Duration and Term Structure of Trade Agreements*

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Abstract

Why are some trade agreements are concluded for a limited period of time while others have the form of evergreen contracts supplemented with a clause requiring an advance termination notice? We employ the recent advances in contract theory to demonstrate that the time structure of the trade agreement is related to the nature of the goods traded and that of the trade-related investments. If the agreement concerns trade in homogenous goods, the fixed-term contract duration is more likely. The fixed-term agreement provides incentives for an initial investment in trade-related infrastructure but leaves the parties the flexibility to reconsider the need for further investment. If the trade agreement covers heterogenous goods, the investment risk is more diversified and the likelihood of overinvestment is lower. Hence the agreement is more likely to be evergreen (with an advance termination notice or an escape clause).

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1 Introduction

The vast majority of WTO members are signatory to one or more regional trade agreements (RTAs) which in the WTO parlance refers to all bilateral, regional or plurilateral trade agreements of a preferential nature. The proliferation of regionalism has continued unabated since the early 1990s. More than 250 bilateral and plurilateral free trade agreements and customs union agreements have been notified to the GATT/WTO up to December 2002, of which 130 were notified after January 1995. Over 170 RTAs are currently in force; an additional 70 are estimated to be operational although not yet notified. By the end of 2005, if bilateral and plurilateral trade agreements reportedly planned or already under negotiation are concluded, their total number might exceed 300.

Trade agreements are rarely permanently binding upon the signatory parties. Since obligations in international law are traditionally viewed as arising only from the consent of states, most bilateral and plurilateral trade agreements and treaties expressly allow a state to withdraw as long as it follows certain procedures of notification. Most trade agreements usually contain a final clause with provisions for the agreement’s duration, termination or for the withdrawal of a party.

While some trade agreements are concluded for unlimited period and allow contracting parties to withdraw from or denounce the agreement at any time by giving an advance notice to the other contracting party (parties), others stipulate trade for a predefined period of time on fixed terms and have no provisions for early withdrawal. The final clauses of the agreements of the second type may contain a non-binding statement about the possibility of renewal based on the mutual consent of the parties. Following the recent contract-theoretic literature we refer to the former type of trade agreements as evergreen with advanced withdrawal (or termination) notice and the latter type as fixed-term.

For example, multilateral and plurilateral agreements which are parts of the WTO compact are evergreen contracts with advance termination notice varying from 12 months (the Agreement on Trade in Civil Aircraft) to 60 days (the International Dairy Agreement and the International Bovine Meat Agreement). Other examples of evergreen trade agreements include the 1992 EC-US Agreement on Trade in Large Civil Aircraft (12-month advance notice), the 2004 Euro-Mediterranean free trade area negotiated among the European Union, Israel, Morocco, Tunisia, Jordan and Egypt (6-month advance notice); the 2001 agreement between Armenia and Kazakhstan (6-month advance notice); the 1997 Agreement on Arab Free Trade Area (12-month advance notice). The examples of fixed term bilateral trade agreements include the 2001 agreement between Turkey and Jamaica (5-year duration); the 1996 Canada-U.S. Softwood Lumber Agreements (5-year duration); a series of rather short-term agreements concluded in the 1960s-90s between India and Tanzania (with the duration ranging from 1 to 2 years) and India and Bangladesh (with the duration from 2 to 3 years) and a series of the Lomé Convention trade and aid agreements between the EU and a number of

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1 The 1992 EC-US Agreement on Large Civil Aircraft stipulated that in exceptional circumstances, a party may terminate the agreement within 15 days following consultations concerning a matter leading to termination. In October 2004, the United States exercised its right to terminate the Agreement by sending a diplomatic note to the European Union’s Council of Ministers.
developing African, Caribbean, and Pacific countries, which were concluded for fixed terms ranging from 5 to 10 years.

A review of final clauses in a large number of bilateral and plurilateral trade agreements suggests that fixed terms agreements are more common between parties whose bilateral trade is mostly in homogeneous goods (e.g., commodities). This is the reason why most of the fixed term agreements either include a commodity exporting developing country as at least one of the parties (e.g., Lomé Convention, India-Bangladesh, and Turkey-Jamaica agreements) or concern sectoral trade between developed countries in a homogeneous commodity (e.g., lumber, oil or gas). On the other hand, evergreen bilateral trade agreements (with advance notice) are characteristic of countries which trade primarily in manufacturing goods and services. That is why bilateral trade agreements between developed countries usually have unlimited duration.

In order to understand the time structure of the trade agreements, one has to explain first why these long-term trade agreements should exist at all. The costs of the long-term agreements are substantial. As desired trade policies may change dramatically over a short period of time because of economic and political shocks, amending the negotiated policy commitments fixed in the agreements may be quite expensive. A consistent argument was provided in a seminal paper by MacLaren (1997) who suggested that the long-term agreements protect incentives for irreversible trade-related investments. Indeed, without trade agreements, such investments are vulnerable to holdup by the trading partner. This logic makes the analysis of trade agreements similar to the incomplete contract theory (Grossman and Hart, 1986, Hart, 1995) and in particular to the theory of incomplete contracts on time (Guriev and Kvasov, 2005). The parties choosing the duration of the contract, have to resolve the incentive-flexibility trade-off. If the trade agreement is too short-term, there will be no incentives to invest; if the trade is too long-term, then it will reduce the parties’ flexibility to react to external shocks; moreover even if the parties renegotiate the outdated agreement, the outcome of renegotiation may provide too strong incentives to invest. The risk of over-investment or of over-specialization is as tangible as that of under-investment. Moreover, this risk is eventually costly for both trade partners, not only for the investing party. Indeed, in a bilateral trade relationship, if the exporting country wants to specialize in a particular export sector, it would require guaranteed terms of market access in the importing country for a sufficiently long period of time to prevent the risk of hold up. The greater the exporting country’s trade-specific investment, the more the importing country has to compensate for the upfront investment costs by expanding the duration of its market-access obligations. This is why many countries are reluctant to sign the long-term trade agreements even if there are substantial mutual benefits. A good example is the ongoing debate among the EU governments on signing an agreement with Russia on the long-term supply of natural gas. The terms of agreement proposed by Russia include a large-scale investment into a gas pipeline which Russia can undertake in exchange for guaranteed long term contracts for the EU imports of Russia’s gas. The EU governments are reluctant to sign the agreement on Russia’s terms because of the uncertainty about the direction of the energy markets.

Therefore the optimal time structure of the trade agreement should take into account the trade-off between
providing efficient incentives for investment today and making sure that there will be no overinvestment tomorrow. Fixed-term contracts and evergreen contracts help resolve this tradeoff in very different ways. The distinction can be illustrated with the following example of a simple bilateral agreement designed to encourage a trade-related investment. The parties sign a free trade agreement for $\theta$ years. Alternatively, they can provide the same incentives if they sign an evergreen contract with $\alpha$ periods advance notice (or with an escape clause). Next year, an investment opportunity may arise. With probability $p$ this investment project will be socially optimal (i.e., it improves the joint welfare of the countries) and should be undertaken, and with probability $q$ it will be socially suboptimal and should not be carried out. With probability $1 - p - q$ there will be no opportunity for undertaking the project.

