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On the bounds of weak manipulability of majoritarian aggregation procedures

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Abstract

A model of upper and lower bounds of weak manipulability of 13 known aggregation procedures, based on majority relation, is proposed. The modified Nitzan-Kelly index is used to evaluate the spread of the degree of manipulability for Kelly's, Gardenfors, and Expected utility weak extensions of agent's preferences over the sets of alternatives. The results are obtained via computer simulations.

Keywords: weak manipulability; majoritarian aggregation procedures; bounds of manipulability;

1. Introduction

Since any non-dictatorial aggregation procedure is manupulable on unrestricted domain, as shown in [1, 2], for the cases of single- and multi-valued choice (see, e.g., [3-5]), the question remains as to what degree the known social choice procedures are subject to manipulability. The problem was first studied in [6-9], where the indices of manipulability were proposed, and the manupulability was studied for the case of Impartial Culture (IC) for which all profiles are assumed to be equiprobable and the alphabetical tie-breaking rule, in which in case of multiple choice the first alternative in alphabetical order is chosen. Then in [10-12] the problem of manipulability is studied in Impartial Anonymous Culture model, in which the set of agents are equally likely, no matter the names of the agents, again under alphabetical tie-breaking rule. To relax the condition of single-valuedness, and not to use any tie-breaking rule, which affects the estimated degree of the manipulability, the extensions of the agents' preferences for the case of multiple-valued choice is presented in [13], and the degree

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of manipulability is studied in [9, 13-15], where sets of considered alternatives could be compared by agents in a strong way, i.e. all possible sets are comparable, for example, leximin or leximax. Apart from that, some of the sets can be assumed to be indistinguishable, which is defined as "weak" extension of the preferences, for example, Kelly's Dominance or Gärdenfors principle. Such weak extensions are defined axiomatically, and, naturally, are partial orders.

2. The model

In the process of producing collective decisions using voting rules, the manipulation takes place if some of the agents can achieve a better collective choice by misrepresenting their preferences.

Consider an example with the profile consisting of 3 groups of agents with preferences over alternatives a, b, and c.

Table 1. A manipulation example

	Agents1 (3 votes)	Agents 2 (2 votes)	Agents 3 (2 votes)
Place 1	a	b	c
Place 2	b	a	b
Place 3	c	c	a

For simplicity, consider the Plurality rule, which counts the number of first places for each alternative and chooses the one with maximum number of such votes. Note that if the preferences are sincere the choice is $\{a\}$, which is worst option for the Agents 3. But if the agents manipulate, and instead of sincere preferences $c \succ b \succ a$, present the ordering $b \succ c \succ a$, then the collective choice will be $\{b\}$, which is better for Agents 3. In the example the choice is single-valued, consists of one alternative, so the resulting alternatives can be compared by agents directly using their preferences. But in general, the result can be multi-valued, which requires to compare sets of alternatives, and construct extended preferences. There are two types of extended preferences, the ones that able to compare all possible sets among each other, and the others which are called weak extended preferences that leave some sets incompatible.

The proposed model compares the possibility of manipulation of two kinds. The first kind of manipulation occurs if the sincere and insincere outcomes of choice can be compared by extended preferences. This can be called upper-bound manipulability. The other kind of manipulation occurs if the sincere and insincere choice sets are upper-bound manipulable, or, in addition, incompatible according to current preference extension. This kind of manipulability can be called lower-bound. The lower-bound manipulability is, naturally, covers more cases of comparison of sets than upper-bound one.

The Nitzan-Kelly index (NK-index) is used to evaluate the degree of manipulability of the aggregation rules [7,8]. The NK-index in general is defined as the share of the manipulable profiles to the total number of the considered profiles, i.e.

$$NK = \frac{d_0}{d_{total}}$$

where d_0 - the number of manipulated profiles, d_{total} - the total number of profiles.

