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Manipulability of majoritarian rules by coalitions with the same first-ranked alternative

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Abstract

The coalitional manipulability of 13 majoritarian aggregation schemes (voting rules) is studied by computational experiments. We consider a special case of coalition formation in which all agents of manipulating coalition report the same first-ranked alternative upon manipulation, i.e. a group of agents misrepresents their preferences and they agree that the same alternative will be on the first place of their insincere preferences. We find out that in most cases one of the least manipulable rules in this framework is Minimal Dominant Set, which was not among the least manipulable rules in the case of individual manipulation.

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1. Introduction

All preferences aggregation procedures are designed to take into account preferences of all agents. Then if someone change her preference, aggregated result may change. There is a problem that the sincere preference is a private knowledge of voter and the rule itself cannot distinguish between the cases when someone submit her sincere or insincere preference. In other words, there may be a case, when someone may purposely submit insincere preference and get better aggregated result. This problem is called manipulability of voting rules and is inevitable for unrestricted domain as was shown by Gibbard [1] and Sattertwaite [2].

The problem of comparing rules by their manipulability was studied in several papers: Chamberlin [3], Nitzan [4], Kelly [5], Lepelley, Mbih [6], Kim, Roush [7], Lepelley, Valognes [8], Favardin et al [9], Favardin,

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Lepelley [10], Slinko [11], Pritchard, Wilson [12], Durand et al [13], Duggan, Schwartz [14], Ching, Zhou [15], Benoît [16], Aleskerov et al. [17-21]. Those papers consider different assumptions about probability of preferences profiles, types of manipulability considered, and the collection of rules under study.

Most of these papers consider positional voting rules and so called tie-breaking rule that reduce multi-valued choice to single-valued one. While latter is considered as simplifying assumption, Aleskerov et al. [17-21] show that it greatly influences the results of estimation.

Our paper is in some sense close to Aleskerov et. al. [21]. We also consider majority-relation based rules, but the coalitional manipulability instead of individual one. As we will show, this change of the model influence the set of least manipulable rules.

The paper is organized as follows. In Section 2 we define a model of estimation of the degree of manipulability. In Section 3 we define rules, and in Section 4 present and discuss the results.

2. Manipulation by coalitions with the same first-ranked alternative

2.1. Multiple choice extensions

We use the similar model as in [17]. We consider $m = 3$ alternatives and $n > 2$ agents (voters). We assume that each agent $i = \overline{1,n}$, has the preference $P_i \in \mathrm{L}$ over alternatives, and the set L stands for all possible linear orders on the set of alternatives.

All agents and their preferences P_i are referred to as a preference profile $\vec{P}$ which is an ordered n-tuple of preferences. The social choice is a result of applying an aggregation procedure on $\vec{P}$. We use the concept of multiple choice meaning that the social choice may consist of one, two or even more alternatives.

In order to allow each agent to compare all possible outcomes of aggregation procedure under multiple choice, we use extended preferences. We assume that every agent i has an extended preference EP_i on the sets of alternatives which is induced by her preference P_i. Thus, each agent can compare all possible multiple choices under the concept of extended preference.

The construction of extended preferences is based on preference extension axioms which have been widely discussed in the literature. The non-exhaustive list of papers includes Barbera [22], Kelly [23], Barbera et al. [24].

In this research we use four ways of constructing extended preferences: Leximax, Leximin, PWorst and PBest. In [17] the preference extensions are defined. All of them allow the agent to choose between two different outcomes of voting and select the best one. Here we present the extended preferences for 3 alternatives. If agent has a preference $a \succ b \succ c$ then extended preferences look as follows:

Leximax: $\{a\} \succ \{a,b\} \succ \underline{\{a,b,c\} \succ \{a,c\} \succ \{b\}} \succ \{b,c\} \succ \{c\}$

Leximin: $\{a\} \succ \{a,b\} \succ \underline{\{b\} \succ \{a,c\} \succ \{a,b,c\}} \succ \{b,c\} \succ \{c\}$

PWorst: $\{a\} \succ \{a,b\} \succ \underline{\{b\} \succ \{a,b,c\} \succ \{a,c\}} \succ \{b,c\} \succ \{c\}$

PBest: $\{a\} \succ \{a,b\} \succ \underline{\{a,c\} \succ \{a,b,c\} \succ \{b\}} \succ \{b,c\} \succ \{c\}$

2.2. The same first-ranked alternative manipulation model

In the general case, manipulation is defined as a situation when an agent or a group of agents misrepresent their preferences and the choice after manipulation is better for her (or them) than it could have been without manipulation.

