Dollarization and the Inflation Threshold*

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Abstract

This paper analyzes the relationship among inflation, dollarization, financial intermediation, and real activity. Empirical evidence suggests non-linearity in the effects of inflation on financial intermediation and real activity, i.e., the existence of an inflation threshold. Evidence also suggests that one way in which inflation affects financial intermediation in high inflation economies is through the substitution of dollars “under the mattress” for savings in domestic banks. Our model captures these empirical regularities. Inflation and real activity are positively related at low levels of inflation. When the inflation rate exceeds certain threshold, however, agents substitute in their portfolios dollars (a non-productive asset of constant value that also provides liquidity services) for deposits issued by domestic banks. This substitution reduces the scale of financial intermediation and the capital investment in the economy. As a consequence, at high levels of inflation the levels of capital stock and output become negatively related to the inflation rate.

KEYWORDS: Dollarization; Inflation; Financial Intermediation
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1 Introduction

The relationship between inflation and economic activity is a classical problem studied by economists. Empirical evidence indicates that a moderate increase in inflation in industrialized countries, where inflation has historically been relatively contained, is either uncorrelated to economic activity or it is slightly positively correlated to it (Bullard and Keating, 1995; King and Watson, 1997; Ahmed and Rogers, 2002; Crosby and Otto, 2000). On the other hand, there is evidence that inflation is negatively correlated with economic activity in developing countries, where inflation has historically been higher than in industrialized economies. Empirical analyses conclude that there exists an inflation threshold beyond which output and inflation are negatively related. The evidence of an inflation threshold is even more striking in case of the effects of inflation on growth. Sarel (1996), Bruno and Easterly (1998), Ghosh and Phillips (1998), Khan and Senhadji (2001), among others, confirm that inflation has no (or almost no) effect on output growth rates when the inflation rate is low, but its impact becomes negative and highly significant when the inflation rate exceeds a certain threshold (estimated to be within the range 10 % - 40 % p.a.).

An inflation threshold has been also identified in the relationship between inflation and financial markets. Boyd, Levine, and Smith, (2001) document a negative correlation between inflation and both the amount of bank lending and stock market activity. In addition, they find that real rates of return are negatively related to inflation in high-inflation countries. Perhaps more importantly for our analysis, they demonstrate that empirically the relation between financial-market activity and inflation strongly supports the presence of a threshold of inflation beyond which financial market activity begins to deteriorate very rapidly. The significance of these findings is clearer when considered in conjunction with the evidence on the link between economic development and the development of financial markets. It has been amply documented that there is a positive correlation between the wealth of nations and their ability to transfer effectively resources from lenders to borrowers, which is the main function of financial markets (King and Levine, 1993; Levine, 1997; Levine et al., 2000). In sum, the evidence establishes a potential, important channel
of transmission from inflation to financial market activity and output.

What is the mechanism of transmission from inflation to financial system and real activity? Evidence indicates that high-inflation economies get “dollarized:” economic agents in these economies start using stable foreign currency, such as US dollars, as a store of value and as a medium of exchange.\(^1\) It has long been recognized that keeping US dollars “under the mattress” can be seen as an internal capital flight which deprives the domestic financial system of loanable funds, and limits the external financing of the real sector (Sahay and Vegh, 1996; Savastano, 1996). However, the substitution between liabilities of the domestic financial system and dollars has not been modeled explicitly.\(^2\)

This study fills this gap in the literature. Our paper constitutes the first theoretical attempt to incorporate the analysis of dollarization into a study of the impact of inflation on financial intermediation and real activity in relation to the observed non-linearities among those variables. Our baseline model exhibits a positive relationship between inflation and output level at low levels of inflation. When inflation exceeds a certain threshold, however, agents divert their savings towards dollars (a non-productive asset of constant value that also provides liquidity services) and away from deposits issued by domestic banks. This financial disintermediation reduces the capital investment in the economy. Hence, at high levels of inflation, the level of the capital stock and output become negatively related to the inflation rate. The baseline framework is set up to easily deliver a positive relationship between inflation and output, as all the seigniorage revenue is rebated to the agents and is saved by them, and productive capital is the highest-return asset available. However, we show that, even in such a framework, there is a threshold of domestic inflation that reverses the inflation-output relationship.

We employ an overlapping generations model in which spatial separation and limited commu-

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\(^1\)We follow Calvo (1996, Chapter 8, p. 153), who defines dollarization as “the use of foreign currency in any of its three functions: unit of account, means of exchange, and, in particular, store of value.” Dollarization experience of various Latin American and transition economies has been described in Feige (2003), Guidotti and Rodriguez (1992), Havrylyshyn and Beddies (2003), Kamin and Ericsson (2003), Sahay and Vegh (1996), Savastano (1996), Van Aarle and Budina (1995).

\(^2\)To the best of our knowledge the only such a model is Duffy, Nikitin and Smith (2005). This paper is reviewed below.
nication generate a transactions role for fiat money (see Townsend, 1987; Champ, et al., 1996; Schreft and Smith, 1997, 1998; and Espinosa-Vega and Yip, 1999). We assume the presence of two currencies: a domestic, possibly high inflation currency with a negative net real rate of return, and a foreign currency, referred to as dollar, which yields a zero net rate of return. Banks emerge endogenously to insure depositors against random liquidity shocks. Unlike private agents, banks are not allowed to hold dollars.\(^3\) Our framework is the first to consider two currencies in a spatial-separation-and-limited-communication framework that preserves the natural asymmetry between dollars and domestic currency.\(^4\) On one hand, dollars are superior as a store of value, and hence they can compete with other assets. On the other hand, the domestic currency is “protected” with the legal restriction (banks cannot hold dollars), so that it cannot be replaced with dollars altogether.