For the considered model there are two NK-indices, lower- and upper-bound, which calculate the share of lower- and upper-bound manipulable profiles. The modified NK-indices are defined as follows

$$NK_{up} = \frac{d_{up}}{d_{total}}$$

where d_{up} is the number of lower-bound manipulated profiles,

$$NK_{lo} = \frac{d_{lo}}{d_{total}}$$

where d_{lo} is the number of manipulated profiles, d_{total} is the total number of profiles.

3. Majoritarian aggregation schemes, extensions of preferences

The aggregation schemes based on the majoritarian relation are considered. Note that the definition for majoritarian relation is as follows. It is said that for two alternatives a and b, a dominates b via majority relation if the number of agents for which a is more preferable than b ($a \succ b$) is greater than the number of agents with the opposite preferences ($b \succ a$). The rules, for which calculations were performed are as follows: Minimal Dominant Set (MDS), Minimal Undominated Set (MUS), Minimal Weakly Stable Set (MWSS), Fishburn's Rule (F), Uncovered Set I, II (US1, US2), Richelson's Rule (R), Copeland's rule I, II, III (C1, C2, C3), k-stable set, k=1,2,3, (1-SS, 2-SS, 3-SS).

The definitions of the aggregation rules under study are presented in [9,14].

Since the aggregation schemes can yield as a choice not only single alternative, but also a set of alternatives, the extended preferences of agents are considered, which allow one to compare sets of alternatives. We consider three weak extensions for which not all sets are compatible. The extensions are the Kelly's dominance (Weak Kelly), the Gärdenfors principle (Weak Gärdenfors), and the Expected Utility dominance (Weak EU) extensions. All of the extensions are defined axiomatically. For a complete discussion see [13]. Here we present the weak extensions for 3 alternatives for agents with preferences $a \succ b \succ c$.

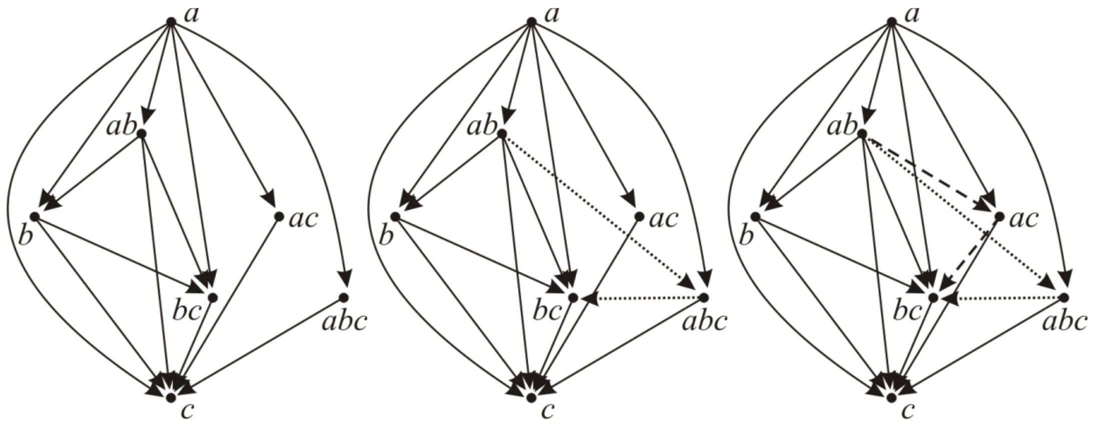

Fig. 1.Kelly's dominance (left), Gärdenfors principle (middle), and Expected Utility dominance (right) extensions for 3 alternatives

Note, that Gärdenfors extension differs from Kelly's in that, in addition, $\{ab\}$ dominates $\{abc\}$, and $\{abc\}$ dominates $\{bc\}$. In EU dominance, in addition to Gärdenfors, $\{ab\}$ dominates $\{ac\}$, and $\{ac\}$ dominates $\{bc\}$.