We study the case of coalitional manipulation, i.e. when manipulation is performed by several agents.

How coalitions can be formed? We consider the case when the coalition is the group of agents who would like to promote the same alternative. It means that in the manipulation attempt they put the same alternative on the first place in their ballots. Manipulation takes place if after the manipulation attempt the resulting choice is better for each member of the coalition than it could be without manipulation.

Consider an example. Let us define a profile consisting of 100 voters with the preferences presented in Table 1. We calculate the choice for the Minimal undominated set rule.

Table 1. An example profile

	Group1	Group2	Group3	Group4	Group5	Group6
1-st place	a	a	b	b	c	c
2-nd place	b	c	c	a	a	b
3-rd place	c	b	a	c	b	a
Number of agents	17	19	16	15	14	19

The *profile under study is presented as the linear orders of groups of agents pointing out how many agents are in each group. The majority relation for this profile is* $\{(a,c),\{c,b\}\}$*. The Minimal undominated set rule yields the choice* $\{a\}$*. But if the Group3, consisting of 16 agents presents instead of* $b \succ c \succ a$ *the insincere preferences* $c \succ b \succ a$ *, the choice of the Minimal undominated set rule will be* $\{c\}$*. The majority relation for the manipulated profile will be* $\{(c,a),\{a,b\}\}$*. That choice* $\{c\}$ *is better for Group3, Group5, and Group6 with the same first alternative c. Note that Group5, and Group6 do not change their preferences, Group3 just joined them on the basis of the first-preferred alternative, but all three groups with the same first-ranked alternative gain from the manipulation.*

2.3. Manipulability index and the computation scheme

In Impartial Culture model (IC) the assumption is that all profiles are equally likely. In Impartial Anonymous Culture (IAC) model, to the contrary, the voting situations are equally likely, i.e. all profiles obtained by permutation of agents correspond to one voting situation. We consider Impartial Culture model.

In the general case with m alternatives, the total number of possible preferences in the form of linear orders is equal to $m!$. Thus, the overall number of all possible profiles for n agents under Impartial Culture will be $(m!)^n$. Nitzan [4] and Kelly [5] introduced the Nitzan-Kelly index (NK) which allows to estimate the degree of manipulability: $NK = \frac{d_0}{(m!)^n}$,where d_0 stands for the number of manipulable profiles.

In our research we define a manipulable profile as a profile where there exists at least one coalition which can successfully manipulate, i.e. can misrepresent their preferences under assumptions discussed in the previous section and obtain a better for them outcome of the aggregation procedure.

In order to estimate the degree of coalitional manipulability, we used computational modeling. The scheme is the following:

1. Generate 1 million profiles
2. For each profile calculate the result of the aggregation procedure
3. Consider all possible coalitions
4. If the coalition satisfies the assumptions, consider all possible manipulation attempts, i.e. all possible ways of misrepresenting the preferences
5. If for a given profile there exist at least one coalition with at least one way of misrepresenting their preferences leading to a better social choice, the profile is marked as a manipulable one

6. If such a situation does not exist, the profile is marked as non-manipulable
7. The NK index is calculated as the share of manipulable profiles

It can be easily seen that the simulation scheme is computationally intensive, especially because of the steps 3-5 where all possible coalitions and all possible manipulation attempts need to be checked for each aggregation procedure.

3. Majoritarian aggregation schemes

We consider the following aggregation rules based on the majority relation, the definitions of the aggregation rules, of upper, lower contours, majority relation are used as noted in [19].