If the trade agreement is fixed term, next year the parties only have $\theta - 1$ years remaining under contract, which represents insufficient incentives for undertaking the investment project if it arrives. If the investment project is socially optimal, the parties have to renegotiate and replace the agreement with a new one for $\theta$ years. Under the evergreen contract, the opposite is true. Next year, the agreement will provide the very same incentives to invest as this year. This means that if the investment is optimal, there is no need to renegotiate. However if it is suboptimal, parties have to scrap the agreement because otherwise the foreign country will invest and over-specialize. Therefore the choice of the agreement will depend on whether $p$ or $q$ is higher. If the risk of overinvestment ($q$ is high) is large, a fixed-term agreement will do better. If the future underinvestment is a potential problem ($p$ is high), the parties will choose the evergreen type of agreement.

Even though renegotiation costs may be substantial they are certainly much less than the potential losses due to the inefficiency of the trade agreement. Therefore the inefficient agreement will be renegotiated. If the trade agreement provides incentives for a ‘value-destroying’ investment, it will be replaced by another agreement once the investment opportunity arrives. If the trade agreement does not provide incentives for undertaking an investment when it is jointly optimal, a new trade agreement will take its place. On the other hand, as the renegotiation costs are not trivial, the choice of the agreement should minimize the likelihood of such renegotiation.

The example above offers an indication of the structure of the agreement which reduces the need for renegotiation. If the risk of overspecialization is relatively more important, we should see the fixed-term trade agreement that provides incentives for investment today but rather discourages investment tomorrow. This is applicable to trade in a homogenous good where investments are typically one-off and bulky. If such an investment turns out inefficient, the welfare cost may be very high for both parties. For example, if a long-term crude oil import agreement provides incentives for upgrading pipelines and oil terminals (i.e., trade-partner-specific and good-specific investments), then the optimally-designed agreement makes sure that the oil exporting party undertakes only those upgrades that are jointly efficient for the parties.

In the case of trade in a differentiated product, the situation is different. Investments in differentiated

\footnote{One has to take into account full economic costs of renegotiation, not only the direct legal costs. The full economic costs are related to the fact that trade negotiations take time; and each day of delays involves forgone gains from trade and unrealized investment opportunities.}
goods are more flexible and less bulky because trade gains are diversified. Therefore, the expected cost of over-investment is lower. Indeed, an investment that enhances gains from trade in one variety today, will enhance gains from trade in another variety tomorrow because investments are specific to the product rather than to the individual variety. For example, human capital investments in a country specializing in outsourced software design can be shared by a large number of software product varieties.

As increasingly common in the recent trade literature, we assume that trade agreements are externally enforced (albeit incomplete) contracts. In principle, our argument could be made in a setting with self-enforced agreements. We choose the former setup for the simplicity’s sake. As we study agreements with clauses spanning over time, solving for equilibria in repeated games would be particularly cumbersome.

The rest of the paper is structured as follows. Section 2 provides a review of the related literature on trade and contract theory. Section 3 describes the setup of the model. In Section 4, we solve for incentives under the fixed-term and the evergreen trade agreements. Section 5 concludes.

2 Related literature

There are several strands of literature that relate to this paper. The tension between the governments’ need to protect irreversible trade-specific investment and the desire to maintain a degree of policy flexibility in the environment with uncertain terms of trade was emphasized in MacLaren (1997) and Bond (2006).

One implication of MacLaren’s and Bond’s analyses is that the more important the trade partner-specific investment, the longer the duration of trade agreements. However, these papers do not provide any insights as to why some trade agreements are concluded for a fixed term and others are evergreen (with advance termination notice or a temporary escape clause).

Another closely related theme in the international economics literature is the effect of uncertainty in the trade environment on the structure of international trade agreements. The earlier papers by Bagwell and Staiger (1990, 2003, 2005), Riezman (1991), Rosendorff and Milner (2001) consider trade agreements negotiated and enforced in the presence of uncertainty about either the trade volume or terms of trade. All of these papers point out that self-enforcing trade agreements will unravel unless countries implement more protectionist policies during periods of trade volume surges to lessen their own incentives to defect. Therefore, these papers interpret periods of high tariffs legitimized by the safeguards and escape clauses of the GATT/WTO legal system not as instances of non-cooperative behavior but rather as an attempt by countries to maintain self-enforcing nature of international cooperation in the environment with volatile trade volume.

Klimenko, Ramey and Watson (2006) consider the role of escape clauses in the environment with the terms-of-trade uncertainty when countries have to rely on exogenous enforcement of trade agreements because continuous renegotiation completely undermines the countries’ ability to sustain self-enforced cooperation.\footnote{MacLaren (1997) models these investments as trade-partner-specific and irreversible specialization of human capital while Bond (2006) studies the case where parties invest in infrastructure to reduce trade costs. While these two setups are somewhat different, the main ideas carry on from one framework to the other one. For simplicity’s sake, we follow Bond’s approach.}
In their setting, the ability of the escape clause to enhance the value of the trade agreement depends on the extent to which the information about the realizations of the stochastic terms-of-trade variable is verifiable by the third party, which adjudicates disputes over alleged violations of trade agreements.

In this paper, we focus on another common clause in trade agreements, the advance notice for unilateral termination. In terms of our theory, the advance notice and the escape clause perform similar functions: they limit opportunities for hold-up and therefore protect the incentives to invest. Even if there is a shock that makes termination mutually beneficial, the advance notice of $\alpha$ periods provide the investing party with a compensation of at least $\alpha$-periods-worth of trade. Conversely, the escape clause of $\beta$ periods gives the investing party a compensation equivalent to the normal trade gains promised to it under the terms of the agreement, which would resume accruing to it after $\beta$ periods of the escape clause. Usually, the next escape clause can be invoked only $\beta$ periods after the previous one ended, so the investing party receives the present value of trade from $\beta$ to $2\beta$ periods from now.

A relatively recent but fast-growing thread in the international economic literature emphasizes contractual incompleteness of international trade agreements which are enforced exogenously. Battigalli and Maggi (2003) examine the role of international agreements on product standards and show how the incompleteness of the trade agreements provides a role for a central dispute settlement mechanism. Horn, Maggi and Staiger (2005) consider trade agreements with endogenous level of contractual incompleteness determined by the contracting costs. Horn (2006) analyzes the role of the National Treatment principle of the WTO in overcoming contractual incompleteness of the international trade agreements that bind tariffs.

