Lectures on relativistic gravity and cosmology.

Lectures 15-16

Alexei A. Starobinsky

Landau Institute for Theoretical Physics RAS

Faculty of Physics, National Research University "Higher School of Economics"

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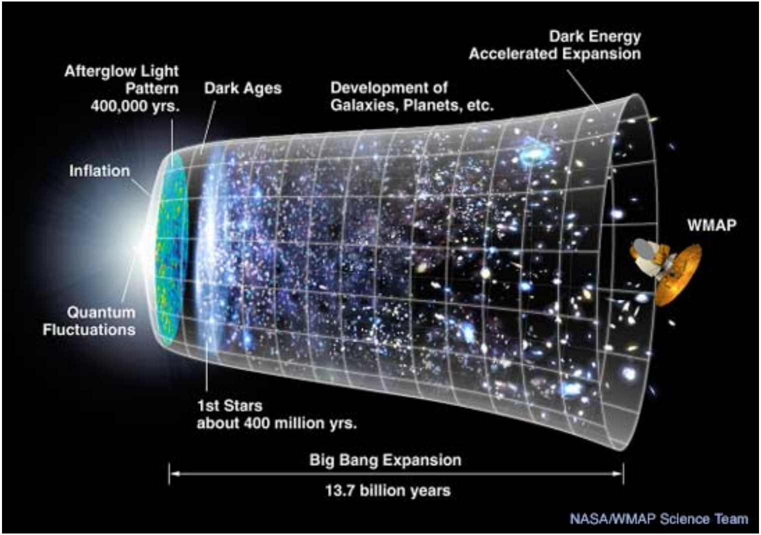
History of our Universe

The large-scale structure of the Universe

Recombination of hydrogen in the Universe

CMB temperature anisotropy

CMB polarization


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
400,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years
NASA/WMAP Science Team

Four principal epochs of the history of our Universe

$H \equiv \frac{\dot{a}}{a}$ where $a(t)$ is a scale factor of an isotropic homogeneous spatially flat universe (a Friedmann-Lemaître-Robertson-Walker background):

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) + small\ perturbations$$

The history of the Universe in one line: four main epochs

$$? \longrightarrow DS \Longrightarrow FLRWRD \Longrightarrow FLRWMD \Longrightarrow \overline{DS} \longrightarrow ?$$

Geometry

$$|\dot{H}| << H^2 \Longrightarrow H = \frac{1}{2t} \Longrightarrow H = \frac{2}{3t} \Longrightarrow |\dot{H}| << H^2$$

Physics

$$p \approx -\rho \Longrightarrow p = \rho/3 \Longrightarrow p \ll \rho \Longrightarrow p \approx -\rho$$

Duration in terms of the number of e-folds $\ln(a_{fin}/a_{in})$

> 60 ~ 55 7.5 0.5

Principal epochs of the Universe evolution – before 1979

The history of the Universe in one line: two principal epochs

$$? \longrightarrow FLRWRD \Longrightarrow FLRWMD \longrightarrow ?$$

Geometry

$$H = \frac{1}{2t} \Longrightarrow H = \frac{2}{3t}$$

Physics

$$p = \rho/3 \Longrightarrow p \ll \rho$$

Nonlinear gravitational instability in the matter dominated Universe

Nonlinear quasi-Newtonian hydrodynamics omitting the assumption $|\delta| \equiv |\frac{\rho - \rho_0}{\rho_0}| \ll 1$, but keeping $|\Phi| \ll 1$ and $|\mathbf{u}| \equiv -\frac{|\nabla V|}{a} \ll 1$. Valid before caustics formation where δ diverges. Beyond caustics, kinetic description of CDM particles and baryons is required.

$$\frac{\triangle \Phi}{a^2} = 4\pi G \rho_0 \delta, \; \dot{\delta} + \frac{\mathrm{div}\,[(1+\delta)\,\mathbf{u}]}{a} = 0, \; (a\mathbf{u})^{\cdot} + (\mathbf{u}\nabla)\mathbf{u} = -\nabla \Phi.$$

Its first integral (the cosmic Bernoulli theorem) is:

$$\dot{V} = \frac{(\nabla V)^2}{2a^2} + \Phi \,.$$

Due to non-linear corrections, Φ becomes time-dependent, but moderately.