Agents decide how to invest their income in the available assets (capital, domestic currency, dollars, and demand deposits). Banks decide how to allocate deposits between capital and domestic currency, with the expectation that a fraction of deposits will be withdrawn early, before capital investments mature, by depositors who face a relocation shock and need money. In this environment, banks have an advantage over individuals, as they can exploit the risk sharing possibilities offered by large numbers of depositors subject to idiosyncratic shocks. Therefore, at low level of inflation, even if the domestic currency is dominated in rate of return by dollars, agents deposit all their income in the banking sector and do not use dollars.\(^5\) However, when inflation exceeds a certain level, the risk sharing benefits afforded by the banking sector are no longer sufficient to induce individuals to deposit all their income in banks. They begin to hold part of their liquidity in the form of dollars, thereby affecting the total amount of resources available for capital formation.

To the best of our knowledge, Duffy, Nikitin and Smith (2005) is the only theoretical paper

\(^3\)The assumption that banks cannot hold foreign currency is a simplifying one, and it is not necessary for the results to obtain as long as banks are not allowed to become completely dollarized, that is, if they face a legal restriction that they hold domestic currency as well. This assumption is consistent with empirical evidence. Governments often limit exposure of the domestic banks to foreign assets.

\(^4\)The only other model with multiple currencies in a spatial-separation-and-limited-communication framework is Machicado (2005) who just assumes that an exogenous fraction of the liquidity demand is in dollars.

\(^5\)Intuitively, the risk sharing benefits overcome the costs of investing in a relatively inferior source of liquidity when inflation is relatively low.
that analyzes substitution between dollars and productive capital. It addresses the dollarization hysteresis paradox; that is, it provides an explanation for why economies may remain dollarized after a successful inflation stabilization. However, Duffy, Nikitin and Smith (2005) analyze dollarization in the sense of asset substitution only, and in their model dollars do not provide liquidity services but are just a store of value.

Our work complements that of Azariadis and Smith (1996) and Choi, Smith and Boyd (1996), who study general equilibrium models in which disintermediation is responsible for the non-linearity of the relationship between inflation and output. However, the economic dimension along which inflation generates detrimental effects in their models is different from ours. In their single-currency models, inflation reduces real returns to savings and exacerbates an informational friction (adverse selection) afflicting the financial system. The severity of the financial market friction is endogenous and varies positively with the rate of inflation. Specifically, higher rates of inflation trigger credit rationing. This rationing limits the availability of investment capital and reduces real activity when inflation exceeds a certain threshold. In our framework, on the other hand, the non-linear relationship between inflation and output is due to a shift of resources into a non-productive liquid asset. Hence, the illiquidity of capital, the stochastic nature of the demand for liquidity, and the properties of optimal risk sharing mechanism are responsible for the threshold level of inflation. The two phenomena, adverse selection and dollarization, are both likely to contribute to the observed non-linear relation between inflation and output observed in reality. This raises an interesting question, which is whether it would be possible to assess the relative importance of the phenomena. The empirical question is a difficult one, as it is difficult to observe directly the amount of information distortion due to adverse selection. However, it is possible to get an accurate estimate of the dollarization of an economy and its relation to inflation and economic performance. One could then residually assess the incidence of the distortion of information in financial markets due to high inflation and its effect on the economy.

Huybens and Smith (1999) also study inflation, financial markets and real activity. They construct a single-currency monetary model in which the negative correlation between financial
intermediation and inflation explains the lower level of real activity under inflation. Two steady state equilibria are identified, one with a relative low and the other with a relative high capital stock. In the high activity state, inflation and real activity are negatively correlated, and real activity and the volume of financial market activity are positively correlated. In addition, they show that the high capital stock steady state may be a saddle for low rates of money growth, and that once the rate of money creation and inflation exceed some critical level, the high activity steady state becomes a source, and so it becomes unapproachable. Therefore, according to this condition, they identify an inflation threshold. Their framework, however, does not allow for dollarization.

The paper proceeds as follows. In the next section we describe the economic environment of the baseline model. In section 3 we prove our main result. Section 4 offers some concluding remarks.

2 The Model

We consider an economy populated by an infinite sequence of two-period lived, overlapping generations of agents. In each period \( t = 1, 2, 3, \ldots \), a continuum of identical agents of unit measure is born in each of two identical locations, which we will refer to as “islands.” Agents derive utility only from old age consumption, and their preferences are described by the function \( U(c) = \log c \). This simple preference structure makes young agents complete savers. The initial old generation is endowed with a stock of fiat money \( M_{-1} \).

Every agent is endowed with one unit of labor in his first period of life and nothing in the second period. Agents supply labor inelastically when young to earn the current real wage \( w \), and are retired when old. In addition to wage income, young agents may receive a transfer of currency from the government.

At the end of the first period of life, each young agent faces a probability \( \pi \) of being relocated to the other island. This idiosyncratic shock implies that at the end of each period a fraction \( \pi \) of the young population is relocated to the other island; therefore, there is no aggregate uncertainty in the economy. We assume that spatial separation and limited communication between islands prevent goods and capital from being transported, and privately issued liabilities from being recognized in
the island where they were not issued. Relocated agents must carry fiat currency, local or foreign, to the other island to purchase consumption. This setup follows, among others, Townsend (1987), Champ et al. (1996), Schreft and Smith (1997, 1998) and Espinosa-Vega and Yip (1999), where money plays a transactions role by overcoming the friction represented by limited communication and spatial separation between islands. The relevant implication of this construction is that money can co-exist with other assets and be dominated in rate of return in equilibrium.