Our paper both builds on and contributes to the contract theory. Starting with Grossman and Hart (1986), the formal theory of holdup has emphasized the role of long-term contracts in protecting incentives for partner-specific investment. Our paper is most closely related to Harris and Holmstrom (1987) and Guriev and Kvasov (2005). Harris and Holmstrom model contract dynamics with positive renegotiation costs. Their rationale for long-term contracts is risk-sharing (between a risk-neutral employer and risk-averse employee) rather than investment incentives. Harris and Holmstrom show that as arrival of new information eventually results in renegotiation and solve for the optimal contract length. Guriev and Kvasov (2005) analyze incomplete contracts in continuous time. For both fixed-term contracts and evergreen contracts, they find the optimal contract duration that resolves the incentive-flexibility tradeoff. If the contract is too short-term, it does not protect incentive to invest. If the contract is too long-term, it reduces the other party’s flexibility: in case a more efficient partner arrives, the other party is constrained by obligations to trade with the investing party. In equilibrium, this inefficiency is renegotiated away, but Guriev and Kvasov show that excessively long-term contracts provide incentives for overinvestment relative to the social optimum. In their basic model (which assumes that there are only two states of nature and renegotiation is costless), these two contract types are equivalent; either can implement the first best. The contribution of our paper is to

\[4\] As discussed in detail in Bagwell and Staiger (2005), the WTO Safeguard Agreement stipulates that if a government imposes escape clause protection in an industry for a period of $\beta$ years, then it cannot reimpose escape clause protection in that industry for the $\beta$ next years.
emphasize the difference between the two types of contracts in a more realistic setting of trade agreements. We show that while the contract duration is chosen to provide incentives for investment at the inception of the contract, the availability of the alternative contract types allows for another degree of freedom. Having two distinct contract types helps to differentiate incentives for investment at the inception of the contract and at the contract’s more mature stages. The fixed-term contract provides weaker incentives for the future investment than for the present investments. The evergreen contract protects the present and the future investments equally well. Therefore investment incentives depend not only on the duration of the contract but on the type of the contract. This is turn implies that the choice of the contract type depends on both the present and the future investment characteristics as well as on the renegotiation costs.

3 Setup

We consider a discrete-time model of trade between two countries, home and foreign. In every period, countries can trade and the foreign country can make a trade-related investment which reduces its cost of exporting to the home country in the periods to come. We begin our analysis by assuming that the foreign country exports a homogenous good (i.e. a commodity). The foreign country’s trade-cost-reducing investment in the homogeneous good industry is ‘bulky’, which is captured by the assumption that in every period the foreign country’s investment is either 0 or 1. This assumption is intended to capture the difference between the specific trade-facilitating investments for the homogeneous good industry and the differentiated good industry. The examples of the former include the construction of large scale transportation and storage facilities for commodities (e.g., ports, oil terminals, oil or gas pipelines, electricity grids). The example of the latter would be an investment which is aimed at reducing the cost of exporting a specific product variety but shared by all differentiated varieties of the product because of their common characteristics. For example, the physical or human capital specific to a given product or service industry is generic with respect to all product or service varieties within the industry. To make the trade-facilitating investment in the differentiated product industry comparable to the bulky investment in the homogeneous good industry, we assume that there is a continuum of varieties of measure 1 and the investment for each variety can be either 0 or 1. Since the varieties are closely related, a trade cost-reducing investment for variety $i$, also reduces the trade cost for variety $j$. Therefore, in each period the expected volume of trade-related investments in the differentiated good industry is between 0 and 1, i.e., exactly as in the homogeneous good industry. But since investing in the differentiated good industry can be undertaken independently for each variety, it does not have the all-or-nothing property of the bulky investment in the homogeneous industry.

\[^{5}\]The difference between the homogenous good and the heterogenous good setups can be seen in the following example. In the case of homogeneous commodities, such as lumber, oil, gas, the investment is usually related to infrastructure and is certainly good specific (gas pipeline is a bulky indivisible investment; also, it cannot be used for selling oil). If we talk about software design or consulting services, each variety is different, but trade-promoting investments (like foreign language skills by the employees or trade fairs etc.) benefit all the varieties.
3.1 Trade and trade-related investments

The stage game is derived from the basic two-country, two-good framework previously considered by Johnson (1953/54), Mayer (1981) and Dixit (1987). We provide only a terse review of the main elements of this framework. The countries home (no *) and foreign ( *) exchange two goods \( x \) and \( y \). The home country exports good \( y \) in exchange for imports of good \( x \) from the foreign country. In this subsection, we assume that both \( x \) and \( y \) are homogenous goods. In the subsection 3.4, we will consider an alternative setup where \( x \) represents a differentiated product with a continuum of possible varieties.

Both countries are large enough to affect the terms of trade through the import tariff, which is the only policy instrument available to the countries’ governments. Although good \( y \) can be shipped costlessly, importing good \( x \) from the the foreign to the home country is costly. The per unit cost of shipping good \( x \) from the foreign to the home country \( v = v(K^*) \) is a decreasing function of capital stock of trade-related infrastructure \( K^* \). When there is an opportunity, the foreign country can increase the stock of the infrastructure \( K^* \) by making an investment. \( K^* \) is specific to the relationship between the home and the foreign countries and cannot be used to reduce trade costs with other potential trade partners. For simplicity we assume that the level of the foreign infrastructure investment is a binary variable: \( \phi = \{0,1\} \). The unit cost of investment is \( c \). The investment opportunities arrive at a Poisson rate \( \sigma \).

We follow the earlier literature on political economy of trade policy (e.g., Baldwin (1987), Bagwell and Staiger (2005)) and assume that each government seeks to maximize a weighted sum of the producer surplus, the consumer surplus and the tariff revenue, with a relatively greater weight on the import-competing producer surplus. Specifically, let \( \gamma > 1 \) denote the weight placed by the domestic government on its import-competing producer surplus. Then the single-period welfare functions of countries given tariff choices \( \tau \) and \( \tau^* \) and the transportation cost \( v(K^*) \) are denoted by \( U(\tau, \tau^*, \gamma, K^*), U^*(\tau, \tau^*, K^*) \). We make a number of conventional assumptions on \( U(\tau, \tau^*, \gamma, K^*) \) and \( U^*(\tau, \tau^*, K^*) \) to ensure the existence of static best response functions that generate Nash equilibria in tariffs. High tariffs \( \tau \) or \( \tau^* \) lead to the autarky outcome, in which welfare levels of both countries are taken to be zero. For lower levels of \( \tau \) and \( \tau^* \), trade volume is positive, and the welfare function of each country is strictly positive, differentiable and strictly quasi-concave in the country’s tariff level.

Let \( \hat{\tau}(\tau^*, \gamma, K^*) \) and \( \hat{\tau}^*(\tau, K^*) \) be the values of \( \tau \) and \( \tau^* \) that maximize the respective welfare functions of the two countries, i.e., the country’s best response tariffs. Given that \( \gamma \) is the weight of the import-competing producer surplus in the home country welfare, it is natural to assume that \( \hat{\tau}_\gamma > 0 \). The Nash equilibrium tariffs are denoted by \( \hat{\tau}^N(\gamma, K^*) > 0 \) and \( \hat{\tau}^{*N}(K^*) > 0 \). We assume that all realizations of \( \gamma \) are sufficiently

For example, following Dixit (1987) we assume that balanced-trade and Marshall-Lerner conditions are satisfied. This ensures that one country’s unilaterally-optimal tariff creates a negative terms-of-trade externality for the other country. Although the phrase “terms-of-trade externality” is rarely used in the parlance of real-world trade-policy negotiators, as Bagwell and Staiger (2002) demonstrate in their recent monograph, the concepts “terms-of-trade gain” and “market-access restriction” describe the single economic experience that occurs when the importing country government raises its import tariff and restricts foreign access to its market.
high that the home country’s Nash equilibrium tariff is prohibitive, i.e., it precludes imports from the foreign
country.\(^7\) (Since we allow that foreign country’s Nash equilibrium tariff can be non-prohibitive, there is some
one-way trade in the Nash equilibrium.)