The first 10 rules are 1. **(MDS)** Minimal dominant set. 2. **(MUS)** Minimal Undominated Set. 3. **(MWSS)** Minimal Weakly Stable Set. 4. **(F)** Fishburn's Rule. 5. **(US1)** Uncovered Set I. 6. **(US2)** Uncovered Set II. 7. **(R)** Richelson's Rule. 8. **(C1)** Copeland's rule I. 9. **(C2)** Copeland's rule II. 10. **(C3)** Copeland's rule III. We also add additional 3 rules 11-13. **(1SS, 2SS, 3SS)** k -stable set. A set is called to be k -stable (kSS), if for each alternative y outside the set, there is an alternative x which dominates y via majority relation in not more than k steps. A step here means a dominating relation. A k -stable set is minimal if no its proper subset is a k -stable set. If there are more than one minimal k -stable sets, their union defines the social choice. For k =1 each alternative outside the set should be just dominated by some alternative inside the kSS set.

Consider the sample profile with the following preferences. For Agent 1 the preference is $a \succ c \succ b$, for Agent 2: $a \succ c \succ b$, for Agent 3: $c \succ b \succ a$, and for Agent 4 the preferences is $b \succ a \succ c$. For this sample profile the rules yield the following choices. Minimal dominant set: $\{a,b,c\}$, Minimal undominated set: $\{a\}$, Minimal weakly stable set: $\{a\}$, Fishburn: $\{a\}$, Uncovered set I: $\{a,c\}$. Uncovered set II: $\{a,b\}$, Richelson: $\{a,c\}$, Copeland I: $\{a\}$, Copeland II: $\{a,c\}$, Copeland III: $\{a\}$, 1-stable set: $\{a,b,c\}$ 2- 3stable sets: $\{a\}$.

4. Results

The degree of manipulability by the same first-ranked alternative coalitions is studied for 3 alternatives and for 3 to 100 agents for corresponding majoritarian aggregation procedures with several extended preferences. In the Table 2 NK index for the case of 4 and 20 agents are provided.

Table 2. The degree of manipulability for selected number of agents.

N	Title	ID	4 agents				20 agents			
			Leximin	Leximax	PWorst	PBest	Leximin	Leximax	PWorst	PBest
1	Minimal dominant set	MDS	0.375	0.167	0.431	0.417	0.285	0.666	0.312	0.742
2	Minimal undominated set	MUS	0.346	0.417	0.486	0.347	0.332	0.735	0.332	0.759
3	Minimal weakly stable set	MWSS	0.346	0.417	0.486	0.347	0.332	0.735	0.332	0.759
4	Fishburn	F	0.346	0.417	0.486	0.347	0.332	0.735	0.332	0.759
5	Uncovered set I	US1	0.527	0.541	0.527	0.597	0.887	0.855	0.887	0.887
6	Uncovered set II	US2	0.347	0.306	0.431	0.291	0.312	0.711	0.312	0.658
7	Richelson	R	0.527	0.541	0.527	0.597	0.887	0.855	0.887	0.887
8	Copeland I	C1	0.361	0.541	0.361	0.541	0.329	0.736	0.329	0.752
9	Copeland II	C2	0.527	0.541	0.527	0.597	0.887	0.855	0.887	0.887
10	Copeland III	C3	0.346	0.417	0.486	0.347	0.332	0.735	0.332	0.759
11	1-stable set	1SS	0.347	0.361	0.431	0.347	0.285	0.744	0.312	0.746
12	2-stable set	2SS	0.346	0.417	0.486	0.347	0.332	0.735	0.332	0.759
13	3-stable set	3SS	0.346	0.417	0.486	0.347	0.332	0.735	0.332	0.759

From Table 2 we can note the following

1) The degree of manipulability is quite high. Share of all manipulable profiles is at least 33%.

2) Different extension axioms provide different results not only in values for a given case but in trend: for most rules for Leximin and PWorst extensions NK index decrease with the number of agents, but for Leximax and PBest the opposite is true. We will discuss it after presenting the figures for other number of agents.