2.1 Production

In each location, perfectly competitive firms produce a single perishable good using physical capital $K_t$ and labor $L_t$. We assume that the production function is Cobb-Douglas:

$$Y_t = AK_t^\alpha L_t^{1-\alpha}, \quad (1)$$

where $A$ and $\alpha$ are known constants, with $0 < \alpha < 1$. For simplicity, we assume full depreciation of capital after production occurs. Because factor markets are perfectly competitive, factor pricing relationships for the intensive production function $y_t \equiv \frac{Y_t}{L_t} = A \left( \frac{K_t}{L_t} \right)^\alpha \equiv Ak_t^\alpha$ are given by

$$\rho_t = A\alpha k_t^{\alpha-1}, \quad (2)$$

and

$$w_t = (1 - \alpha) Ak_t^\alpha, \quad (3)$$

where $\rho$ represents the gross return on capital. Note that in each island the aggregate capital is equal to the capital stock per worker, because $L_t = 1$ in each period.

2.2 Monetary Policy

The domestic monetary authority follows a simple money growth rule by setting the gross money growth rate $\sigma \geq 1$ so that

$$M_{t+1} = \sigma M_t. \quad (4)$$

\footnote{Note that because the islands are identical, the population flows are symmetric and the population in both islands is constant over time.}
Newly printed bills are given to young agents as lump-sum transfers. In other words, seigniorage revenue is rebated to young agents. Real seignorage revenue is

\[ g_t = \frac{M_t - M_{t-1}}{P_t} = \left( 1 - \frac{1}{\sigma} \right) m_t, \]

where \( m_t \equiv \frac{M_t}{P_t} \) indicates time-\( t \) real balances.

2.3 Dollars

Besides capital and domestic currency, young agents have access to a third asset, called “dollars.” Dollars are an unproductive asset of constant real value,\(^7\) which agents can buy or sell for consumption goods. Holding dollars is identical to storing a consumption good (if storage were possible), with one important exception: like the domestic currency and unlike any other asset or good, dollars can be carried by agents relocated to another island.\(^8\)

2.4 Financial Intermediation

As in Champ et al. (1996) and Schreft and Smith (1997, 1998), in this economy competitive banks arise endogenously to provide insurance against liquidity (relocation) risk. A common interpretation of banks in this environment is as coalitions of identical agents whose objective is to maximize the expected utility of a representative member. Banks act competitively in the sense that (i) they take rates of return on capital, money, and dollars as given; (ii) they choose rates of return on deposits to maximize the expected utility of the representative depositor; and (iii) they earn zero profits. Both banks and individuals must take portfolio allocation decisions before uncertainty is realized, but banks have a simple advantage over individuals. Because a (known) fraction of agents \( \pi \) is relocated every period, they can pool risk. In this particular setting, banks will know that a fraction \( \pi \) of their depositors will withdraw at the end of the first period of their life and will demand fiat currency.

\(^7\)We assume that the world (US) inflation is zero, and that the purchasing power parity holds.

\(^8\)The label “dollar” is used to indicate a liquid, safe, and universally accepted asset, and of course other interpretations would be possible, for example gold.
We assume that banks cannot hold dollars in their portfolios, but only domestic currency and capital investment. This assumption is essential to ensure positive demand for domestic currency when there is inflation in the domestic economy. Although it would be possible to partially relax this assumption, some restriction on the portfolio of banks would be necessary to insure a positive demand for the domestic currency. The assumption, however, fits the practice of many countries, in which governments protect their seigniorage base by restricting the use of foreign currency or foreign currency assets by domestic intermediaries. We also assume that young agents face no such legal restriction.

2.5 Overview of the Model and Timing of Events

We begin by providing an overview of the main features of the model through a time-line of events to illustrate what economic forces are at work before solving the model formally.

At the beginning of period $t$, a new generation is born. Young agents supply labor to firms, production takes place, and capital and labor are paid their marginal products. In addition, the government may make a transfer of newly printed currency to young agents. At this point, young agents, old agents, and banks, are free to trade with one another. All trade must occur locally, within each location. Note that some old agents, who were relocated from the other island, will supply money while others, who remained in the same location, withdraw from the bank to consume. At the end of the trading process, old agents will consume while all money and the remaining supply of goods will be in the hand of young agents and banks.

Next, young agents and banks make their investment decisions. Young agents decide how much to deposit in the bank and how much to invest in dollars. It is important to note that, for the cases of interest in our analysis, it is never optimal for individuals to invest directly in a portfolio of capital and local currency: in such investments banks have an advantage because they pool risk across a large population of depositors. One way to see this clearly is to note that any individual direct portfolio can be replicated by banks, while individuals cannot replicate all the return profiles of bank deposits. Moreover, any direct investment in capital would be lost in case of relocation.
Hence, the portfolio allocation decision of individuals is limited to bank deposits and dollars. Banks, on the other hand, are prevented from holding dollars, and they have to allocate deposits between physical capital and domestic currency. Banks need domestic currency to provide for relocated depositors who withdraw at the end of the current period.

After portfolio allocations take place, all trading opportunities cease. At this point, uncertainty is realized. Agents who are hit by the relocation shock go to their bank, withdraw, and move to the other island carrying currency and possibly dollars. The next period begins and the cycle is repeated indefinitely.

3 Solution of the Model

In this section we prove the main results of the paper: under low inflation, dollars do not circulate, and the standard Mundell-Tobin effect holds, in that there is a positive equilibrium relation between inflation and the capital stock. As the inflation rate increases, however, individuals stop depositing their entire income and start holding dollars directly. In addition, when this event occurs the relationship between inflation and capital stock is reversed: an increase in the rate of inflation decreases the equilibrium capital stock.

Formally, the analysis of the equilibrium proceeds as follows. First, we establish two conjectures about the relationship between the demand for dollars by young agents and the inflation rate: 1) When the money supply growth rate (and therefore, inflation rate) is low, young agents do not use dollars. As a consequence, they deposit all their income with competitive banks.9 2) When there is high inflation, young agents split their income between dollars and bank deposits.