4. The manipulability

The evaluation of the degree of manipulability is performed in the following way. The 1 million of profiles are generated. For each of the profiles the result of the aggregation procedures is calculated, and all possible manipulation attempts are performed. Then the share of the manipulable profiles is calculated as upper- and lower-bound NK indices.

4.1. Upper and lower bounds of manipulability

The lower- and upper-bound manipulability differs drastically for odd and even number of agents. The typical dynamics of the degree of manipulability is presented on the figures. One can see on the charts for 5 alternatives for EU weak preference extension that for odd number of agents only MUS and C3 procedures have considerably higher level of lower-bound manipulability. While for even number of agents, on the contrary, MDS rule has low level of lower-bound manipulability.

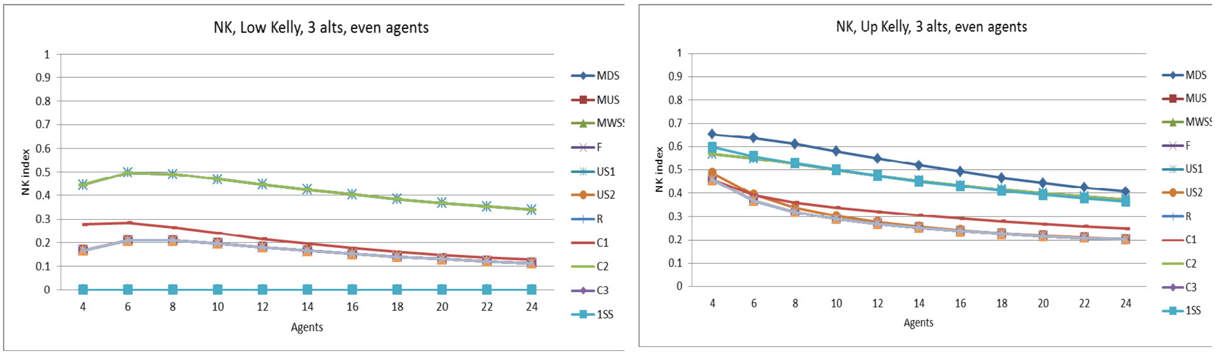

Fig. 2. For even number of agents, lower-bound (left), and upper-bound (right) manipulability for Kelly dominance extension for 3 alternatives.

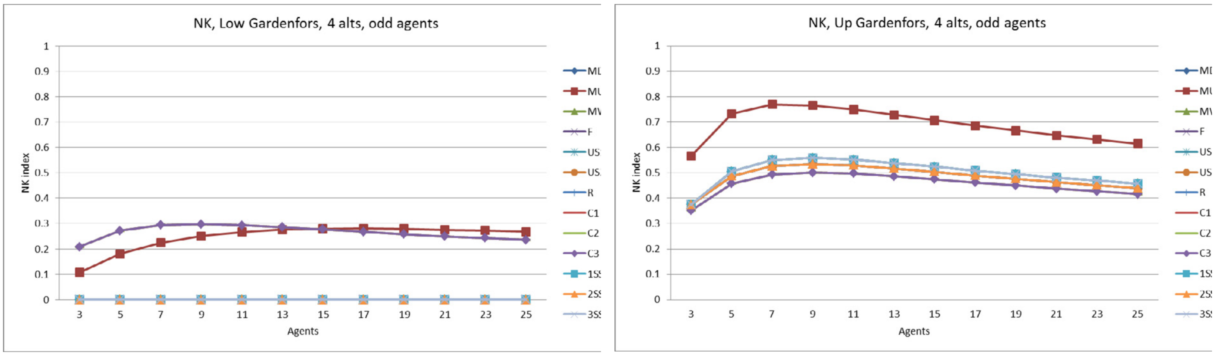

Fig. 3. For odd number of agents, lower-bound (left), and upper-bound (right) manipulability for Gärdenfors principle extension for 4 alternatives.