The large-scale structure of the Universe

First galaxies began to form at redshifts $z \leq 13$ (spectroscopically-confirmed), but most of them were formed at $z \leq 3$. For present scales exceeding few Mpc, there are no gravitationally bound objects, but the spatial distribution of galaxies and galaxy clusters (dubbed the large-scale structure of the Universe) is very non-trivial up to scales exceeding ~ 300 Mpc where it becomes quasi-homogeneous.

The first non-linear large-scale structures formed in the Universe were caustics (also dubbed the Zeldovich pancakes) which form on two-dimensional surfaces when $\delta_{lin}(\mathbf{r}, t) \approx 1$. However, they are unstable and soon decay leaving the net of one-dimensional filaments (the cosmic web). In the intersection of filaments (nodes of the web), rich clusters of galaxies are located. The typical distance between nodes is ~ 100 Mpc, and the typical thickness of filaments is ~ 10 Mpc. Between filaments, large voids are located in which there are very few galaxies.

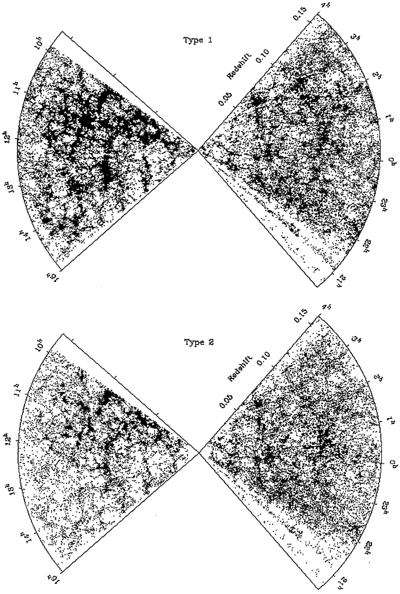


Figure 19. Redshift slices for different spectral types: type 1 corresponds to E/S0, type 2 to Sa/Sb, type 3 to Sc/Sd and type 4 to Irr.

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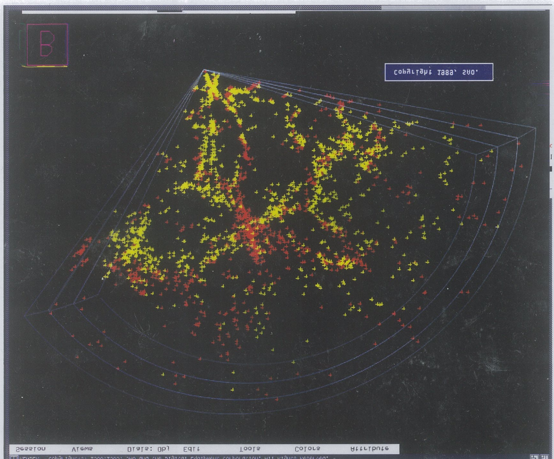
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Caustics formation in the Zeldovich approximation

Equations for non-linear quasi-Newtonian hydrodynamics can be solved exactly in the case of planar symmetry - motion depending on one Cartesian coordinate (x) only. The method of solution: transition from the Eulerian description to the Lagrangian one. In GR terms, this corresponds to the transition from the longitudinal gauge to the comoving one which coincides with synchronous gauge in the case of dust-like matter. For $a(t) \propto t^{2/3}$,

$$x = s - \frac{3t^2}{2a^2}\Phi_0'(s), \quad u \equiv a\left(\frac{dx}{dt}\right)_s = -\frac{t}{a}\Phi_0'(s), \quad V = t\Phi\,,$$

$$\Phi = \Phi_0(s) - \frac{3t^2}{4a^2}\Phi_0'^2(s), \ \rho_m = \rho_0\left(\frac{\partial x}{\partial s}\right)_t^{-1} = \frac{1}{6\pi G t^2\left(1 - \frac{3t^2}{2a^2}\Phi_0''(s)\right)}.$$

$\rho = \infty$ when $(\delta\rho/\rho)_{lin} = 1$.