Under each conjecture, we analyze the steady state equilibrium, define its characteristics, and

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9A simple way to gain intuition about the economy with low inflation is to imagine the formally equivalent borderline case in which there is no inflation on the domestic currency. Then, domestic currency and dollars are perfect substitutes, and the economy is like one in which dollars do not exist. If young agents held dollars, they would forfeit a higher return on productive capital.

Now, think of an economy where there is a very low rate of inflation. Even though the return on dollars is now marginally higher than the return on domestic currency, and so relocated agents would be better off with dollars, in expected utility terms agents are better off depositing all their wage income with banks and gain the return on capital. This of course as long as the difference between the return on dollars and on the domestic currency is not large.
study the relationship between money supply growth rate and per capita capital stock. Then, we verify the existence of a unique threshold of money supply growth rate, denote by $\sigma^*$, beyond which young agents demand dollars: when $\sigma > \sigma^*$, the demand for dollars is positive; otherwise, dollars do not circulate in the domestic economy. All the analysis is limited to steady-state equilibria.

3.1 The Basic Model: Equilibrium under Low Inflation

Under low inflation, agents deposit all their first-period income with competitive banks. In this case, the economy is identical to one in which dollars do not exist, as the demand for dollars is always zero. In other words, in this section we consider economies for which the gross return on domestic currency is not much below unity, while the marginal product of capital is strictly above unity. Formally,

$$\frac{1}{\sigma^*} \leq \frac{1}{\sigma} \leq 1 < \rho,$$

where $\sigma^*$ is the threshold money growth rate defined above, the existence of which will be verified below, and $\rho$ is the marginal product of capital.

Individuals in such an economy solve a very simple problem, and deposit all their income in a bank which, as mentioned above, can be thought of as a coalition of agents which maximizes the expected utility of its representative member. Hence, a bank maximizes the expected utility of the representative young agent given by:

$$U_t = \pi \log c^{rl}_{t+1} + (1 - \pi) \log c^{st}_{t+1} = \pi \log (d_t R^{rl}_t) + (1 - \pi) \log (d_t R^{st}_t).$$

(6)

Here, $c^{rl}_{t+1}$ denotes the consumption of the agent if he is relocated, and $c^{st}_{t+1}$ the consumption of the same agent if he does not change location; $R^{st}_t$ represents the rate of return promised by the bank to “non-movers,” while $R^{rl}_t$ is the rate of return promised by the bank to “movers;” finally, $d_t$ is the per capita deposit with the bank, which in this case is the entire individual income, $w + g$, real wage plus government transfer.

Let the fraction of bank deposits invested in physical capital be $q_t$. The fraction held in domestic currency then will be $1 - q_t$. In addition, let $R^{dn}_t$ represent the rate on return on domestic currency,
which in a steady-state equilibrium is simply equal to the reciprocal of the money growth rate $\frac{1}{\sigma}$.

A bank maximizes $U_t$ subject to two resource constraints. First, a bank knows that a fraction $\pi$ of depositors will withdraw at the end of the period, and to each depositor, per unit of deposit, the bank has promised a return of $R^\text{rl}_t$. Therefore, after the relocation shock is realized a bank finances with money returns to depositors for $\pi R^\text{rl}_t$. Because the fraction of bank’s assets held in currency is $(1 - q_t)$ and the return on money is $R^m_t$, solvency of the bank requires that

$$\pi R^\text{rl}_t = (1 - q_t) R^m_t,$$

which can alternatively be written as

$$R^\text{rl}_t = \frac{(1 - q_t) R^m_t}{\pi}.$$  \hfill (7)

The second constraint simply states that agents who are not relocated, of whom there are $1 - \pi$, each promised a gross return of $R^\text{st}_t$ per unit deposited, must be paid using the return on capital. The implication is that

$$(1 - \pi) R^\text{st}_t = q_t \rho_t,$$

or equivalently

$$R^\text{st}_t = \frac{q_t \rho_t}{1 - \pi}.$$  \hfill (8)

Banks maximize $U_t$ with respect to $q_t$, $R^\text{st}_t$ and $R^\text{rl}_t$, subject to (7) and (8). Note that there is no aggregate uncertainty in the economy, therefore banks do not face uncertainty about the individual relocation probability $\pi$. Substituting (7) and (8) into the objective function of the banks $U_t$ and deleting the unnecessary constants in the maximization problem by exploiting the properties of logarithms, we obtain this equivalent simple problem for the bank:

$$\max_{q_t} \pi \log(1 - q_t) + (1 - \pi) \log q_t.$$

It is readily verified that the solution to the bank’s optimization problem sets $q_t = 1 - \pi$, or $1 - q_t = \pi$. This solution means that a bank will invest in currency a share of deposits equal to the share of population which gets transferred to a different location. Likewise, a bank will invest
in capital a share of deposits equal to the share of population which is not relocated. Notice that this solution implies that \( c^t_{t+1} = d_t R^t_{t} \), and \( c^t_{t+1} = d_t \rho_t \) in equilibrium. Hence, agents are promised a rate of return which cannot be achieved by agents individually: relocated agents get the return they would have received if they had known that they were going to be relocated and held only currency, while non-movers get the return they would have received if they had known that they were not going to be relocated and invested only in capital. The particular form of the solution depends on the level of risk aversion associated with log preferences, and the fact that deposits are the dominant investment asset. We will show that with high inflation the portfolio allocation of the bank will be different.

