Lectures on relativistic gravity and cosmology.

Lectures 17-18

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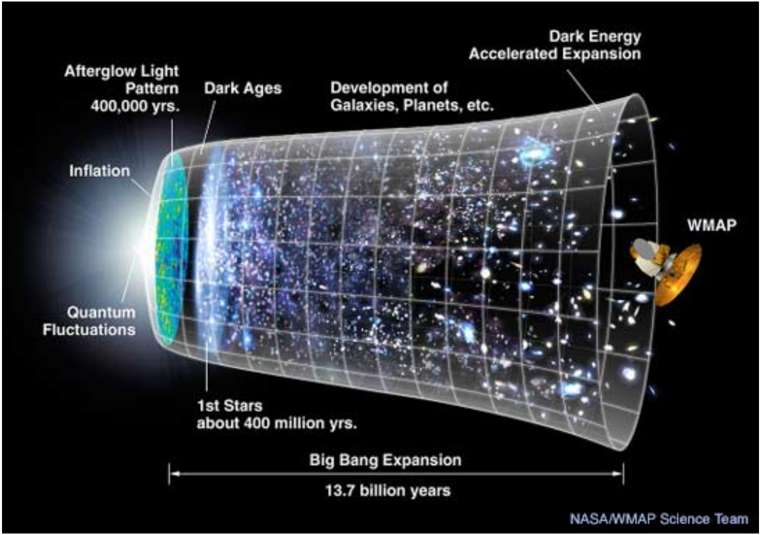
Present spectrum of matter perturbations

Baryon acoustic oscillations

The Sunyaev-Zeldovich effect

Big Bang nucleosynthesis


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
400,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years
NASA/WMAP Science Team

Initial conditions for the early Universe at the radiation dominated stage

In the super-Hubble regime ($k \ll aH$) in the coordinate representation:

$$ds^2 = dt^2 - a^2(t)(\delta_{\alpha\beta} + h_{\alpha\beta})dx^\alpha dx^\beta, \quad \alpha, \beta = 1, 2, 3,$$

$$h_{\alpha\beta} = 2\xi(\mathbf{r})\delta_{\alpha\beta} + \sum_{a=1}^{2} g^{(a)}(\mathbf{r})\, e^{(a)}_{\alpha\beta},$$

$$e^{\alpha(a)}_{\alpha} = 0, \; g^{(a)}_{,\alpha}\, e^{\alpha(a)}_{\beta} = 0, \; e^{(a)}_{\alpha\beta}\, e^{\alpha\beta(a)} = 1.$$

ξ describes primordial scalar perturbations, g – primordial tensor perturbations (primordial gravitational waves (GW)).

The most important quantities:

$$P_\xi(k), \; n_s(k) - 1 \equiv \frac{d \ln P_\xi(k)}{d \ln k}, \; r(k) \equiv \frac{P_g}{P_\xi}.$$

$\xi(\mathbf{k})$ are Gaussian and delta-correlated:

$$\langle \xi(\mathbf{k}_1)\xi(\mathbf{k}_2)\rangle = P_\xi(k)\,\delta^3(\mathbf{k}_1 - \mathbf{k}_2),$$

$$\langle \xi(\mathbf{r}_1)\xi(\mathbf{r}_2)\rangle = \int \frac{dk}{k}\,\frac{\sin kr}{kr}\,P_\xi(k),\;\; r = |\mathbf{r_1} - \mathbf{r_2}|\,,$$

$$P_\xi(k) = (2.10 \pm 0.03)\cdot 10^{-9}\left(\frac{k}{k_0}\right)^{n_s-1},$$

$$k_0 = 0.05\ \mathrm{Mpc}^{-1},\quad n_s - 1 = -0.035 \pm 0.004\,.$$

Transfer function for density perturbations in the cold dark matter component at the matter dominated stage ($a(t) \propto t^{2/3}$, $\Phi = -\frac{3}{5}\xi$): $\delta_m = \frac{9k^2t^2}{10a^2}\,\xi(k)\,c(k)$ where
$c(k) = 1,\;\; \frac{k}{a(t_0)} \ll \frac{k_{eq}}{a(t_0)} \approx 0.01\,\mathrm{Mpc}^{-1} \sim H_0\sqrt{z_{eq}}\,,$
$c(k) \propto \ln k/k^2,\;\; k \gg k_{eq}.$