The Zeldovich approximate solution for 3D motion:

$$\mathbf{r} = \mathbf{s} - \frac{3t^2}{2a^2}\nabla\Phi_0(\mathbf{s}), \quad u = -\frac{t}{a}\nabla\Phi_0(\mathbf{s}), \quad V = t\Phi\,,$$

$$\Phi = \Phi_0(\mathbf{s}) - \frac{3t^2}{4a^2}(\nabla\Phi_0(\mathbf{s}))^2, \quad \rho_m = \rho_0(t)\left(\mathrm{Det}\left(\frac{\partial\mathbf{r}}{\partial\mathbf{s}}\right)_t\right)^{-1}.$$

The solution is not exact (deviation in the second order of perturbation theory), but has a reasonable accuracy $\sim (20-30)\%$ at the moment of first caustics formation.

For comparison: in the case of the spherically-symmetric collapse of dust, ρ diverges in the centre when $\delta_{lin} = \frac{3}{20}(12\pi)^{2/3} \approx 1.686$.

The calculation of evolution of the large-scale structure of the Universe beyond caustics requires N-body simulations.

Quasi-equilibrium thermodynamic theory of recombination

1. Before recombination ($T = T_\gamma(1+z) > T_{rec}$), primordial plasma consisting of usual matter - photons, electrons and protons with $\approx 25\%$ (by mass) admixture of α-particles - is ionized. Main interaction between particles: the Thomson scattering

$$e + \gamma \to e + \gamma, \quad \sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 \approx 6.65 \cdot 10^{-25}\,\mathrm{cm}^2.$$

In this regime, $\sigma_T n_e t \gg 1$, so plasma is in the state of the thermal equilibrium.

2. After recombination ($T < T_{rec}$): electrons are captured by protons forming H atoms. Photons become practically free after that: the optical depth since recombination up to the present time $\tau \approx 0.054 \pm 0.007$ (arXiv:1807.06209) (the later result of the same collaboration $\tau = 0.051 \pm 0.006$ (arXiv:2007.04997)).

The condition for the thermal equilibrium of the chemical reaction $p + e \leftrightarrow H + \gamma$ is:

$$\mu_p + \mu_e = \mu_H \, .$$

The electroneutrality condition: $n_e = n_p$. Also $n_p + n_H = n_b$ from the baryon number conservation. Using the expression for an ideal non-relativistic gas in thermal equilibrium:

$$n_e = g_e \left(\frac{m_e T}{2\pi \hbar^2} \right)^{3/2} e^{(\mu_e - m_e)/T} , \;\; n_p = g_p \left(\frac{m_p T}{2\pi \hbar^2} \right)^{3/2} e^{(\mu_p - m_p)/T} ,$$

$$n_H = g_H \left(\frac{m_H T}{2\pi \hbar^2} \right)^{3/2} e^{(\mu_H - m_H)/T} ,$$

where the statistical weights are $g_e = g_p = 2, \; g_H = 4$, we get the Saha formula:

$$\frac{n_p^2}{n_H} = \left(\frac{m_e T}{2\pi \hbar^2} \right)^{3/2} e^{-I/T}$$

where $I = m_p + m_e - m_H = 13.6 \, \mathrm{eV}$ is the ionization energy.

Let us introduce the degree of ionization $\alpha_p = \frac{n_p}{n_p + n_H} = \frac{n_p}{n_b}$.

$$\frac{\alpha_p^2}{1-\alpha_p} n_b = \left(\frac{m_e T}{2\pi\hbar^2}\right)^{3/2} e^{-I/T} = 2.4 \cdot 10^{15}\ T^{3/2} \exp\left(-\frac{158000}{T}\right),$$

where T is in ^0K. Recombination occurs when $\alpha_p = 1/2$.
Numerical solution of this equation gives
$T_{rec} \approx 3800\,\mathrm{K},\ z_{rec} \approx 1400$. Note that $T_{rec} \ll I$ due to the small ratio of n_b to n_γ.

