Marshallian Surplus for Non-Quisilinear Preferences: Partial Equilibrium Setting[1]

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Despite its ubiquity it is now widely accepted that Consumer's Surplus should not be used as a welfare measure… [Slesnik, 1998]

Modern mainstream economics, intermediate and advanced micro textbooks teach us after classical Marshall, Hicks, Samuelson etc. contributions that Marshallian surplus - area to the left from the textbook demand function - represents exact change in consumer' money metric utility only in special quasilinear case, when marginal utility of income is every time unity. In the general case, it is recommended therefore to use Hicksian equivalent or compensated variations as a measures of utility change, keeping in mind Marshallian surplus as a useful, but rather poor, approximation of true utility change. Our papers shows that in one-dimensional case when only price of the good changes, area to the left of Walrasian demand curve every time represents *exact*, not approximate, change in consumer' indirect money metric utility function, for rather general class of differentiable preferences.

1. Introduction

Beginning from [Dupuit, 1844] developed by [Marshall, 1890, 1920], [Hicks, 1943] and numerous well-known economic theorists, consumer' surplus concept was one of the most old and extensively discussed notions in the economic analysis[3]. It has attractive applied value and a simple expositional beauty. Consumer' surplus is applied in traditional cost-benefit analysis, international trade analysis, partial equilibrium welfare analysis, tax reform evaluation – the scope of application is enormous (see [Currie et al., 1971] for an early survey of applications). However, a bunch of

[1] Author is indebtful to Richard Ericson, (scientific advisor of this research), to Viktor M. Polterovich, Jiandong Ju, Konstantin Sonin, Alexey Savvateev and Sergey Kovalev for very helpful comments and advices and also participants of December 2003, July 2004, December 2004, December 2005 EERC workshops, participants of April 2007 and April 2008 Higher School of Economics International Conferences in Moscow for discussions and recommendations. Related research in consumer surplus theory was supported by Human Capital Foundation (Contract № 18) and Economics Education and Research Consortium (Grant R03-1421). We thank also Fulbright Program for providing us productive research environment in US.

[2] Author can be reached at moskalionov@mail.ru. See also our more elaborated analysis of the aggregate surplus properties in the abstract general equilibrium model at http://ssrn.com/abstract=1093715.

[3] Total number of references for consumer surplus theory is huge; it is more than 200 nowadays. For example, [Suzumura, 1985] discovered 181 references for this topic.

criticism towards consumer' surplus concept had essentially narrowed the possible field of its direct theoretical and practical application. Two strains of skepticism against surplus concept are well-known. First, area to the left of Walrasian (which sometimes also called Marshallian or ordinal, or uncompensated) demand curve[4] works as an exact representation of money metric utility change only when marginal utility of numeraire (or money) is every time constant and equal to unity, as was proved in [Samuelson, 1942]. This condition is satisfied, as criticism suggests, only in the case of *quasilinear preferences*, when consumer' surplus is path-independent in a full space of possible adjustment paths. As well known, quasilinear (or parallel) preferences imply that demand for all other goods except numeraire (which price should be normalized to unity) should be independent from consumer' income changes – a very strong assumption indeed[5]! Second criticism states that surplus is generally path-dependent in multi-dimensional case. In classical papers, [Chipman and Moore, 1976, 1980] clearly demonstrated that consumer surplus is path-independent in a rather general class of adjustment paths only when preferences are quasilinear or homothetic, depending on respective restriction in the budget space. All these conditions are cases which are never observed in reality. Does it mean that lovely consumer surplus is an example of scientific fiction, "useless theoretical toy", as Samuelson stated? Does it mean that we should through away all introductory economics textbooks as providing our students with a bulk of pseudo-scientific trash?

Economic theory generally provided two major solutions so far. First, different approximations to "true utility measures" were proposed (since classical papers by [Harberger, 1971] and [Willig, 1976]); second, derivations of equivalent or compensated, variations were extensively discussed (see classical procedure by [Hausman, 1981] and later research). Even in recent papers, see ex. [Hillinger, 2001], it is stated actually that Marshallian surplus is, by itself, meaningless stuff, and only approximations to surplus change (that is, welfare triangles) should be used in practice.

Our paper contributes to some kind of defense of Marshallian surplus technique. We develop the so-called "*ordinal approach"* (or, better to say, semi-ordinal, semi-cardinal approach) to consumer surplus theory and claim that surplus should not be generally path-independent (in a large space of possible adjustment paths). Neither we claim that consumer' preferences should be quasilinear in order for surplus to be correct money metric utility measure. Developing independently original [Zaiac, 1979] and [Stahl, 1980, 1983][6] consumer surplus theories, we show rigorously[7],

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[4] [Mas-Collell, Whinston and Green, 1995] named it *area variation measure*.