The Nash equilibrium welfare levels are \(U^N(\gamma)\) and \(U^*N(\gamma)\). The joint welfare of the two countries is
given by \(\tilde{U}(\tau, \tau^*, \gamma, K^*) \equiv U + U^*\) (hereinafter we will use tilde for the joint variables). We assume that
\(\tilde{U}_\tau < 0, \tilde{U}_{\tau^*} < 0\), so that freer trade increases the joint welfare. The jointly optimal home tariff is strictly
positive, albeit non-prohibitive, for all realizations of \(\gamma > 1\): \(\tau^E(\gamma) > 0, \tau^*E(\gamma) = 0\) (the superscript “\(E\)”
stands for “efficient”).

Given our interpretation of \(\gamma\), it is natural to assume that its reduction implies a lower jointly-optimal
tariff \(\tau^E > 0\) and a greater volume of import in the home country, which increases the marginal effect of
the foreign infrastructure investment on the home country welfare: \(\frac{\partial U}{\partial K^{*}} < 0\). The foreign country
can invest either \(\Delta K^* = 0\) or \(\Delta K^* = 1\) per period; the cost of investment is \(c\Delta K^*\).

The parties discount the future at the common discount rate \(\rho\). (We assume that the capital stock does
not depreciate; non-trivial depreciation rate would simply be added to \(\rho\))

The countries’ marginal per period payoffs from the investment are \(u(\tau, \tau^*, \gamma) = \frac{\partial U^*}{\partial \tau \partial K^{*}}\) and
\(u^*(\tau, \tau^*) = \frac{\partial U^*}{\partial K^{*}}\). The joint per period payoff is given by \(\tilde{u}(\tau, \tau^*, \gamma) \equiv u(\tau, \tau^*, \gamma) + u^*(\tau, \tau^*)\). We
introduce a linearization \(U^*(\tau, \tau^*, K^*) = U^*(\tau, \tau^*, K^0) + \frac{\partial U^*}{\partial K^{*}}(K^* - K^0) + o(K^* - K^0)\) and
assume that maximum per period investment \(\Delta K^* = 1\) is small compared to \(K^*\). This assumption allows us to
neglect the higher-order terms of the Taylor expansion in the neighborhood of \(K^0\). Therefore the effect of
investment on future payoffs will be linear.\(^8\)

During each period, parties can renegotiate the previously concluded agreements. The cost of rene-
gotiation is \(k\) per period. Since this cost is small relative to joint gains from amending the agreement,
renegotiation will always happen in equilibrium. However, because the renegotiation costs are not trivial,
the parties choose the contract that minimizes these costs.

Finally, we assume that all the bargaining power belongs to the home country.

### 3.2 Uncertainty

The home country’s domestic political economy parameter \(\gamma\) changes over time. For simplicity, we assume
that this parameter has only two realizations: \(\gamma\) can be high \(\gamma = \gamma^P\) (i.e., consistent with the protectionist
\(^7\)This is for the technical simplicity’s sake only. Our results extend to general cases in which there are Nash equilibriums with
non-prohibitive tariffs. However, the more general cases require more notation and additional modeling details.

\(^8\)In principle, one can argue for either convex or for concave relationship between the foreign country’s investment and
welfare. On one hand, the investment cost functions are usually convex. On the other hand, the effect of the infrastructure
investment on the welfare is likely to be concave – the more we have invested the greater amount is traded, hence the higher
return to investment. As it is hard to determine the nature of the ultimate effect of \(K^*\), we use a linear function as the first
approximation. A non-linear relationship would imply similar results but require more cumbersome derivations.

Linearization also simplifies the role of depreciation. If the linearity assumption holds, depreciation does not affect incentives
to invest.
stance of the home government) or low \( \gamma = \gamma^L < \gamma^P \) (i.e., consistent with the liberal trade-policy stance of the home government). In the latter case, liberal trade policy is globally optimal (i.e., maximizes joint welfare of the two countries), while if \( \gamma = \gamma^P \), the global optimum involves higher trade barriers.

We distinguish between three states of nature: “Good”, “Medium”, and “Bad” \((G, M, B)\), respectively. In both \(G\) and \(M\) states, \( \gamma = \gamma^L \), while in the state \(B, \gamma = \gamma^P\). The difference between the \(G\) and \(M\) states is that there is no direct transition between states \(G\) and \(B\). Essentially, if the state is \(G\), everyone knows that protectionist preferences are unlikely. While if the present state is \(M\), the state \(B\) is likely to arrive next period.

Formally speaking, we consider a Markov process where the transitions between the three states occur at given rates. The transitions from state \(M\) to states \(G\) and \(B\) take place at the rates \(\mu_G, \mu_B\), respectively. For simplicity, we assume that transitions to \(M\) out of both \(G\) and \(B\) states occur at the same rate \(\lambda\). Each row in the transition matrix below represents the probability distribution of the state in the next period \(s_{t+1}\) given the current state \(s_t\):

<table>
<thead>
<tr>
<th>(s_t)</th>
<th>(s_{t+1} = G (\gamma_{t+1} = \gamma^L))</th>
<th>(s_{t+1} = M (\gamma_{t+1} = \gamma^L))</th>
<th>(s_{t+1} = B (\gamma_{t+1} = \Gamma))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(G)</td>
<td>(1 - \lambda)</td>
<td>(\lambda)</td>
<td>(0)</td>
</tr>
<tr>
<td>(M)</td>
<td>(\mu_G)</td>
<td>(1 - \mu_G - \mu_B)</td>
<td>(\mu_B)</td>
</tr>
<tr>
<td>(B)</td>
<td>(0)</td>
<td>(\lambda)</td>
<td>(1 - \lambda)</td>
</tr>
</tbody>
</table>