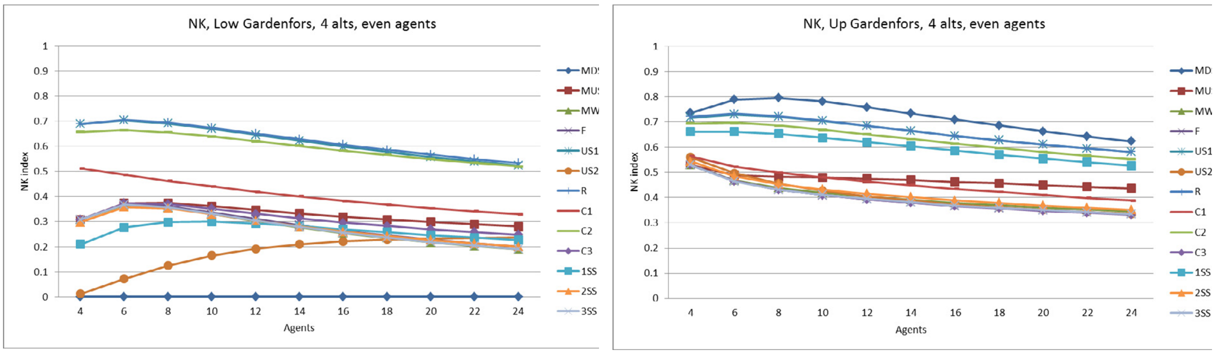

Fig. 4. For even number of agents, lower-bound (left), and upper-bound (right) manipulability for Gärdenfors principle extension for 4 alternatives.

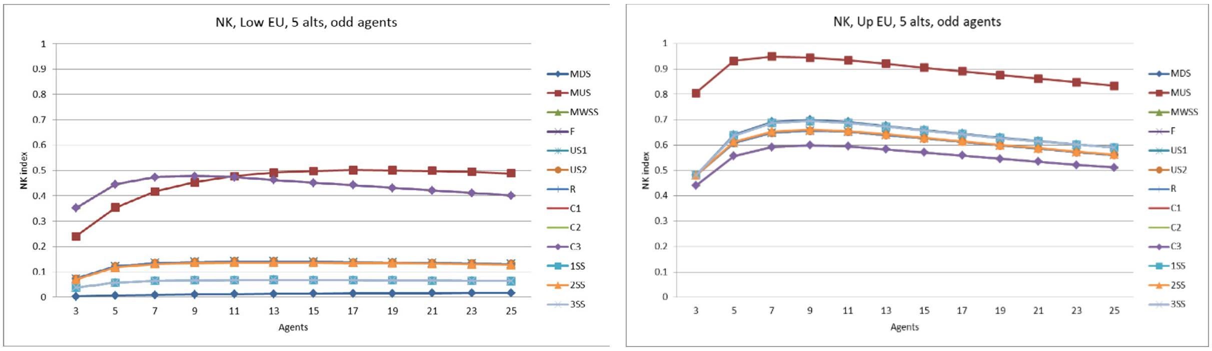

Fig. 5. For odd number of agents, lower-bound (left), and upper-bound (right) manipulability for Expected Utility extension for 5 alternatives.

