial black holes (PBHs) with masses in the range $(10^{17} - 10^{23})$ g. PBHs with larger masses are possible, too, but they may not constitute the whole DM.

Dark energy

DE is more general than Λ, it provides the remaining part in the Friedmann equation.

$$\Omega_{DE} \approx 0.7.$$

Now the whole FRLW universe is used as a test particle. Quantitative and internally self-consistent definition of its effective EMT - through gravitational field equations conventionally written in the Einstein form:

$$\frac{1}{8\pi G}\left(R_\mu^\nu - \frac{1}{2}\delta_\mu^\nu R\right) = T^\nu_{\mu\,(vis)} + T^\nu_{\mu\,(DM)} + T^\nu_{\mu\,(DE)}\,,$$

$G = G_0 = const$ - the Newton gravitational constant measured in laboratory. In the absence of direct interaction between DM and DE: $T^\nu_{\mu\,(DE);\nu} = 0$.
The left side can be determined from observations, e.g. from $D_L(z)$. Then, after subtracting $T^\nu_{\mu\,(vis)}$ and $T^\nu_{\mu\,(DM)}$ determined at the previous steps, $T^\nu_{\mu\,(DE)}$ follows.

Possible forms of DE

1. Physical DE.

New non-gravitational field of matter. DE proper place – in the rhs of gravity equations.

2. Geometrical DE.

Modified gravity. DE proper place – in the lhs of gravity equations.

3. Λ - intermediate case.

Observations: $T_{\mu (DE)}^{\nu}$ is very close to $\Lambda\delta_{\mu}^{\nu}$ for the concrete solution describing our Universe.

$$< w_{DE} >= -1.03 \pm 0.03$$

(arXiv:1807.06209), where $w_{DE} \equiv p_{DE}/\rho_{DE}$.

$w_{DE} > -1$ – normal case,
$w_{DE} < -1$ – phantom case,
$w_{DE} \equiv -1$ – the exact cosmological constant ("vacuum energy").

Possible candidates for DE apart from Λ

1. Physical DE: a scalar field with some interaction potential minimally coupled to gravity. Dubbed quintessence in the case of DE in the present Universe.

$$\mathcal{L}_\phi = \frac{1}{2}\phi_{,i}\phi^{,i} - V(\phi)\,.$$

If $\phi = \phi(t)$, then

$$\rho_{DE} = \frac{\dot{\phi}^2}{2} + V,\;\; p_{DE} = \frac{\dot{\phi}^2}{2} - V,\;\; w_{DE} + 1 = \frac{\dot{\phi}^2}{2V}\,.$$

Scalar field can mimic DE if $\dot{\phi}^2 \ll V(\phi)$ for a time period exceeding H^{-1}, and then $H^2 \approx \frac{8\pi G}{3}(\rho_m + V(\phi))$. For $V(\phi) = \frac{m_\phi^2 \phi^2}{2}$, this requires $m_\phi \ll H$. DE phantom behaviour is not possible in this case.

2. Geometric DE: a specific form of $f(R)$ gravity $\mathcal{L}_g = \frac{f(R)}{2}$ in the range of R where $df/dR \approx 2f/R$.

3. Mixed DE: a scalar-tensor gravity.

$$\mathcal{L} = -\frac{1}{2}R\phi^2 + \frac{1}{2}\phi_{,i}\phi^{,i} - V(\phi)\,.$$

In the latter two cases, the effective gravitational constant depends on R or ϕ. Also DE phantom behaviour is possible.

In the dispute between Plato and Democritus, Plato was right by 70%, and Democritus by 30%.

Three fundamental cosmological constants

1. Baryon to photon ratio.

$$\frac{n_b}{n_\gamma} = 6.01 \times 10^{-10} \, \frac{\Omega_b}{0.0045} \, \left(\frac{2.725}{T_\gamma(\mathrm{K})}\right)^3 \left(\frac{70}{H_0}\right)^2 .$$

Beyond it: a theory of baryogenesis.

2. Baryon to total non-relativistic matter density.

$$\frac{\rho_b}{\rho_m} = 0.167 \, \frac{\Omega_b}{0.05} \, \frac{0.3}{\Omega_m} .$$

Beyond it: a theory of dark matter.

3. Energy density of present dark energy.

$$\rho_{DE} = 6.44 \times 10^{-30} \, \frac{\Omega_{DE}}{0.7} \, \left(\frac{H_0}{70}\right)^2 \, \mathrm{g/cm^3} ,$$

$$\frac{G^2 \hbar \rho_{DE}}{c^5} = 1.25 \times 10^{-123} \, \frac{\Omega_{DE}}{0.7} \, \left(\frac{H_0}{70}\right)^2 .$$

Beyond it: a theory of present dark energy.

The standard LambdaCDM cosmological model

The standard ΛCDM model (neglecting radiation).

$$a(t) = a_1 \left(\sinh \frac{3H_1 t}{2}\right)^{2/3}, \ \Lambda = 3H_1^2 = 3H_0^2\,(1-\Omega_m)$$

with $\Omega_m \approx 0.3,\ H_1 \approx 0.84 H_0 \approx 60\,\mathrm{km/s/Mpc}.$

$$H^2(z) = H_0^2\left[1-\Omega_m+\Omega_m(1+z)^3\right].$$

The stage of acceleration begins at $t = t_{ac}$ when

$$\ddot{a} = 0,\ z_{ac} = \left(\frac{2(1-\Omega_m)}{\Omega_m}\right)^{1/3} - 1 \approx 0.67,\ \Omega_m(z_{ac}) = 2/3\ .$$

The age of the Universe: $t(H) = \frac{1}{3H_1}\ln\frac{H+H_1}{H-H_1}$. For $H_0 = 70$ km/s/Mpc, $t_0 \approx 0.96 H_0^{-1} \approx 13.5\,\mathrm{byl.\ y},\ t_{ac} \approx 7.3\,\mathrm{byl.\ y}.$