3) Rules US1, R and C2 coincide and significantly differ from the others in terms of degree of manipulability.

This can be explained as follows – they are the only rules that use lower contour set in their definitions. The typical manipulation strategy for the coalition is to put the deviated preferences in the ballot, so that there will be a tie in the majority relation, and US1, R and C2 rules will give a better social choice. As soon as we consider all possible coalitions, the share of profiles where a coalition can obtain a tie in the majority relation is high and exceeds 80% for 20 agents and 90% for 100 agents.

Let us show the results for other number of agents. At Figure 1 one can observe changes in NK index for Leximax extension. As one can see there is a periodical structure in indices, which is consistent with the results of [21], where the case of individual manipulability was considered. Moreover, for the case of odd number of agents all the rules give the same results. This can be easily explained because for the odd number of agents majority relation is always complete. In order to have more representable results, on Figures 2-4 we present results separately for even and odd number of voters.

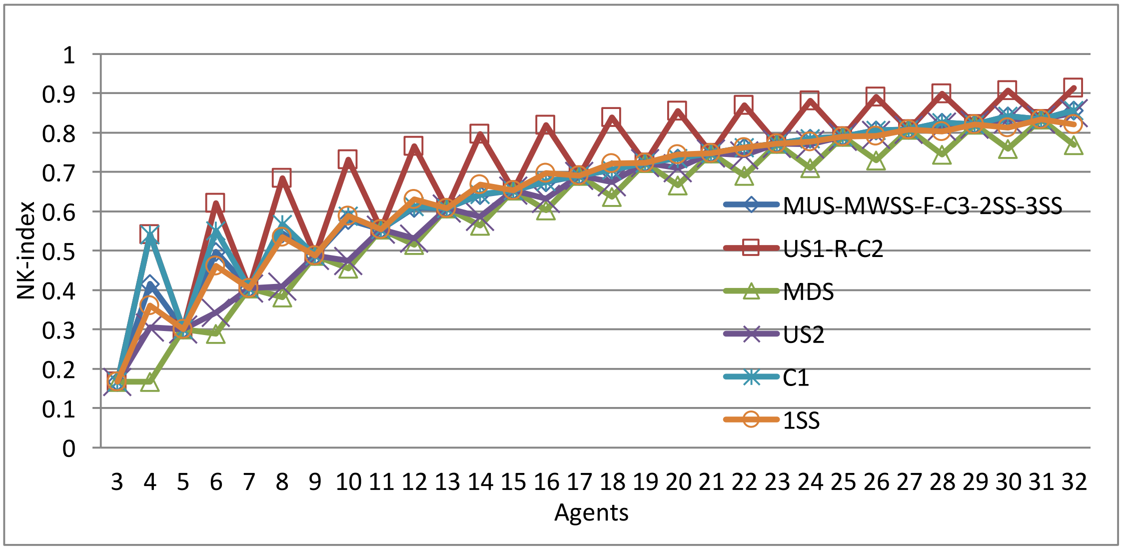

Fig. 1. NK index for Majoritarian rules, Leximax extension, IC, 3 alternatives, 3-32 agents

First, we can see that results for PBest and Leximax are similar to each other but strongly differ from the results for Leximin and PWorst extensions. One of the explanations is the difference between extension methods. If the preference of an arbitrary agent is $a \succ b \succ c$, then for the PBest and Leximax extensions the sets $\{a,b,c\}$ and $\{a,c\}$ are better than $\{b\}$, while for Leximin and PWorst extensions the opposite is true. It seems that in most cases manipulation takes place between the sets $\{b\}$ and either $\{a,b,c\}$ or $\{a,c\}$. Manipulation between $\{a,b,c\}$ and $\{a,c\}$ is less possible, that is why Figures 2 and 3, and Figures 4 and 5 look similar. At the same time, only difference between Leximax and PBest is observed in the relations between $\{a,b,c\}$ and $\{a,c\}$. As we can see from the figures this difference may influence the least manipulable rules for each case: for Leximax it is Minimal Dominant Set and for PBest it is Uncovered Set II.