Baryon acoustic oscillations (BAO)

Rms linear density fluctuation in the sphere with the radius $8(H_0/100)\,\mathrm{Mpc} = 11.4\,(H_0/70)\,\mathrm{Mpc}$ is:

$$\sigma_8 = \sqrt{<\delta^2>} = 0.811 \pm 0.006\,.$$

Slow growth of δ to smaller scales.

Due to the presence of baryons coupled to photons before recombination, $c(k)$ becomes slightly modulated with the relative amplitude $\propto \rho_b/\rho_m$ and the frequency $c_s(\eta_{rec})\eta_{rec}$ in the k-space. This leads to a feature in the correlation function of density perturbations. New kind of distance estimator arises:

$$D_V(z) = \left[(1+z)\,D_A^2\,\frac{cz}{H(z)}\right]^{1/3}, \quad D_A = \frac{D_L}{(1+z)^2}\,.$$

The smallness of BAO - one more proof of the existence of non-baryonic dark matter.

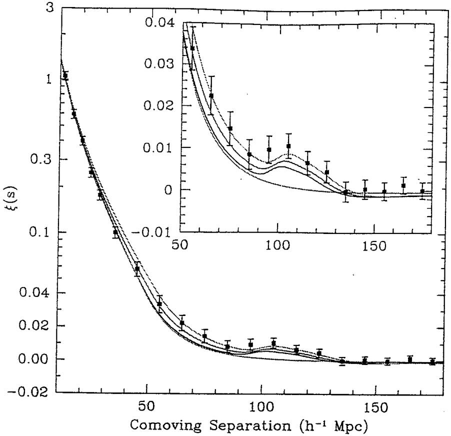


FIG. 2.— The large-scale redshift-space correlation function of the SDSS LRG sample. The error bars are from the diagonal elements of the mock-catalog covariance matrix; however, the points are correlated. Note that the vertical axis mixes logarithmic and linear scalings. The inset shows an expanded view with a linear vertical axis. The models are $\Omega_m h^2 = 0.12$ (top, green), 0.13 (red), and 0.14 (bottom with peak, blue), all with $\Omega_b h^2 = 0.024$ and $n = 0.98$ and with a mild non-linear prescription folded in. The magenta line shows a pure CDM model ($\Omega_m h^2 = 0.105$), which lacks the acoustic peak. It is interesting to note that although the data appears higher than the models, the covariance between the points is soft as regards overall shifts in $\xi(s)$. Subtracting 0.002 from $\xi(s)$ at all scales makes the plot look cosmetically perfect, but changes the best-fit χ^2 by only 1.3. The bump at $100h^{-1}$ Mpc scale, on the other hand, is statistically significant.

Necessary condition for galaxy formation

$$\sqrt{P_\xi}\left(\frac{t_\Lambda}{t_{eq}}\right)^{2/3} \gtrsim 1\,.$$

It is also necessary for stars, planets and life appearance. Thus, the four fundamental cosmological constants

$$A_1 = 2.1\times10^{-9},\ A_2 = 6.01\times10^{-10},\ A_3 = 0.167,\ A_4 = 1.25\times10^{-123}$$

should satisfy the inequality

$$\left(\frac{m_p}{M_{Pl}}\right)^4\left(\frac{A_2}{A_3}\right)^4\frac{A_1^{3/2}}{A_4} \gtrsim 1\,.$$

The left-hand side is equal to 0.46, so it is satisfied "just so". However, the account of the logarithmic growth of δ during the radiation dominated stage makes this number more than unity.