[5] Samuelson [Samuelson, 1942] called the restrictions of preferences needed for path-independent consumer surplus to be relevant concept as "highly unrealistic".

[6] These theories are not well-known to economics profession in general. Even New Palgrave Dictionary of Economics doesn't site their research! See [Takayama, 1986].

[7] More in-depth analysis of the problem one can find in our papers, see [Moskalionov, 2008], [Москальонов, 2007]. Complete analysis of the surplus theory in the simple exchange economy will appear on www.eerc.ru and posted on author' page at http://ssrn.com/author=434208 .

that area to the left of the ordinal textbook demand curve **exactly**, and *not approximately*, represents true change in individual money metric utility such that classical cardinal interpretations of the surplus by [Dupuit, 1844] can be every time applied for the rather general class of differentiable preferences[8].

2. The Model

There are generally N goods. At each moment t consumer is endowed with income $y(t)$ which can be also interpreted as endowment of numeraire, price vector is $p(t) = (p_1(t),...p_N(t))$[9]. Consumption of the good i in social state t is indexed as $x_i(t)$**,** so consumption bundle is $x(t) = (x_1(t),...x_N(t))$. Consumer is endowed with preference relation $\succsim$ which is a binary relation on $\mathbb{R}_+^N \times \mathbb{R}_+^N$. We have standard

Assumption 1. *Consumer' preference relation* $\succsim \subset \mathbb{R}_+^N \times \mathbb{R}_+^N$ *is:*

1. *Complete on* $X \times X$, $X = \mathbb{R}_+^N$*:* $\forall x, y \in X \quad y \succsim x$ *or* $x \succsim y$ *;*
2. *Transitive on* $X \times X$ *:* $\forall x, y, z \in X \quad x \succsim y$ *and* $y \succsim z \Rightarrow x \succsim z$*;*
3. *Strictly convex on* $X \times X$ *:* $\forall x, y \in X \quad y \succsim x,\ y \neq x \Rightarrow \alpha y + (1-\alpha)x \succ x \quad \forall 0 < \alpha < 1$*;*
4. *Closed set relative to* $X \times X$ *;*
5. *Strictly monotonous on* $X \times X$ *:* $\forall x, y \in X \quad y \geq x \Rightarrow y \succ x$[10].

Sometimes we will also use assumption that $\succsim$ is C^r differentiable [Mas-Colell, 1985]. By Debreu Theorem [Debreu, 1959], see also [Mas-Colell, 1985], there exists a strictly quasiconcave increasing utility function $u(x)$ that represents $\succsim$ on X , $u : X \to \mathbb{R}^1$. Moreover, there exists a space of mathematically admissible strictly quasiconcave increasing utility functions on X , this space is denoted as $U(\succsim)$. If $\succsim$ is C^r - differentiable, then $u(x)$ is also C^r - differentiable, - well known fact. A space of C^r - differentiable strictly quasiconcave increasing utility representations for $\succsim$ is denoted as $U^C(\succsim)$. Walrasian demand function at each point t is a well-defined (see [Mas-Collell *et al.*, 1995]) continuous function: $x(t) = f(p(t), y(t)) = Arg \max_{x(t) \geq 0} u(x(t))$ s.t. $y(t) \leq p(t)x(t)$, which is C^{r-1} differentiable for C^r -differentiable utility without critical points at each regular point of demand

[8] For sure, if cardinal-ordinal economic theorist does not have any specific individual preferences in the choice of specific cardinal representation from the full space of possible utility representations!

[9] If good k is numeraire then $p_k(t) \equiv 1$ for $\forall t \in [0,1]$.

[10] Standard notation is: $x \geq y \iff x_i \geq y_i \ \forall i$ and $x \neq y$.

(see Proposition 2.7.2 in [Mas-Colell, 1985], [Samuelson, 1947]). Here $p(t)x(t)$ is a scalar product of two vectors. An indirect utility function $v:\mathbb{R}^N_{++}\times\mathbb{R}^1_{++}\to\mathbb{R}^1$ is defined in usual way as $v(p(t),y(t))=u(x(p(t),y(t)))$. We will need one more

Assumption 2. *Consumer' indirect utility function* $v(p,y)$ *is* C^1 *at each* $(p(t),(y(t))$.