Now, note that total per capita deposits are the sum of wage income and the seigniorage transfer payment:

\[
d_t = w_t + g_t = w_t + (1 - \frac{1}{\sigma})m_t = w_t + (1 - \frac{1}{\sigma})(1 - q_t)d_t.
\]

Solving the last equation for \( d_t \) and taking into account (3), we get

\[
d_t = \frac{(1 - \alpha)[Ak^t_{t}]}{1 - (1 - \frac{1}{\sigma})\pi}.
\]

Because capital formation is equal to the fraction of deposits invested in capital, we have that \( k_{t+1} = (1 - \pi)d_t \), and the law of motion for the per capita capital stock is given by

\[
k_{t+1} = \frac{(1 - \pi)(1 - \alpha)[Ak^t_{t}]}{1 - (1 - \frac{1}{\sigma})\pi}.
\]

Define \( \psi \equiv \frac{(1-\pi)(1-\alpha)\Delta}{1-(1-\frac{1}{\sigma})\pi} \). For a given value of \( \sigma \), \( \psi \) is a positive constant. Hence the dynamic properties of equation (11) are the same as the properties of the standard Diamond (1965) model. Clearly, in this case the steady-state equilibrium will not exist for all parameter values, as in steady state the return on capital must be above unity. However, it is easy to show examples of parameter configurations for which the steady-state equilibrium exists.

Lemma 1 summarizes the characteristics of the steady state equilibrium, and Proposition 1 establishes the main comparative statics result.
Lemma 1. Assume that $A_0\psi^{-1} > 1$. The dynamic equation (11) has a unique positive steady state, $k = \psi^{1/(1-\alpha)}$. This steady state is stable.

Proposition 1. If young agents do not demand dollars as a store of value, an increase in the steady-state money supply growth rate raises the steady-state per capita capital stock.

The statements can be easily verified by direct computation. Intuitively, an increase in the inflation rate raises the per capita seigniorage revenue, and it increases the capital investment, because the investment is a fixed share of the deposit of the young, $d_t$.

3.2 Equilibrium under High Inflation

In this section and the next we illustrate the main result of the paper. As inflation increases beyond a certain threshold, it leads to disintermediation, direct holdings of dollars, and lower capital formation.

Relative to the case studied in the previous section, the return on money is lower, and therefore deposits become less attractive relative to dollars, even if the return on capital is higher. This is because relocated agents do not have use for capital. When deposits are less attractive, agents split their first-period income between the deposit in a bank and dollar holdings. Here we solve the model under this assumption, and in the next section confirm that indeed there is a unique threshold of inflation beyond which the solution studied in this section applies, and below which the solution of the previous section applies. The individual budget constraints are now given by

$$c_t^s = (w_t + g_t - d_t) + d_t R_t^d,$$

for an agent who is not hit by the liquidity shock, and, for a relocated agent,

$$c_t^r = (w_t + g_t - d_t) + d_t R_t^d,$$

where $w_t + g_t - d_t$ is the dollar holding of a representative agent. Recall that the gross real return on dollar holdings is set to unity.
The expected utility of the representative agent is now given by

\[ U_t = \pi \log \left( (\omega_t + \gamma_t - d_t) + d_t R_{vl}^t \right) + (1 - \pi) \log \left( (\omega_t + \gamma_t - d_t) + d_t R_{vl}^t \right). \]  

(Banks and individuals maximize the same expected utility. Banks decide how to allocate deposits between capital and domestic currency, and take as given rates of return on all assets as well as their deposits. Individuals decide how much of their income to invest in dollars and how much to deposit in a bank, again taking as given all rates of return in the economy, including bank deposits. Formally, a bank maximizes (14) with respect to \( q_t \) subject to the same budget constraints of the previous sections, equations (7) and (8), taking \( \omega_t, \gamma_t, d_t, \) and \( k_t \) as given. The representative agent maximizes the same objective function with respect to \( d_t \) taking \( k_t, q_t, \omega_t \) and \( \gamma_t \) as given.

Substituting (7) and (8) into (14), differentiating it with respect to \( q_t \), and taking into account (12) and (13), we get the first-order optimality condition of the competitive bank:

\[ \frac{\partial U_t}{\partial q_t} = \pi \frac{d_t R_{vl}^t}{\omega_t} \left[ -\frac{d_t R_{vl}^m}{\pi} \right] + (1 - \pi) d_t \rho_t = 0. \]  

(15)

In steady state this condition simplifies to

\[ \frac{R_{vl}^m}{c_{vl}^m} = \frac{\rho}{c_{vl}^m}. \]  

(16)

Substituting (7) and (8) into (14), differentiating it with respect to \( d_t \), and taking into account (12) and (13), we obtain the first-order optimality condition of the representative agent:

\[ \frac{\partial U_t}{\partial d_t} = \pi \frac{d_t R_{vl}^m}{\omega_t} \left[ \frac{R_{vl}^m (1 - q_t)}{\pi} - 1 \right] + (1 - \pi) \rho_t q_t = 0, \]  

(17)

which simplifies in the steady state to

\[ \frac{\pi - R_{vl}^m (1 - q)}{c_{vl}^m} = \frac{\rho q - (1 - \pi)}{c_{vl}^m}. \]  

(18)

In addition to the individual’s and the bank’s optimality conditions, in equilibrium the markets for money, capital, and dollars have to clear. The dollar market clears residually by virtue of the assumption that there is a perfectly elastic supply of dollars and that the dollar value is fixed.
relative to the consumption good. The market clearing conditions for domestic currency and capital
are given, in the steady state, by

$$m = d(1 - q), \quad (19)$$

and

$$k = dq. \quad (20)$$

These conditions simply state that the quantity of money in the economy has to equal the fractions
of banks’ deposits held in cash, while capital formation equals the fraction of banks’ deposit invested
in capital.