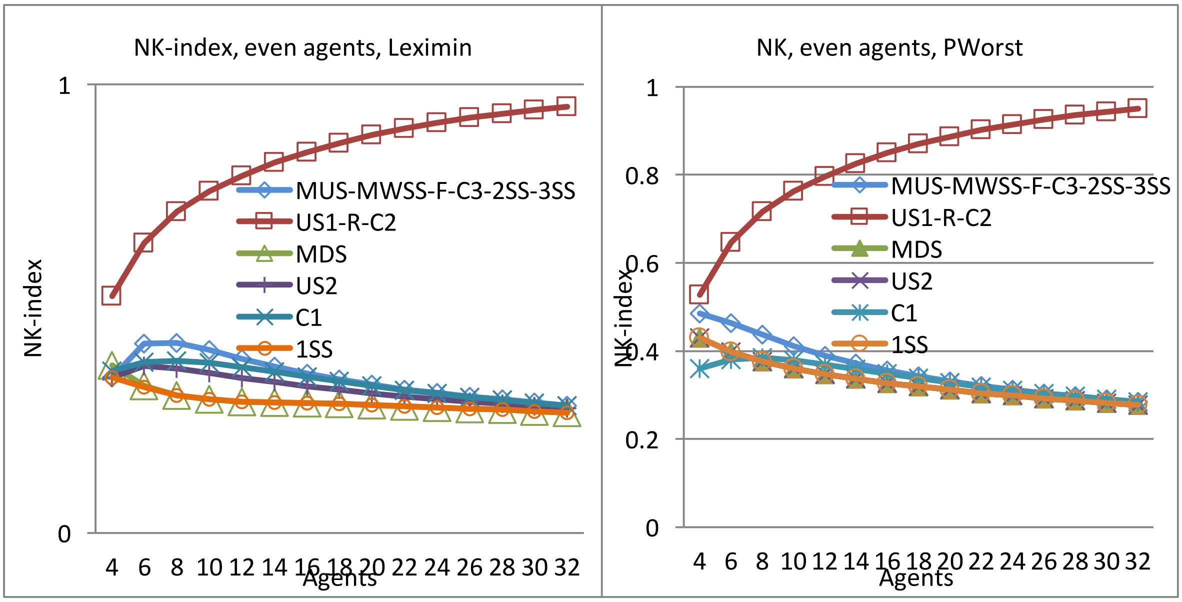

Fig. 2. NK index for Majoritarian rules for even number of agents, Leximin extension (left), PWorst3 extension (right)

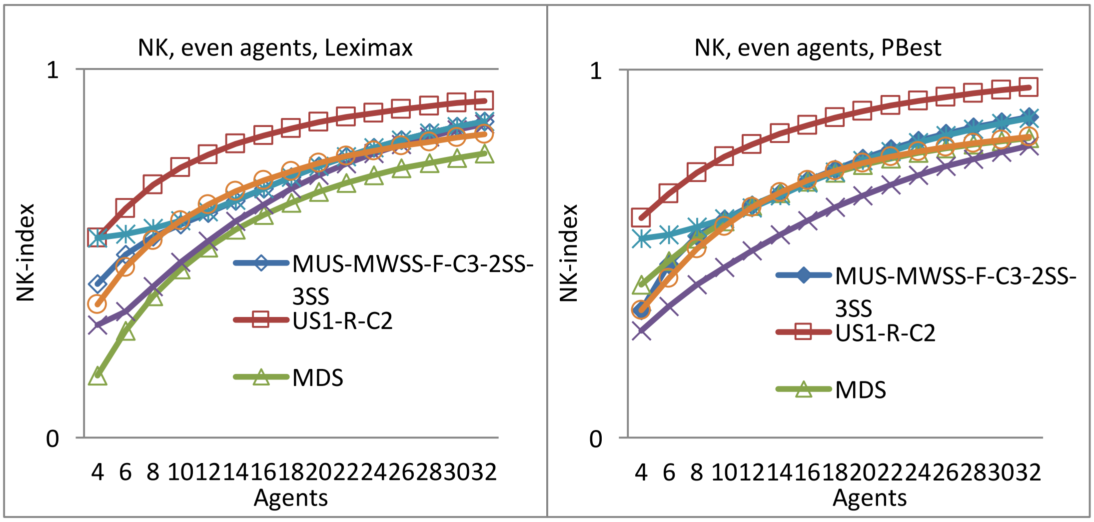

Fig. 3. NK index for Majoritarian rules for even number of agents, Leximax extension (left), PBest3 extension (right)