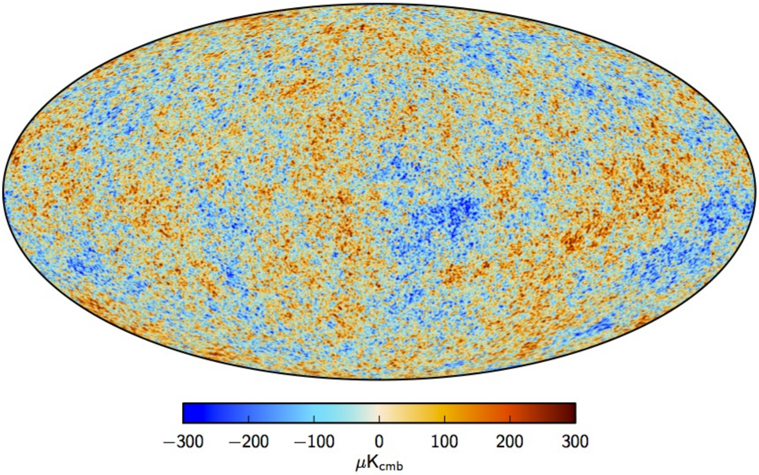
However, in fact recombination is somewhat delayed due to departure from exact thermal equilibrium caused by the following reasons.
1) The most efficient recombination of electrons occurs not to the ground state $n = 1$ but to the excited states of hydrogen, from which they cascade very quickly down to the first excited state with n = 2.

2) Atoms in the $2p$ state quickly decay to the $1s$ state by emitting a Lyman-α photon, but due to cosmological redshifting, there exist a small probability that this photon will not be captured by another H atom in its ground state.
3) Atoms in the $2s$ state decay much more slowly by emission of two photons ($\Gamma_{2s} \approx 8.2\ \mathrm{s}^{-1}$).
4) Atoms in the $n = 2$ state may be also re-ionized by ambient CMB photons before they reach the ground state.
The most recent account of all possible channels of recombination leads to $T_{rec} \approx 3000\,\mathrm{K},\ z_{rec} \approx 1100$. This corresponds to $t \approx 10^{13}\,\mathrm{s}$ from the end of the inflationary stage. At this moment the Universe is already at the matter dominated stage.

Another very important result is that the duration of recombination (the thickness of the last scattering surface (LSS)) is small: $\Delta z_{rec}/z_{rec} < 0.1$. This gives a possibility to consider it as instantaneous in many (though not all) cases.

CMB temperature anisotropy

Planck-2015: P. A. R. Ade et al., arXiv:1502.01589



CMB temperature anisotropy

Three contributions: the Sachs-Wolfe effect, the Silk effect and the Doppler effect. Assuming instantaneous recombination (that is a good approximation for the multipole number $l \lesssim 500$):

$$\frac{\Delta T(\theta, \phi)}{T} = -\frac{1}{2} \int_{\eta_{rec}}^{\eta_0} \left(\frac{\partial h_{\alpha}^{\beta}(\eta, \mathbf{r})}{\partial \eta} \right)_{r=\eta_0-\eta} e^{\alpha} e_{\beta} \, d\eta +$$

$$+ \left(\left(\frac{\delta \rho}{3(\rho + p)} \right)_{\gamma} + u_{\gamma}^{\alpha} e_{\alpha} \right) (\eta = \eta_{rec}, r = \eta_0 - \eta_{rec}),$$

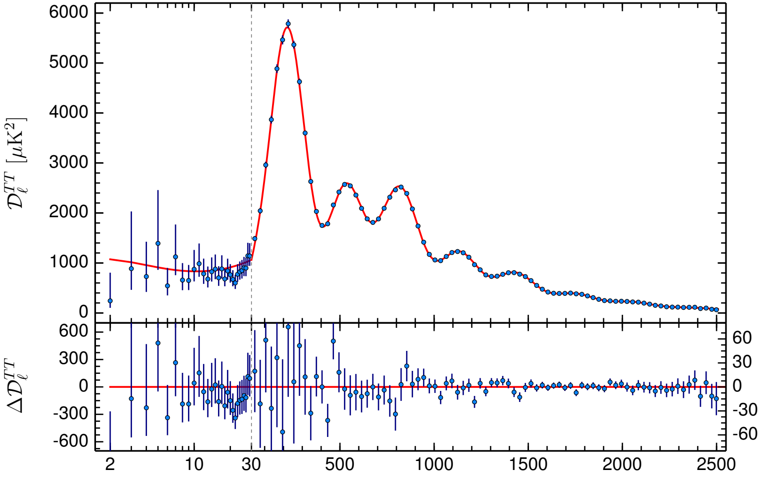
where $e^{\alpha} = -\frac{x^{\alpha}}{r}$ is the tangent vector along the light ray trajectory.