We use \(p_{t,s}\) to denote the probability of being in state \(s = G, M, B\) at time \(t\). Given the initial distribution \((p_0,G, p_0,M, p_0,B)\), these probabilities are given by:

\[
\begin{align*}
p_{t,G} &= \bar{p}_G + (p_0,G - \bar{p}_G) (1 - \lambda)^t + \frac{\mu_G}{\mu_G + \mu_B} \left[(1 - \lambda)^t - (1 - \lambda - \mu_G - \mu_B)^t \right], \\
p_{t,M} &= \bar{p}_M + (p_0,M - \bar{p}_M) (1 - \lambda)^t + \frac{\mu_B}{\mu_G + \mu_B} \left[(1 - \lambda)^t - (1 - \lambda - \mu_G - \mu_B)^t \right], \\
p_{t,B} &= \bar{p}_B + (p_0,B - \bar{p}_B) (1 - \lambda)^t + \frac{\mu_B}{\mu_G + \mu_B} \left[(1 - \lambda)^t - (1 - \lambda - \mu_G - \mu_B)^t \right],
\end{align*}
\]

where \(\bar{p}_s\) denotes the steady state distribution:

\[
(\bar{p}_G, \bar{p}_M, \bar{p}_B) = \left( \frac{\mu_G}{\lambda + \mu_G + \mu_B}, \frac{\lambda}{\lambda + \mu_G + \mu_B}, \frac{\mu_B}{\lambda + \mu_G + \mu_B} \right).
\]

The steady state probabilities \(\bar{p}_s\) can be derived in either of two ways. First, these probabilities are the limit distribution for \(t \to \infty\) : \(\bar{p}_s = \lim_{t \to \infty} p_{t,s}\). Alternatively, \(\bar{p}_s\) is the eigenvector of the transition matrix: if the present state is \(p_{t,s} = \bar{p}_s\), then it will be the same next period \(p_{t+1,s} = \bar{p}_s\).

Note that we introduce three states even though there are only two realizations of the home country political economy parameter \(\gamma\). This makes the structure of uncertainty sufficiently rich to separate trade policy and investment decisions. While trade is optimal in states \(G\) and \(M\), investment will only take place in state \(G\). In state \(G\), it is optimal both to set low tariffs and to invest (as high level of \(\gamma\) is relatively unlikely to occur in the future). In state \(B\), it is optimal to set higher tariffs and trade at a lower level so
that investment does not pay off. In the intermediate state $M$, parties trade at the same level as in the state $G$ (as the level of $\gamma$ is low) but do not invest (as the protectionist preferences $\gamma = \gamma^P$ are likely to arrive in the future).

We will also introduce another source of uncertainty: the availability of investment opportunity. Investment at time $t$ is only possible if there is an investment opportunity. We assume that investment opportunity is available with probability $\sigma$; there is no investment opportunity with probability $1 - \sigma$. The arrivals of investment opportunities are independent across time periods.

### 3.3 Timing

The timing is as follows.

- Period $t$ begins. State transition is realized. Parties observe the state $s = G, M, B$ and the political economy parameter $\gamma = \gamma^P, \gamma^L$. Investment opportunity arrives with probability $\sigma$ or does not arrive with probability $1 - \sigma$.

- Parties choose whether to trade according to an agreement signed in previous periods or to renegotiate. The renegotiation may replace the existing agreement with a new long-term or spot trade agreement, or the Nash equilibrium tariffs. Renegotiation incurs cost $\kappa$.

- If there is an investment opportunity, the foreign country decides whether to invest.

- Trade occurs. Period $t$ ends.

### 3.4 First best

We will first consider the first best for the homogenous good case, then extend the analysis to the setup where $x$ is a differentiated product.

**Homogenous good**

The first best level of trade depends on the current state of nature. The jointly optimal tariffs are $\tau^E(\gamma), \tau^*E(\gamma)$. The level of trade is higher in the states $G$ and $M$ (when $\gamma = \gamma^L$).

Let us now solve for the optimal investment decision (contingent upon the arrival of an investment opportunity). Investment raises welfare in all states, but the immediate effect of investment is lower in the state $B$ (when $\gamma$ is high) than in the state $M$ and $G$ (and it is the same in $M$ and $G$ states). We denote the joint per-period return to investment in these states by $\tilde{u}^L \equiv \tilde{u}(\tau^E(\gamma^L), \tau^*E(\gamma^L), \gamma^L)$ and $\tilde{u}^P \equiv \tilde{u}(\tau^E(\gamma^P), \tau^*E(\gamma^P), \gamma^P)$, respectively. As assumed above, the joint return to investment is higher under liberal trade policy: $\tilde{u}^L > \tilde{u}^P$.

The decision to invest should take into account the expected global returns to investment which include the returns in the current state as well as the future transitions to other states of nature. Let $\tilde{W}_s$ be the
effect of investment on the expected social returns to investment where \( s = G, M, B \) is the initial state. Once the investment opportunity arrives in the state \( s \), investment is optimal whenever \( \tilde{W}_s > c \). By definition,

\[
\tilde{W}_G = \frac{1}{1 + \rho} \left[ \bar{u}^L + (1 - \lambda)\tilde{W}_G + \lambda\tilde{W}_M \right]
\]

\[
\tilde{W}_M = \frac{1}{1 + \rho} \left[ \bar{u}^L + (1 - \mu_G - \mu_B)\tilde{W}_M + \mu_G\tilde{W}_G + \mu_B\tilde{W}_B \right]
\]

\[
\tilde{W}_B = \frac{1}{1 + \rho} \left[ \bar{u}^P + (1 - \lambda)\tilde{W}_B + \lambda\tilde{W}_M \right]
\]

where \( \rho > 0 \) is the discount rate.

This system has the following unique solution:

\[
\tilde{W}_M = \frac{\bar{u}^L(\rho + \lambda + \mu_G) + \bar{u}^P\mu_B}{\rho(\rho + \lambda + \mu_G + \mu_B)}
\]

\[
\tilde{W}_G = \frac{\bar{u}^L[(\rho + \lambda + \mu_G)(\rho + \lambda) + \rho^2] + \bar{u}^P\mu_B\lambda}{\rho(\rho + \lambda + \mu_G + \mu_B)(\rho + \lambda)}
\]

\[
\tilde{W}_B = \frac{\bar{u}^L(\rho + \lambda + \mu_G)\lambda + \bar{u}^P[\mu_G(\rho + \lambda) + (\rho + \lambda + \mu_G)\rho]}{\rho(\rho + \lambda + \mu_G + \mu_B)(\rho + \lambda)}
\]

One can easily check that \( \bar{u}^L > \bar{u}^P \) implies \( \tilde{W}_B < \tilde{W}_M < \tilde{W}_G \). In state \( G \), the expected returns to investment are high, as the parties expect relatively long period under low tariffs; in the states \( B \) and \( M \), longer periods of protectionism are more likely.

**Differentiated goods**

Now consider the case of the differentiated product. The per unit cost of trade depends on the aggregate stock of capital \( K^* = \int_0^1 K^{e_i} di \). As there is a continuum of product varieties, exactly \( \bar{p}_s \) per cent of the varieties are in the state \( s = G, M, B \) is . Whenever there is an investment opportunity for variety \( i \), the investment costs \( c \) and leads to the expected joint returns of \( \tilde{W} = \sum_{s=G,M,B} \bar{p}_s \tilde{W}_s \).

Throughout the paper we assume that the parameters are such that investment is optimal if the good is differentiated, or if the good is homogenous and the state is \( G \). If the good is homogenous, but the state is \( M \) or \( B \), there should be no investment.