The steady-state pricing equilibrium relations determine rates of return on money, capital and
labor:

$$R^m = \frac{1}{\sigma}; \quad (21)$$

$$\rho = \alpha A k^{1-\alpha}; \quad (22)$$

$$w = (1 - \alpha) \left[ A k^{\alpha} \right]. \quad (23)$$

Finally, the budget constraints of government, banks, and individuals determine the remaining
steady-state equilibrium variables:

$$g = \left( 1 - \frac{1}{\sigma} \right) m \quad (24)$$

determines the real value of the transfer. Equilibrium returns on bank deposits are given by

$$R^{st} = \frac{q\rho}{1 - \pi}, \quad (25)$$

and

$$R^{st} = \frac{(1 - q)R^m}{\pi}. \quad (26)$$

Steady state consumption is

$$c^{st} = (w + g - d) + dR^{st}; \quad (27)$$
and

\[ c^{rl} = (w + g - d) + dR^{rl}. \] (28)

The competitive equilibrium of the economy in steady state is fully described by equations (16) and (18)-(28). Together, these twelve equations determine the equilibrium value of the twelve endogenous variables \( c^m, c^{st}, R^{st}, R^{rl}, R^m, \rho, q, k, w, g, m \) and \( d \).

Combining (16) and (18) yields

\[ \rho(R^m - \pi) = R^m(1 - \pi). \] (29)

Considering equation (22), equation (29) can be rewritten as

\[ \rho = \frac{1 - \pi}{1 - \pi \sigma} = \alpha Ak^{\alpha - 1}. \] (30)

The interpretation of (30) is straightforward: the levels of capital stock and output are negatively related to the steady-state inflation rate. Hence, an increase in \( \sigma \) raises the return on capital and reduces the capital stock. Proposition 2 summarizes the main finding of the subsection.

**Proposition 2.** If young agents use dollars as a store of value, then the steady-state level of per capita capital stock depends negatively on the growth rate of the nominal money supply.

### 3.3 The Inflation Threshold

To complete the steady-state analysis, we need to show that there exists a unique threshold value of the money supply growth rate, \( \sigma^* \), and associated rate of inflation, such that the demand for dollars by young agents is positive for money growth rates higher than \( \sigma^* \), but there is no demand for dollars for money growth rates lower than \( \sigma^* \). Thus, in this section we show that the analysis of section 3.1 applies when the money growth rate is below the threshold \( \sigma^* \), while the analysis of section 3.2 applies when the money growth rate exceeds the threshold \( \sigma^* \).

We proceed in four steps. First, we show that there exists a unique value of \( \sigma \) such that, under the restriction that young agents deposit all their income \( w + g = d \) with banks, the system (16), (18)-(28) has a unique solution. In other words, young agents deposit all their income with
competitive banks at this value of the money growth rate, but a marginal change in the money growth rate and associated inflation rate may generate a positive demand for dollars. Second, we show that at this level of the money growth rate, the share of domestic currency in banks’ portfolios is \( \pi \), while the share of productive capital is \((1 - \pi)\). The first and second steps together prove that the solution that we obtain by imposing the restriction \( w + g = d \) in the system represented by equations (16), (18)-(28) is indeed the same (unique) steady-state equilibrium obtained in section 3.1. Third, we show that dollar holdings depend positively on the money growth rate at this critical level if and only if the share of bank deposits invested in capital depends positively on \( \sigma \). Finally, we verify that at this threshold, the capital share indeed positively depends on the money growth rate; that is, banks increase the share of their portfolio invested in physical capital as individuals increase their holding of dollars. Together, the four steps show that the demand for dollars is positive at high levels of inflation, while at lower levels of inflation agents deposit all their wage income with competitive banks, and there is no demand for dollars. Therefore the conjectures made in sections 3.1 and 3.2 hold.

To simplify the presentation and provide a precise summary of the discussion above, we state our results in a sequence of lemmas, and relegate their proofs to the appendix. Lemma 2 below establishes the uniqueness of the money growth threshold.

**Lemma 2.** Assume that \( \frac{\alpha}{(1 - \alpha)(1 - \pi)} > 1 \). Then, under the restriction \( w + g = d \), there exists a unique value of \( \sigma \), denoted \( \sigma^* \), for which a solution to the system (16), (18)-(28) exists, and this solution is unique.

Lemma 3 summarizes the main result on banks’ portfolio allocation at the money growth threshold. It says that the shares of capital and currency at the threshold are exactly the same as they are in the low-inflation equilibrium. However, the result of Lemma 3 is not trivial. Unlike in section 3.1, we derive this result while allowing for dollar holdings: young agents have the opportunity to substitute bank deposits with dollars, but choose to hold deposits only.

**Lemma 3.** Suppose that \( \sigma = \sigma^* \). Then the share of productive capital in competitive banks’
portfolios is $q = 1 - \pi$ and the share of domestic currency is $1 - q = \pi$.

Lemma 4 shows that at the threshold, dollar holdings depend positively on the money growth rate if and only if the share of bank deposits invested in capital depends positively on $\sigma$.

**Lemma 4.** Define dollar holdings by $\delta \equiv w + g - d$. Suppose that $\sigma = \sigma^*$. Then the inequality $\frac{\partial \delta}{\partial \mu} < 0$ holds if and only if $\frac{\partial q}{\partial \mu} < 0$.

Lemma 5 verifies that, at the threshold, the capital share depends positively on the money growth rate.

**Lemma 5.** Suppose that $\sigma = \sigma^*$. Then $\frac{\partial q}{\partial \mu} < 0$.

When $\sigma > \sigma^*$ the share of currency in the bank portfolios is smaller than $\pi$. Intuitively, banks take into account that relocated agents hold a part of their wealth in dollars, which are transportable, reduce the share $1 - q$ of domestic currency that yields low return, and increase the share of capital, $q$.