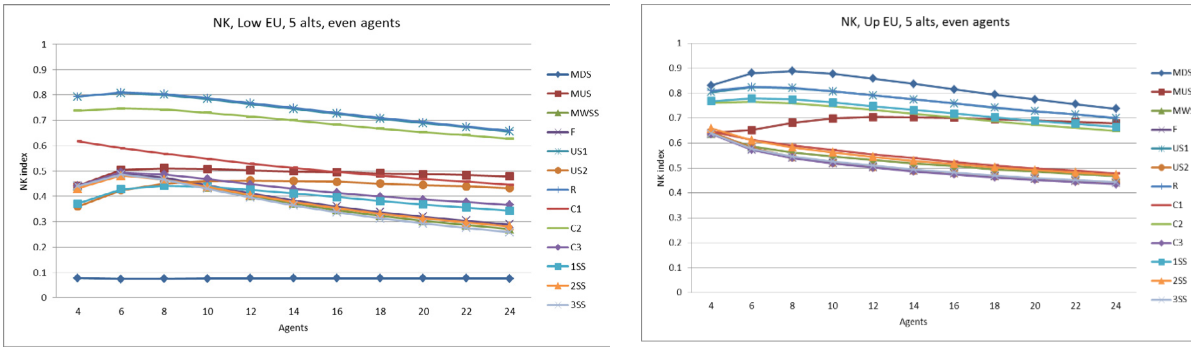

Fig. 6. For even number of agents, lower-bound (left), and upper-bound (right) manipulability for Expected utility extension for 5 alternatives.

To show the difference among the procedures, for the considered extensions we present the degree of manipulability for lower and upper bound in the following tables.

Table 2. The degree of manipulability for lower- and upper-bounds manipulability for all extensions, procedures for 5 alternatives and 19 agents

	19 agents					
	Kelly		Gärdenfors		EU	
	Low	Up	Low	Up	Low	Up
MDS	0	0.63	0	0.63	0.02	0.63
MUS	0.49	0.88	0.49	0.88	0.50	0.88
MWSS	0	0.63	0.05	0.63	0.07	0.63
F	0	0.60	0.04	0.60	0.14	0.60
US1	0	0.60	0.04	0.60	0.14	0.60
US2	0	0.60	0.04	0.60	0.14	0.60
R	0	0.60	0.04	0.60	0.14	0.60
C1	0.39	0.55	0.43	0.55	0.43	0.55
C2	0.39	0.55	0.43	0.55	0.43	0.55
C3	0.39	0.55	0.43	0.55	0.43	0.55
1SS	0	0.63	0.05	0.63	0.07	0.63
2SS	0	0.60	0.05	0.60	0.13	0.60
3SS	0	0.63	0.05	0.63	0.07	0.63

Table 3. The degree of manipulability for lower- and upper-bounds manipulability for all extensions, procedures for 5 alternatives and 20 agents

	20 agents					
	Kelly		Gärdenfors		EU	
	Low	Up	Low	Up	Low	Up
MDS	0	0.78	0	0.78	0.08	0.78
MUS	0.46	0.70	0.47	0.69	0.49	0.69
MWSS	0.27	0.50	0.29	0.49	0.30	0.49
F	0.29	0.47	0.30	0.46	0.32	0.46
US1	0.62	0.73	0.67	0.73	0.69	0.73
US2	0	0.48	0.33	0.46	0.44	0.46
R	0.62	0.73	0.68	0.73	0.69	0.73
C1	0.46	0.50	0.47	0.50	0.47	0.50
C2	0.65	0.67	0.65	0.67	0.65	0.67
C3	0.37	0.47	0.39	0.45	0.39	0.45
1SS	0	0.73	0.34	0.70	0.37	0.69
2SS	0.26	0.51	0.30	0.49	0.31	0.49
3SS	0.27	0.48	0.29	0.46	0.29	0.46

4.2. Aggregation procedures with minimal manipulability

As the number of agents increases, a particular group of procedures has minimal degree of manipulability in lower- and upper-bound manipulation models. We assume here, that if procedures are closer than 0.01 to the minimal level of manipulability, then they are considered minimally manipulable.

For 3 alternatives for odd number of agents the degree of manipulability is equal to zero for all procedures and all extensions for both lower and upper bounds. For 3 alternatives for even number of agents the minimally manipulated procedures are presented in the following tables.