For $\ell \lesssim 1.5\sqrt{z_{rec}} \approx 50$, neglecting the Silk and Doppler effects, as well as the Integrated Sachs-Wolfe (ISW) effect due to the presence of radiation and dark energy (resulting in deviation of $a(t)$ from a pure power law $t^{2/3}$), and using the results for λ and μ for scalar perturbations in the synchronous gauge in the regime $k\eta \ll 1$ derived earlier, we get:

$$\frac{\Delta T(\theta,\phi)}{T} \approx -\frac{1}{5}\,\xi(r=\eta_0-\eta_{rec},\theta,\phi) \approx \frac{1}{3}\,\Phi(r=\eta_0-\eta_{rec},\theta,\phi)\,.$$

CMB temperature anisotropy multipoles

$$\Delta T(\theta, \phi) = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi), \;\; D_l^{TT} = \frac{l(l+1)}{2\pi(2l+1)} \sum_m |a_{lm}|^2.$$



Information from CMB temperature anisotropy

1. The Universe was isotropic and homogeneous at $z = z_{rec}$ with better than 10^{-5} accuracy.
2. No large quadrupole $\rightarrow$ no large-scale decaying modes.
3. The first acoustic peak is at $l \approx 220 \rightarrow$ no spatial curvature (for comparison, it would be at $l \approx 600$ for $\Omega_{curv} = 0.7$).
4. Existence of acoustic oscillations with the asymptotic period $l_{ac} = \frac{\pi(\eta_0 - \eta_{rec})}{c_s \eta_{rec}} \approx 302$. The acoustic scale $\theta_{ac} = \frac{\pi}{l_{ac}} = (1.0411 \pm 0.0003) \cdot 10^{-2}$. Their existence is due to the absence of decaying modes at super-Hubble scales before recombination that results in the acoustic waves in the sub-Hubble regime being standing, not running: $\Phi(\mathbf{k}) = \Phi^*(-\mathbf{k}) \propto \sin(kc_s\eta + const)$, $c_s(\eta_{rec}) \approx 0.453$, $k \rightarrow \frac{l+1/2}{\eta_0 - \eta_{rec}}$.
5. The height of third acoustic peak strongly grows with $n_b \rightarrow$ accurate measurement of n_b.
6. It becomes possible to measure the primordial Fourier spectrum of scalar perturbations $\xi(\mathbf{k})$ and their statistics.

New cosmological parameters relevant to the initial spectrum of perturbations

The primordial spectrum of scalar perturbations has been measured and its deviation from the flat spectrum $n_s = 1$ has been discovered (using the multipole range $\ell > 40$):

$$\langle \xi^2(\mathbf{r}) \rangle = \int \frac{P_\xi(k)}{k}\, dk, \;\; P_\xi(k) = (2.10 \pm 0.03)\cdot 10^{-9} \left(\frac{k}{k_0}\right)^{n_s - 1},$$

$$\langle \xi(\mathbf{r}) \rangle = 0, \;\; k_0 = 0.05 \ \mathrm{Mpc}^{-1}, \quad n_s - 1 = -0.035 \pm 0.004 \, .$$

Two new fundamental observational constants of cosmology in addition to the three known ones (baryon-to-photon ratio, baryon-to-matter density and the cosmological constant). However, the simplest inflationary models predicted the second constant in terms of the quantity $N_H = \ln \frac{k_B T_\gamma}{\hbar H_0} \approx 67.2$. Statistics of ξ - Gaussian with excellent accuracy.

CMB linear polarization

New effect: generation of linear CMB polarization during recombination through Thomson scattering of CMB photons on electrons. The effect is of the relative order of 10^{-6}: $C_l^{EE} \propto C_l^{TT}(\Delta z_{rec}/z_{rec})^2$.

No circular polarization.

Only the E-mode has been found, no primordial B-mode of polarization which is produced by tensor perturbations (primordial gravitational waves which are standing in the sub-Hubble regime, too, similar to scalar perturbations). The present upper limit on their power spectrum is

$$r \equiv \frac{P_g}{P_\xi} < 0.028$$

(arXiv:2208.00188). The expected correlation between CMB temperature anisotropy and E-model polarization has been discovered, too.

CMB E-mode polarization multipoles