**Assumption A1.** The parameters are such that:

\[
\tilde{W}_M < c < \tilde{W}_G. \quad (2)
\]

This assumption implies \( \tilde{W}_G > c \) since \( \tilde{W}_G > \tilde{W} \).

The assumption allows us to focus on the most interesting case. Otherwise, either \( \tilde{W} < c \), and there is no need for investment in the differentiated good case (and the parties are better off not signing any trade agreement), or \( \tilde{W}_M > c \), and the state \( M \) is not different from the state \( G \) as in both states it is optimal to set low tariffs and invest both states \( M \) and \( G \).

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9This formula assumes that infrastructure is used randomly across sectors in different states. If the capital is allocated in a non-random way, the condition (2) would be even less demanding.
4 Trade agreements and investment

Suppose an investment opportunity arises. We first derive the incentives to invest under different contract types starting with the case of a homogenous good. Then we solve for the differentiated good case. After that, we compare the renegotiation costs for each case for different types of contracts. Notice that we assume that in the case of the differentiated good, the tariff fixed in the trade agreement applies to all varieties of the differentiated good.

4.1 Null contract

We shall first consider the case of a null contract: countries do not conclude a long-term agreement. The terms of trade are negotiated on the spot, since the home country is assumed to have full bargaining power, the foreign country’s payoff is equivalent to its payoff under the Nash equilibrium. The return to investment is therefore trivial and foreign country does not invest. As we will show below as long as renegotiation costs are low relative to the returns to investment, the null contract is outperformed by other contracts.

4.2 Fixed term agreement

The parties sign a contract to trade for \( \theta \) periods with the tariffs \( \tau, \tau^* \). (If \( \theta \) is not an integer, trade in the last period occurs with a probability \( \theta - \text{int}(\theta) \)). Under this contract, the foreign country’s payoff does not depend on \( \gamma \); therefore the foreign country’s returns to investment only depend on the contracted tariffs \( \tau, \tau^* \) and not on the state \( s \).

Let \( u^* = u^*(\tau, \tau^*) \) be the one-period return to investment and \( V^*_\theta \) the expected discounted returns to investment received by the foreign country given the contract with tariffs \( \tau, \tau^* \) and the duration \( \theta \):

\[
V^*_\theta = \frac{1}{1 + \rho} [u^* + V^*_{\theta-1}] = u^* \frac{1 - (1 + \rho)^{-\theta}}{\rho} + (1 + \rho)^{-\theta} V^*_\theta.
\]

As argued above, the foreign country’s payoff under the null contract is trivial \( V^*_0 = 0 \) (regardless of the state \( s = G, M, B \)). Hence the foreign country’s payoff under the contract with tariffs \( \tau, \tau^* \) and the duration \( \theta \) is

\[
V^*_\theta = u^* \frac{1 - (1 + \rho)^{-\theta}}{\rho}.
\]

The minimum duration \( \bar{\theta} \) that provides the incentive to invest given should solve \( V^*_\bar{\theta} = c \).

Proposition 1 The minimum duration of the fixed-term contract which provides a sufficient incentive for investment is

\[
\bar{\theta} = \frac{\ln \left( \frac{u^*}{c - u^*} \right)}{\ln (1 + \rho)}.
\]

One period into the lifetime of the fixed-term agreement with the duration \( \bar{\theta} \) (and tariffs \( \tau, \tau^* \)), it no longer provides sufficient incentives to invest \( V^*_\bar{\theta} = c > V^*_\bar{\theta-1} \).
Under this trade agreement, the foreign country is happy with its terms and wants to continue with the same tariffs even if the state is $B$ and trade on the same terms is jointly inefficient. Therefore, once the state is $B$, the home country will immediately ask for renegotiation and will compensate the foreign country for scrapping the agreement.

### 4.3 Evergreen agreement

Now consider a contract that provides an ongoing protection for investment with a requirement that the unilateral withdrawal from the treaty has to be preceded by a notification of the other party $\alpha$ periods in advance of the withdrawal.\(^{10}\) Thus, the contract stipulates that the parties trade under the tariffs $\tau, \tau^*$ indefinitely; the home country has the right to terminate the agreement at time $t$ by sending the foreign country a written notice at time $t - \alpha$.

As long as the state is $G$, there is no need to renegotiate; renegotiation would be a zero-sum game. Therefore, until the state is $B$, the parties continue to trade under the terms of the contract. Once state $B$ arrives, the contract is renegotiated. If the state is $M$ and there is no new investment opportunity, there is also no need to renegotiate. But if an investment opportunity arrives in state $M$, the parties do need to renegotiate; otherwise the foreign country would invest which would be jointly inefficient. Therefore, the home country offers to terminate the trade agreement and pays the compensation equivalent to the foreign country’s payoff from another $\alpha$ periods worth of trade under the terms of the agreement (therefore the foreign country obtains $V^*_\alpha$ as defined by (3)).

Now consider the foreign country’s investment decision under the evergreen contract. Suppose that the investment opportunity arrives. The foreign country’s returns to investment are as follows. First, $u^*$ for each period until the state $B$ (with $\gamma = \gamma^P$) arrives. Second, a payment equivalent to the payoff from $\alpha$ periods of continued trade under the terms of the agreement — after this state has arrived. Third, the payments equivalent to the outside option (i.e., the Nash outcome) after the good state returns. The latter effect is trivial as the home country has all the bargaining power and would capture the entire surplus from the resuming high level of trade.

Therefore the foreign country’s returns to investment $V^*_{\alpha,s}$, $s = G, M, B$ are as follows:

\[
\begin{align*}
V^*_{\alpha,G} &= \frac{1}{1 + \rho} \left[ u^* + (1 - \lambda)V^*_{\alpha,G} + \lambda V^*_{\alpha,M} \right] \\
V^*_{\alpha,M} &= \frac{1}{1 + \rho} \left[ u^* + (1 - \sigma)(1 - \mu_B - \mu_G)V^*_{\alpha,M} + (1 - \sigma)\mu_G V^*_{\alpha,G} + (\sigma + (1 - \sigma)\mu_B) V^*_{\alpha} \right] \\
V^*_{\alpha,B} &= V^*_{\alpha}
\end{align*}
\]

where $V^*_{\alpha}$ is the return to investment under a fixed term contract of length $\alpha$, and $u^*$ is the foreign country’s one period return to investment given that trade takes place under the contracted tariffs $\tau, \tau^*$. In state $B$, it is jointly efficient to terminate the contract. In order for the foreign country to agree to the immediate

\(^{10}\)An alternative setting is the evergreen contract with a unilateral escape clause (like in Bagwell and Staiger, 2005). The analysis would be similar but more cumbersome. Therefore we focus on the advance termination notice.
termination, the home country has to compensate the foreign country with a one-time payment $V^*_\alpha$ for the foregone $\alpha$ periods worth of returns of investment under the trade agreement. In state $G$, the foreign country will receive one-period worth of returns $u^*$ and the expected returns for the next period ($V^*_\alpha,G$ with probability $1 - \lambda$ and $V^*_\alpha,M$ with probability $\lambda$). In the intermediate state, $M$, the parties have to take into the account the need to renegotiate the contract in case there is an investment opportunity. If there is no investment opportunity (probability $1 - \sigma$), the parties continue to trade under the present contract until the bad state arrives. If this happens (unconditional probability $(1 - \sigma)\mu_B$), parties terminate the contract and the home country pays the foreign country a lump-sum payment $V^*_\alpha$. If the investment opportunity arrives, the contract provides the foreign country with incentives to invest. On the other hand, if the investment opportunity does arrive in state $M$, seizing this opportunity is jointly inefficient. Hence, it is optimal to pay $V^*_\alpha$ to the foreign country and terminate the contract immediately.