The Sunyaev-Zeldovich effect

One more effect in CMB that arises after recombination, after the formation of rich galaxy clusters and hot gas inside them at recent redshifts.
Its origin: inverse Compton scattering of low-energy CMB photons on relativistic electrons of hot gas.
For $\mathcal{E}_e \gg m_e$ and isotropic distribution of photons, the average energy of scattered photons

$$\mathcal{E}_{\gamma,\mathrm{scat}} = \mathcal{E}_\gamma \, \frac{4\mathcal{E}_e}{3m_e} \, .$$

The outcome of the effect: distortion of the Planck energy spectrum of CMB photons in the direction of rich galaxy clusters:

$$\Delta I_\nu = \frac{2T_\gamma^3}{h^2} y g(x), \quad x = \frac{h\nu}{T_\gamma}, \quad y = \frac{\sigma_t}{m_e} \int T_e n_e \, dl \, ,$$

$$g(x) = x^4 e^x \, \frac{x \coth \frac{x}{2} - 4}{e^x - 1} \, ,$$

where y is the Comptonization parameter (the integral is taken along the line of sight), n_e and T_e are the electron concentration and temperature respectively, and σ_t is the Thomson cross-section. $g(x)$ is negative for small x and positive for large x.

A powerful method to discover rich galaxy clusters and to investigate their spatial structure, additional to X-ray observations.

Neutrino decoupling

Radiation dominated stage:

$$a(t) \propto t^{1/2}, \quad \rho = \frac{3}{32\pi G t^2} = \frac{4.47 \cdot 10^5}{t^2} \frac{\mathrm{g}}{\mathrm{cm}^3} \equiv \kappa \rho_\gamma \,.$$

$$T(^\circ\mathrm{K}) = 1.52 \cdot 10^{10} \kappa^{-1/4} t^{-1/2}, \quad T(\mathrm{MeV}) = 1.31 \kappa^{-1/4} t^{-1/2} \,,$$

where t is in seconds. For the mixture of photons, electrons, positrons and 3 types of neutrinos and antineutrinos: $\kappa = 1 + \frac{7}{4} + \frac{21}{8} = 5.375$.

The condition for a particle to be in the thermal equilibrium in the expanding Universe: $\tau \equiv (\sigma n v)^{-1} \ll t$. The particle decouples if $\tau \sim t$.

Neutrino decoupling:

$$e^+ + e^- \leftrightarrow \nu_e + \bar{\nu}_e, \quad \sigma_\nu = \frac{G_F^2 \mathcal{E}^2}{\hbar^4}, \quad G_F \approx 10^{-49}\,\mathrm{erg} \cdot \mathrm{cm}^3, \quad t_{\nu_e} \sim 0.2\,\mathrm{s}\,.$$

Electron-positron annihilation and neutrino temperature

Electrons and positrons annihilate to photons adiabatically when $T \sim m_e \approx 0.5\,\mathrm{MeV},\ t \sim 3\,\mathrm{s}$. As a result, the photon temperature increases compared to the neutrino one. From the entropy conservation ($S = \frac{4\rho}{3T}$ for an ultrarelativistic gas):

$$T^3 + \frac{7}{4}T^3 = T^3_{\gamma,new},\ \ T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma,$$

$$\rho_\nu = \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} \rho_\gamma = 0.227\rho_\gamma\,.$$

Thus, after that $\kappa = 1.68$.

Neutron-to-proton ratio and BBN

$$e^+ + n \leftrightarrow p + \bar{\nu}_e, \quad \nu_e + n \leftrightarrow p + e^- .$$

At high temperature ($T > 2\,\mathrm{MeV}$):

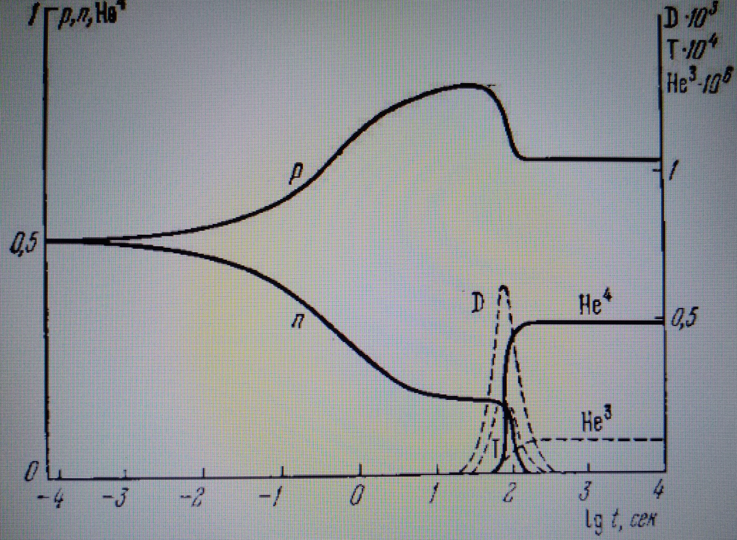
$$\frac{n_n}{n_p} = \exp\left(-\frac{\Delta m}{T}\right), \quad \Delta m = 1.28\,\mathrm{MeV} .$$