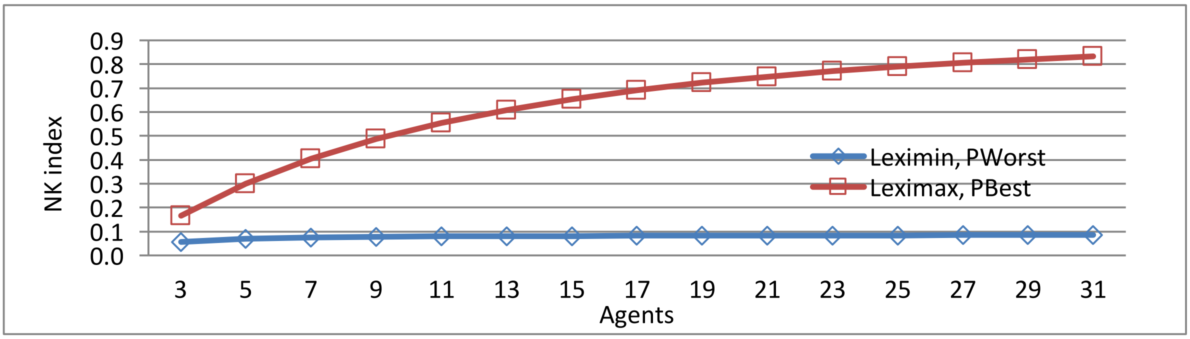

Fig. 4. NK index for Majoritarian rules, odd number of agents, all extensions.

Second, let us discuss the difference in changes of the index between Leximax and Leximin estension cases. For the case of 100 agents NK index for Leximax is almost 95% while for Leximin it is about 20%. The main explanation is the difference in the ordering of social choices in these extended preferences. As we know from Section 2.1., for Leximax $\{a,b,c\}$ is on the 3-rd place and it is more preferable than $\{b\}$ which is on the 5-th place. For Leximin the situation is opposite, i.e. $\{a,b,c\}$ is on the 5th place and is less preferable than $\{b\}$ on the 3rd place. The manipulation attempt where a coalition deviates the preferences in order to obtain choice $\{a,b,c\}$ is considered successful under Leximax and unsuccessful under Leximin. This pattern explains the approximate 75% difference in the values of NK index.

We can aggregate the results in the Table 3. The selection of the number of agents represents the general tendency. The rules with exactly the same NK indices are presented in parenthesis, like (MUS-MWSS-F-C3-2SS-3SS).

Table 3. Minimal manipulating majoritarian rules.

Agents	4	6	8-30	40-60	70-100
Leximin	(MUS-MWSS-F-C3-2SS-3SS) US2	1SS	MDS, 1SS	MDS, 1SS	(MUS-MWSS-F-C3-2SS-3SS) MDS, US2, C2, 1SS
Leximax	MDS	MDS	MDS	MDS	MDS
PWorst	C1	C1	MDS, US2, 1SS	MDS, US2, 1SS	(MUS-MWSS-F-C3-2SS-3SS) MDS, US2, C1, 1SS
PBest	US2	US2	US2	MDS	MDS, 1SS

While we cannot point out the best rule for all cases, we can find some specific rules, which are the least manipulable for each case. The best rules for small number of voters and most cases are Uncovered set II, Minimal Dominant Set and for some cases Copeland I. For large number of voters the best rules are Minimal Dominant Set and 1-Stable set. In this way, the obtained results differ from the results for individual manipulation [21], where Minimal dominant set was not among least manipulable rules.

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