Thus, we have shown that at the threshold level of inflation an increase in the rate of inflation (or reduction in rate of return on domestic currency) lowers the currency share in the portfolio of banks, that is $\frac{\partial q}{\partial \sigma} > 0$. Moreover, a reduction in the return on domestic currency from the threshold level makes deposits less attractive, and therefore an increase in inflation increases direct dollar holdings. Formally, $\frac{\partial \delta}{\partial \mu} < 0$, or $\frac{\partial \delta}{\partial \sigma} > 0$. Proposition 3 summarizes the main findings of this subsection.

**Proposition 3.** Assume that $\frac{\alpha}{(1-\alpha)(1-\pi)} > 1$. Then there exists a unique value of the money supply growth rate, $\sigma^*$, such that $\delta = 0$ for $\sigma \leq \sigma^*$, but $\delta > 0$ for $\sigma > \sigma^*$.

In words, the demand for dollars $\delta$ is positive when the rate of money growth exceeds $\sigma^*$, but not when $\sigma \leq \sigma^*$. Intuitively, when the money growth rate (and hence the inflation rate) exceed that threshold $\sigma^*$, the existence of which we just proved, agents divert their savings towards dollars and away from deposits issued by domestic banks. This financial disintermediation reduces the capital investment in the economy, even though banks increase the share of deposits invested in physical capital. Hence, at high levels of inflation, the levels of capital stock and output become
negatively related to the inflation rate.

We close this section by noting that the results are in large part robust to a change in the assumption that seignorage revenue is rebated to individuals. If, instead, we assume that the government uses seignorage to finance its own consumption, all the main results of the model hold with one important exception. When inflation is low and there is no demand for dollars, a change in the money growth rate does not affect capital accumulation and output. An increase in the rate of inflation in this case amounts to a simple transfer of resources from individuals to the government, and the Mundell-Tobin effect disappears. At high levels of inflation, however, an increase in the rate of inflation lowers the equilibrium capital stock.

4 Conclusions

We analyze the relationship among inflation, dollarization, financial intermediation, and real activity. We incorporate the analysis of dollarization into a study of the impact of inflation on financial intermediation and real activity.

We show that, at low levels of inflation, there is a positive relationship between inflation and the level of real activity. When the inflation rate exceeds certain threshold, agents substitute in their portfolios dollars (a non-productive liquid asset of constant value) for deposits issued by domestic banks. This substitution reduces the scale of financial intermediation and the capital investment in the economy. As a consequence, at high levels of inflation the levels of capital stock and output become negatively related to the inflation rate.

The model that we studied can be extended in several directions. First, we have concentrated on steady-state analysis, but an interesting question concerns the effect of inflation on the dynamics of the economy. A related question is the study of an endogenous growth version of the model, to analyze how changes in the inflation rate affect the growth rate of the economy through the substitution between dollars and bank deposits. Another interesting exercise would be to partially relax the constraint on the asset side of the balance sheet of banks, and allow them to hold dollars. This would introduce the possibility of varying regulation of the banking sector, for example in terms
of reserve requirements and suspensions of payments, and evaluating its consequences. Finally, we have assumed an infinitely high transportation cost for capital, but different assumptions would be possible. Allowing the transportation of capital across island at a cost would introduce a new dimension to the analysis and permit to evaluate how the liquidity of capital affects the threshold of inflation beyond which inflation generates a negative impact on economic activity.

References


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Appendix

Proofs

Proof of Lemma 3: It is convenient to find the threshold value of $R^m$ rather than $\sigma$.

If

$$w + g = d, \quad (31)$$

equations (27) and (28) can be rewritten as

$$c^m = \frac{dR^m(1 - q)}{\pi} \quad (32)$$

and

$$c^{st} = \frac{d\rho q}{1 - \pi} \quad (33)$$

The system (16), (18)-(24), (31)-(33) is the system of 11 endogenous variables: $c^m, c^{st}, \rho, q, k, w, g, m, d, R^m$ and $\sigma$.\(^{10}\) Combining (16), (18), (32) and (33) we get

$$q = 1 - \pi. \quad (34)$$

At the threshold, the optimal share of deposits held in currency is the same as when the inflation is low and dollars are not used, i.e. it is equal to the share of relocated agents in the economy.

Taking into account (34) and successively eliminating $\sigma, c^m, c^{st}, \rho, q, w, g, m$ and $d$ from the system, we obtain a system of just two equations with 2 unknowns, $k$ and $R^m$

$$k^{1-\alpha} = \frac{A(1 - \alpha)(1 - \pi)}{1 - \frac{\pi(1 - R^m)}{1 - \pi}} \quad (35)$$

$$\pi \left( \frac{1}{R^m} - 1 \right) = (1 - \pi) \left( 1 - \frac{1}{A\alpha k^{\alpha - 1}} \right). \quad (36)$$

The first equation describes a negative relationship between $k$ and $R^m$. The second equation describes a positive one. Therefore, the system (35)-(36) can have at most one solution. Figure\(^{10}\)Now $\sigma$ becomes one of the endogenous variables, because we search for its threshold value so that equation (31) is satisfied.

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1 shows equations (35) and (36) graphically in the \((R^m, \rho)\) plane.\(^{11}\) The downward-sloping curve
represents equation (36). When \(R^m = 1\), both sides of (36) are equal to zero and hence, \(\rho = A\alpha k^{\alpha-1} = 1\) as well. On the other hand, \(1 - \pi\) is the upper bound on the right-hand side of (36). Hence, it represents the upper bound on the left-hand side as well. Therefore, \(R^m\) asymptotically
approaches \(\pi\) when \(\rho\) goes to positive infinity.