Using the condition $\tau = t$ with $\tau(\mathrm{s}) \approx 10^5\, T_9^{-5}$, one gets $T = 0.86\,\mathrm{MeV}$, $t \approx 1\,\mathrm{s}$, $Y_{^4\mathrm{He}} = \frac{2n_n}{n_n+n_p} = 0.37$. Exact calculation gives $Y_{^4\mathrm{He}} = 0.245$ for Ω_b producing the best fit to the deuterium abundance. Observations: $Y_{^4\mathrm{He}} = 0.25 \pm 0.005$.

At $t \sim 100\,\mathrm{s}$ BBN reactions begin and light elements ^{4}He, D, ^{3}He, ^{7}Li are synthesized.

$$n+p \to D+\gamma, \; D+D \to T+p \to {}^4\mathrm{He}+\gamma, \; T+D \to {}^4\mathrm{He}+n ,$$

$$D + D \to {}^3\mathrm{He} + n \to T + p, \; D + p \to {}^3\mathrm{He} + \gamma .$$


1
p, n, He⁴
0,5
0
D·10⁵
T·10⁴
He³·10⁵
1
0,5
p
n
D
T
He⁴
He³
-4
-3
-2
-1
0
1
2
3
4
lg t, сек

Also the decay of a free neutron with the characteristic time $\tau_n \approx 878$ s (arXiv:2106.10375) should be accounted for.
The primordial deuterium abundance is very sensitive to Ω_b (it decreases with the growth of Ω_b) . For $\Omega_b \approx 0.05$,
$\frac{n_D}{n_H} = (2.50 \pm 0.05)\, 10^{-5}$.
Observations: $\frac{n_D}{n_H} = (2.53 \pm 0.03)\, 10^{-5}$.

$$\frac{n\left({}^3\mathrm{He}\right)}{n_H} \approx 10^{-5}, \quad \frac{n\left({}^7\mathrm{Li}\right)}{n_H} \approx 3 \cdot 10^{-10}.$$

The earlier process: generation of baryon asymmetry, if we want to begin evolution of the Universe from a matter-antimatter symmetric state. Requires 3 necessary conditions:

1. Non-conservation of the baryon charge.
2. CP-violation.
3. Temporal breaking of thermal equilibrium.

Restrictions on additional particles and the Universe anisotropy from BBN

Two possible mechanisms which may effectively increase κ.
1. Additional types of relativistic matter decoupled from baryons at $t \sim 1$ s (e.g., sterile neutrino).
2. Anisotropy of cosmological expansion at $t \sim 1$ s (the large scale decaying mode of GW).
Then the moment of neutron decoupling t_* shifts:
$t_* \sim T_*^{-2}\kappa^{-1/2}, \; T_* \propto \kappa^{1/6}, \; t_* \propto \kappa^{-5/6}$.
Increase of κ leads to strong increase of $Y_{^4He}$. Observations: $|\delta\kappa/\kappa| \lesssim 5\%$. One more argument against the existence of the fourth (sterile) neutrino in standard cosmology.

Universe was isotropic at $t \sim 1$ s with at least a few percent accuracy. From this it follows that quasi-homogeneous anisotropy (decaying mode of GW in the $k = 0$ limit) is absent at $t = t_{rec} \sim 10^{13}$ s with better than z_{rec}/z_{BBN} accuracy.