The solution to the system is as follows:

$$V^*_{\alpha,G} = \frac{u^* \rho + \lambda + \sigma + (\mu_B + \mu_G)(1 - \sigma) + [1 - (1 + \rho)^{-\alpha}] \frac{\lambda}{\rho} (\sigma + \mu_G(1 - \sigma))}{\rho + \lambda + \sigma + (\mu_B + \mu_G)(1 - \sigma) + \frac{\lambda}{\rho} ((\sigma + \mu_G(1 - \sigma))}$$

where we used the expression for $V^*_\alpha$ from (3).

To find the minimum termination notice time $\bar{\alpha}$ which provides sufficient investment incentives, we need to solve $V^*_{\alpha,G} = c$.

**Proposition 2** The minimum advance termination notice time $\bar{\alpha}$ of the evergreen contract to provide incentives for investment is\textsuperscript{11}

$$\bar{\alpha} = \ln \left( \frac{u^* \rho + \lambda + \sigma + (\mu_B + \mu_G)(1 - \sigma)}{\rho + \lambda + \sigma + (\mu_B + \mu_G)(1 - \sigma) + \frac{\lambda}{\rho} ((\sigma + \mu_G(1 - \sigma))} \right) - \theta = \ln \left( \frac{1 + \rho^\alpha \lambda + \sigma + (\mu_B + \mu_G)(1 - \sigma)}{\lambda (\sigma + \mu_G(1 - \sigma))} \right) \ln (1 + \rho).$$

### 4.4 The optimal contract

Let us now calculate the expected renegotiation costs under each contract. Suppose that the parties sign the fixed-term contract (in the good state given an investment opportunity). Once it is signed, it provides sufficient incentives to invest in the first period of the contract’s duration. In the subsequent periods, four contingencies can arise. First, another investment opportunity may arrive, then the parties need to sign a new fixed-term contract. Second, the state $M$ can arrive; no renegotiation is needed since the remaining duration of the existing fixed-term contract is insufficient to provide incentives for investment. Hence, the foreign country will not invest. Third, the bad state may eventually arrive. Then the parties renegotiate. Fourth, the state may remain good but no investment opportunity arrives; there is no need for renegotiation.

Now, let us consider the evergreen contract. Here the situation is different. If an investment opportunity does arrive in state $G$, there is no need for renegotiation. But if an investment opportunity arrives in state

\textsuperscript{11}See Guriev and Kvasov (2005) for a detailed intuition for why the optimal advance notice $\bar{\alpha}$ is shorter than the optimal duration of the fixed-term contract $\bar{\theta}$.  

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$M$, then the parties renegotiate to rule out the investment; they replace the evergreen contract with a null contract. The same happens if state $B$ arrives.

We can now compare the expected renegotiation costs under the fixed term contract and under the evergreen contract.

**Proposition 3** In the case of homogenous good, the parties will choose the fixed-term contract whenever the good state of nature is sufficiently less likely than the intermediate state (i.e., $\lambda$ is sufficiently high, and/or $\mu_G$ is sufficiently low).

The Proof is relegated to the Appendix.

4.5 Trade in differentiated goods

In the case of trade in differentiated goods, the analysis is straightforward. At each period, there are investment opportunities for $\sigma > 0$ varieties. The fixed-term contract should therefore be constantly renegotiated. Expected renegotiation costs are $\kappa$ per period.

Under the evergreen contract, if the investment is not too costly, the parties are better off not renegotiating at all and just allowing all investment opportunities to be exploited. Assumption A1 above (2) implies that the parties are better off to allow investments whenever an investment opportunity arrives. Therefore in the case of differentiated goods, the parties prefer the evergreen contract.

Notice, that in our model the termination notice is never used. This is explained by the fact that we consider a continuum of differentiated good varieties, of which exactly $\bar{p}_B = \frac{\mu_B}{\lambda + \mu_G + \mu_B}$ are in state $B$; the probability of many more varieties being in the protectionist state is trivial. In reality, if there is a discrete set of goods and there is a positive (albeit small) probability that all (or very many) of them are in the state $B$; then the home country may sometimes use the advance notice to terminate the agreement.

5 Concluding remarks

In this paper we argue that the duration and the time structure of trade agreements are driven by the nature of trade covered by the agreement and the trade-related investments that the agreements are supposed to provide incentives for. Whenever choosing the structure of the trade agreement, parties have to address the incentives-flexibility trade-off. If the agreement is set to expire too soon, it does not protect irreversible investment in trade-related infrastructure. If the agreement is meant to last a very long time, it reduces the flexibility of the trading parties to change the terms of trade if there is a change in preferences or other economic parameters. Essentially, as the trade agreement becomes inefficient, the parties have to renegotiate it. However, as renegotiation is costly, it is in the interest of both parties to choose a trade agreement that would minimize the expected renegotiation costs. We show that this trade-off is resolved through the use of two instruments: duration of the agreement (short-term vs. long-term) and the time structure (fixed-term agreement vs. evergreen agreement with an advance termination notice).
If the parties trade in a homogenous good, they need an agreement that protects incentives for bulky irreversible investments. Therefore, the parties choose a fixed-term trade agreement. Once there is an opportunity for a new investment project, they conclude another fixed-term agreement etc. If the parties trade in differentiated goods, there are likely to be continuous opportunities for trade-facilitating investments. Therefore, there is a need for ongoing provision of incentives to invest. Therefore parties opt for an evergreen contract with a termination notice.

This prediction seems to be in line with available anecdotal evidence. Trade agreements that cover trade in homogenous goods are normally fixed-term, while the agreements that concern trade in differentiated goods and services are mostly evergreen. Yet, further research is needed to test this prediction in a more systematic way.
Appendix: Proofs

Proof of Proposition 3.