Table 4. Minimally manipulated procedures for lower-bound manipulability for 3 alternatives

Lower-bound manipulation 3 alts	Even number of agents
Low Kelly	MDS, US2, 1SS
Low Gärdenfors, Low EU	MDS

Table 5. Minimally manipulated procedures for upper-bound manipulability for 3 alternatives

Upper-bound manipulation 3 alts	Even number of agents
Up Kelly, Up Gärdenfors, Up EU	MUS, MWSS, F, US2, C3, 2SS, 3SS

The least manipulable procedures for 4 procedures for lower bound are as follows.

Table 6. Minimally manipulated procedures for lower-bound manipulability for 4 alternatives

Lower-bound manipulation 4 alts	Odd number of agents	Even number of agents
Low Kelly	MDS, MWSS, F, US1, US2, R, 1SS, 2SS, 3SS	MDS, US2, 1SS
Low Gärdenfors	MDS, MWSS, F, US1, US2, R, 1SS, 2SS, 3SS	MDS
Low EU	MDS, MWSS, 1SS, 3SS	MDS

And the least manipulable procedures for 4 alternatives for upper bound are the same for all extensions.

Table 7. Minimally manipulated procedures for upper-bound manipulability for 4 alternatives

Upper-bound manipulation 4 alts	Odd number of agents	Even number of agents
Up Kelly, Up Gärdenfors, Up EU	C1, C2, C3	F, US2, C3, 3SS

The minimally manipulable procedures for 5 alternatives are presented in the tables.

Table 8. Minimally manipulated procedures for lower-bound manipulability for 5 alternatives

Lower-bound manipulation 5 alts	Odd number of agents	Even number of agents
Low Kelly	MDS, MWSS, F, US1, US2, R, 1SS, 2SS, 3SS	MDS, US2, 1SS
Low Gärdenfors, Low EU	MDS	MDS

Table 9. Minimally manipulated procedures for upper-bound manipulability for 5 alternatives

Upper-bound manipulability 5 alts	Odd number of agents	Even number of agents
Up Kelly, Up Gärdenfors, Up EU	C1, C2, C3	F, US2, C3, 3SS

5. Conclusion

Within the framework of the bound weak manipulability model, for 3 alternatives for odd number of agents the degree of manipulability is equal to zero in all cases. For even number of agents for 3 alternatives for lower-bound extensions, MDS is the least manipulable procedure for all of the extensions along with US2, 1SS for lower-bound Kelly. For upper-bound case the procedures MUS, MWSS, F, US2, C3, 2SS, 3SS are least manipulable for all three extensions.

For 4 alternatives, MDS, MWSS, F, US1, US2, R, 1SS, 2SS, 3SS procedures are least manipulable for lower bound Kelly and Gärdenfors, for odd number of agents, the MDS rule is minimally manipulable for lower-bound Gärdenfors and EU extensions. While for the rest lower-bound Kelly for even number of agents the least manipulable rules are MDS, US2, 1SS, and MDS, MWSS, 1SS, 3SS are minimally manipulable for even number of agents for lower-bound EU. For upper-bound manipulability C1, C2, C3 are minimally manipulable for even number of agents and F, US2, C3, 3SS for odd number of agents for all three extensions.

For 5 alternatives the Minimal Dominant Set (MDS) aggregation procedure is less manipulable according to lower-bound degree of manipulability both for odd and even number of agents, as the number of agents increases. And it is the only minimally manipulable procedure for Low Gärdenfors and Low EU manipulability. For Low Kelly manipulability, the group of procedures MDS, MWSS, F, US1, US2, R, 1SS, 2SS, 3SS is minimally manipulable for odd number of agents, and the group of MDS, US2, 1SS is minimally manipulable for even number of agents. As for the upper-bound manipulability model, for all three weak preference extensions, Kelly's, Gärdenfors, and EU, for odd number of agents the group of C1, C2, C3 is minimally manipulable for odd number of agents, and the group of F, US2, C3, 3SS is minimally manipulable for even number of agents. So, the Copeland's rule III (C3) has the minimal upper-bound manipulability for 5 alternatives for odd and even number of agents as number of agents increases.

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