The upward-sloping curve represents (35). When \(R^m = 1\), \(k = [A(1-\alpha)(1-\pi)]^{1/(1-\alpha)}\) and \(\rho = \frac{\alpha}{(1-\alpha)(1-\pi)}\). Therefore,
\[
\frac{\alpha}{(1-\alpha)(1-\pi)} > 1
\]
is the sufficient condition for the existence of a solution to the system (35)-(36) and hence, for the
existence of the dollarization threshold. Q.E.D.

Proof of Lemma 4: Taking into account that \(w + g = d\), and combining (16), (18), (25), (26),(27)
and (28), we get
\[
\frac{c^{st}}{c^{vl}} = \frac{R^{st}}{R^{vl}} = \frac{\rho}{R^m} = \frac{\rho q - (1 - \pi)}{\pi - R^m(1 - q)} = \frac{q \rho \pi}{(1 - \pi)(1 - q) R^m}.
\]
The last equality can be simplified as
\[
\frac{q \pi}{(1 - \pi)(1 - q)} = 1.
\]
Cross-multiplying and collecting terms yield
\[q = 1 - \pi.\]
In words, the share of productive capital in banks’ portfolios is \(1 - \pi\), and the share of domestic
currency is \(1 - q = \pi\). Q.E.D.

Proof of Lemma 5: Combining (16),(18),(27) and (28), eliminating \(c^m\) and \(c^{st}\) and using the notation
\(\delta \equiv w + g - d\), we get
\(^{11}\)Equation (24) describes a unique negative relationship between \(\rho\) and \(k\). Hence, a monotonic positive (negative)
relationship between \(R^m\) and \(k\) implies a monotonic negative (positive) relationship between \(\rho\) and \(R^m\).
\[ \frac{R^m}{\rho} = \delta + \frac{dR^m(1-q)}{\pi} \delta + \frac{dq}{1-\pi} \]  

(38)

Solving the last equation for \( \delta \) yields

\[ \delta = \frac{d\rho R^m(q + \pi - 1)}{\pi(1-\pi)(R - R^m)}. \]  

(39)

Differentiating with respect to \( R^m \) yields

\[ \frac{\partial \delta}{\partial R^m} = \frac{\partial}{\partial R^m} \left( \frac{d\rho R^m}{\pi(1-\pi)(\rho - R^m)} \right) (q + \pi - 1) + \frac{d\rho R^m}{\pi(1-\pi)(\rho - R^m)} \frac{\partial q}{\partial R^m}. \]

Given that the first term of the last expression is equal to zero when \( q = 1 - \pi \), we conclude that \( \frac{\partial \delta}{\partial R^m} < 0 \) if and only if \( \frac{\partial q}{\partial R^m} < 0 \). Q.E.D.

**Proof of Lemma 6:** Equation (39) can be rewritten as

\[ (w + g - d)(\rho - R^m) + \frac{dR^m \rho(1-q)}{\pi} = \frac{dR^m \rho q}{1-\pi}. \]

Substituting (23) for \( w \), (24) for \( g \), and dividing through by \( (dq) \), we get

\[ \left[ (1-\alpha)A k^{\alpha-1} + R^m - 1 - \frac{R^m}{q} \right] (\rho - R^m) + \frac{\rho R^m(1-q)}{\pi q} = \frac{\rho R^m(1-\pi)}{1-\pi}. \]

(40)

Totally differentiating the last equation, we get

\[ \left[ (1-\alpha)A k^{\alpha-1} + R^m - 1 - \frac{R^m}{q} \right] (\frac{d\rho}{dR^m} - dR^m) + \\
\frac{1}{q} \left[ \rho R^m \left( \frac{1}{q^2} \right) dq + \frac{\rho}{q} dR^m + \frac{R^m}{q} d\rho - \rho dR^m - R^m d\rho \right] = \frac{1}{1-\pi} [\rho dR^m + R^m d\rho]. \]

(41)

Evaluating (41) at the threshold and taking into account that

\[ \left[ (1-\alpha)A k^{\alpha-1} + R^m - 1 - \frac{R^m}{q} \right] = \frac{w+g-d}{k} = 0 \]

and that

\( q = 1 - \pi \), we obtain after some algebraic manipulations:
\[
\left(\rho - R_m\right)\frac{R_m}{q}\left(-\frac{\rho R_m}{\pi q^2}\right)\, dq = \left(\rho - R_m\right)\left(-1 + \frac{1}{q}\right)\, dR_m + \left(\rho - R_m\right)A(1 - \alpha)^2k^{\alpha-2}\, dk. \tag{42}
\]

Hence,
\[
\frac{\partial q}{\partial R_m} = \left(\rho - R_m\right)\left(-1 + \frac{1}{q}\right)\left(\rho - R_m\right)R_mq^2 - \rho R_m\pi q^2 + \left(\rho - R_m\right)A(1 - \alpha)^2k^{\alpha-2}\, \frac{\partial k}{\partial R_m} < 0. \tag{43}
\]

The last inequality holds because
\[
\frac{\partial k}{\partial R_m} > 0, 12 \left(\rho - R_m\right)\frac{R_m}{q^2} - \frac{\rho R_m}{\pi q^2} = \frac{\rho R_m}{q^4} - \frac{\rho R_m}{\pi q^2} - \frac{(R_m)^2}{q^2} < \frac{\rho R_m}{q^2} \left(1 - \frac{1}{\pi}\right) < 0 \text{ and } \frac{1}{q} - 1 > 0. \quad \text{Q.E.D.}
\]

\footnote{This follows from equations (16) and (18) in the same way as in the proof of Proposition 2 (section 4 of the paper).}
Existence and Uniqueness of the Inflation Threshold

Figure 1

\[ \rho \]

\[ (35) \]

\[ (36) \]