Denote $R^{FT}_{s}$ the expected renegotiation costs under the fixed-term given the state is $s$, and $r_s$ the expected renegotiation costs if there is no contract. Then

\[
R^{FT}_{G} = \frac{1}{1+\rho} \left[ \sigma (1-\lambda) (\kappa + R^{FT}_{G}) + (1-\sigma) (1-\lambda) R^{FT}_{G} + \lambda R^{FT}_{M} \right]
\]

\[
R^{FT}_{M} = \frac{1}{1+\rho} \left[ (1-\mu_G - \mu_B) R^{FT}_{M} + \mu_G R^{FT}_{G} + \mu_B R^{FT}_{B} \right]
\]

\[
R^{FT}_{B} = \frac{1}{1+\rho} [\kappa + (1-\lambda) r_B + \lambda r_M]
\]

\[
r_G = \frac{1}{1+\rho} [\sigma (1-\lambda) (\kappa + \min\{R^{FT}_{G}, R^{EG}_{G}\}) + (1-\sigma) (1-\lambda) r_G + \lambda r_M]
\]

\[
r_M = \frac{1}{1+\rho} [(1-\mu_G - \mu_B) r_M + \mu_G r_G + \mu_B r_B]
\]

\[
r_B = \frac{1}{1+\rho} [(1-\lambda) r_B + \lambda r_M]
\]

Now consider the evergreen contract. Here the situation is different. If an investment opportunity does arrive in the state $G$, there is no need for renegotiation. But an investment opportunity arrives in state $M$, then the parties need to renegotiate to rule out the investment. Also, renegotiation happens once the state $B$ has arrived.

\[
R^{EG}_{G} = \frac{1}{1+\rho} \left[ (1-\lambda) R^{EG}_{G} + \lambda R^{EG}_{M} \right]
\]

\[
R^{EG}_{M} = \frac{\sigma}{1+\rho} [\kappa + (1-\mu_G - \mu_B) r_M + \mu_G r_G + \mu_B r_B] +
\]

\[
+ \frac{1-\sigma}{1+\rho} [(1-\mu_G - \mu_B) R^{EG}_{M} + \mu_G R^{EG}_{G} + \mu_B R^{EG}_{B}]
\]

\[
R^{EG}_{B} = \frac{1}{1+\rho} [\kappa + (1-\lambda) r_B + \lambda r_M]
\]

Let us solve for $\delta_s = R^{EG}_{s} - r_s$ and $\Delta_s = R^{FT}_{s} - R^{EG}_{s}$. In the end of the day we are interested in the parameter constellation that imply $\Delta_G < 0$. Let also us introduce $\bar{\delta} = (1-\mu_G - \mu_B) \delta_M + \mu_G \delta_G + \mu_B \delta_B$.

We can see right away that $\Delta_B = 0$ and $\delta_B = \frac{\kappa}{1+\rho}$. Now let’s write the equations for the remaining $\delta_s$ and $\Delta_s$

\[
\Delta_G = \frac{1}{1+\rho} [\sigma (1-\lambda) \kappa + (1-\lambda) \Delta_G + \lambda \Delta_M]
\]

\[
\Delta_M = \frac{1}{1+\rho} [(1-\mu_G - \mu_B) \Delta_M + \mu_G \Delta_G - \sigma \kappa + \sigma \bar{\delta}]
\]

\[
\delta_G = \frac{1}{1+\rho} [(1-\lambda) (1-\sigma) \delta_G + \lambda \delta_M - \sigma (1-\lambda) \kappa - \sigma (1-\lambda) \min\{\Delta_G, 0\}]
\]

\[
\delta_M = \frac{1}{1+\rho} [\bar{\delta} (1-\sigma) + \sigma \kappa]
\]
The first two equations imply

\[ \Delta_G = \frac{\kappa ((1 - \lambda)(\rho + \mu_G + \mu_B) - \lambda) + \lambda \delta}{\rho (\rho + \lambda + \mu_G + \mu_B) + \lambda \mu_B} \]  

(4)

\[ [\rho + \sigma + \lambda (1 - \sigma)] \delta_G = \frac{1}{1 + \rho} \left[ \delta (1 - \sigma) + \sigma \kappa \right] - \sigma (1 - \lambda) \kappa - \sigma (1 - \lambda) \min \{ \Delta_G, 0 \} \]

\[ \delta_G = \frac{\delta - (1 - \mu_G - \mu_B) \frac{1}{1 + \rho} \left[ \delta (1 - \sigma) + \sigma \kappa \right] - \mu_B \sigma}{\mu_G} = \frac{[\rho + \sigma + (\mu_G + \mu_B)(1 - \sigma)] \delta - \sigma \kappa (1 - \mu_G - \mu_B) - \mu_B \kappa}{(1 + \rho) \mu_G} \]

Therefore

\[ [\rho + \sigma + \lambda (1 - \sigma)] \left[ [\rho + \sigma + (\mu_G + \mu_B)(1 - \sigma)] \delta - \sigma \kappa (1 - \mu_G - \mu_B) - \mu_B \kappa \right] \]

\[ = \lambda \mu_G \delta (1 - \sigma) + \lambda \mu_G \sigma \kappa - \sigma (1 - \lambda) \kappa (1 + \rho) \mu_G - \sigma (1 - \lambda) (1 + \rho) \mu_G \min \{ \Delta_G, 0 \} \]

When is the fixed-term contract preferred? Consider the case \( \Delta_G < 0 \). Then

\[ [\rho + \sigma + \lambda (1 - \sigma)] \left[ [\rho + \sigma + (\mu_G + \mu_B)(1 - \sigma)] \delta - \sigma \kappa (1 - \mu_G - \mu_B) - \mu_B \kappa \right] \]

\[ = \lambda \mu_G \delta (1 - \sigma) + \lambda \mu_G \sigma \kappa - \sigma (1 - \lambda) \kappa (1 + \rho) \mu_G - \sigma (1 - \lambda) (1 + \rho) \mu_G \Delta_G \]

\[ \delta \left( [\rho + \sigma + \lambda (1 - \sigma)] [\rho + \sigma + (\mu_G + \mu_B)(1 - \sigma)] - \lambda \mu_G (1 - \sigma) \right) \]

\[ = \kappa (\lambda \mu_G \sigma - \sigma (1 - \lambda) (1 + \rho) \mu_G + [\rho + \sigma + \lambda (1 - \sigma)] [\sigma (1 - \mu_G - \mu_B) + \mu_B]) \]

\[ - \sigma (1 - \lambda) (1 + \rho) \mu_G \Delta_G \]

We need to substitute \( \delta \) from (4), find \( \Delta_G \) and check that \( \Delta_G < 0 \). This holds if and only if

\[ 0 > \lambda \mu_G \sigma \left[ \lambda - (1 - \lambda) (1 + \rho) \right] + \lambda [\rho + \sigma + \lambda (1 - \sigma)] [\sigma (1 - \mu_G - \mu_B) + \mu_B] + \sigma [(1 - \lambda) (\rho + \sigma + \mu_G + \mu_B) - \lambda] [\rho + \sigma + \lambda (1 - \sigma)] [\rho + \sigma + (\mu_G + \mu_B)(1 - \sigma)] - \lambda \mu_G (1 - \sigma) \]

This is the case if \( \lambda \to 1, \sigma \to 1 \)

\[ 0 > \mu_G + [\rho + 1] [1 - \mu_G] + [-1] [\rho + 1] [\rho + 1] = \]

\[ = -\rho \mu_G - [\rho + 1] \rho \]

Q.E.D.
References