The assumption on differentiability of indirect utility function is not as restrictive as differentiability of direct utility $u(x)$ (see ex. [Chipman and Moore, 1976]) but is extremely helpful in the abstract consumer surplus theory. There are a lot of examples when direct utility is not differentiable everywhere on X but indirect utility is differentiable on a budget space $\mathbb{R}^N_{++}\times\mathbb{R}^1_{+}$: famous one is a case of perfect complements. A space of C^1 indirect utility functions defined on the budget space $\mathbb{R}^N_{++}\times\mathbb{R}^1_{+}$ for $\succsim$ on $X\times X$ is denoted as $V^C(\succsim)$. Obviously, $V^C(\succsim)=U^C(\succsim)(x(p,y))$ for admissible pairs (p,y).

Adjustment path is a (piecewise) C^1 parameterization $\psi:[0,1]\to\mathbb{R}^N_{++}\times\mathbb{R}^N_{++}\times\mathbb{R}^1_{+}$. In other words, we will consider only those trajectories $\psi(t)=(p(t),x(t),y(t))$ along which $x(t)>0$ (one exception is paths that start from the origin), it also guarantees us regularity of demand along such paths (for differentiable direct utilities). A path $\psi:[0,1]\to\mathbb{R}^N_{++}\times\mathbb{R}^N_{++}\times\mathbb{R}^1_{+}$ connects two social states $s=(p(0),x(0),y(0))$ and $s'=(p(1),x(1),y(1))$ if $\psi(0)=s$ and $\psi(1)=s'$, we denote such path as $\psi(s,s')$. At each point t of so defined path $\psi(t)$ with C^r utility function FOC for consumer utility maximization problem holds: $\nabla u(x(t))=\lambda(t)p(t)$ where λ is consumer' Lagrange multiplier[11]. It should be also noted, that under our assumptions Antonelli' equation [Antonelli, 1886] or Roy's identity (in vector form) holds: $x(p(t),y(t))=-[\partial v(p(t),y(t))/\partial p(t)]/[\partial v(p(t),y(t)/\partial y(t)]$ at each point t of adjustment path. Marshallian generalized consumer' surplus for move from state 0 to state 1 can be calculated using theory of curve integrals of the second type as follows[12]:

$$CS(0,1)=y(1)-y(0)-\sum_{i=1}^{N}\int_{p_i(0)}^{p_i(1)}x_i(t)dp_i(t) \tag{1}$$

$$=y(1)-y(0)-\sum_{i=1}^{N}\int_0^1 x_i(t)\frac{dp_i(t)}{dt}dt \tag{2}$$

$$=y(1)-y(0)-\sum_{i=1}^{N}\int_{\psi(s,s')}x_i(t)dp_i(t)=CS(\psi(s,s')),$$

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[11] Corner solutions are eliminated from analysis by the definition of adjustment path; at the origin FOC is well defined, by Assumption 1.

[12] Russian definition, applies to integrals along piecewise smooth paths, in the western mathematical literature, line integrals are used (for piecewise linear paths). The difference is minor.

if $\psi(0)=s$ and $\psi(1)=s'$. A path $\psi:[0,1]\rightarrow\mathbb{R}^N_{++}\times\mathbb{R}^N_{++}\times\mathbb{R}^1_+$ *is monotonous* (MAP) if marginal Marshallian surplus $\partial CS(t)/\partial t=\sum_{i=1}^N p_i(t)\partial x_i/\partial t$ has constant sign along interval $[0,1]$: $\partial CS(t)/\partial t=\sum_{i=1}^N p_i(t)\partial x_i/\partial t>0, or<0, or=0$ for $\forall t\in[0,1]$. A path is *piecewise monotonous* (PMAP) if along some segments of $[0,1]$ $\partial CS(t)/\partial t=0$, along other ranges of $[0,1]$ it has the same constant sign. CS is *path-independent* in a set of paths connecting two points $\Psi(s,s')$ if for any two different paths connecting any two social states: $\psi^1(s,s')\in\Psi(s,s')$, $\psi^2(s,s')\in\Psi(s,s')$, $\psi^1\neq\psi^2$, we have: $CS(\psi^1(s,s'))=CS(\psi^2(s,s'))$.

3. Main Results

In this section we work only with paths which go along some fixed Walrasian demand curve: when only one price (say, of the first good) is changing: $\psi(t)=(p_1(t),\overline{p}_{i\neq1},x(t),\overline{y})$. We have at shot:

Lemma 1. *CS is path-independent in a subset of all admissible paths that go along fixed Walrasian demand curve for the first good.*

Proof: straightforward from (1): if $\dot{y}(t)=0,\ \dot{p}_{i\neq1}(t)=0$ then (upper dots are derivatives with respect to *t* as usual) $CS(0,1)=\int_0^1 x_1(t)[\partial p_1(t)/\partial t]dt=\int_{p_1(0)}^{p_1(1)} x_1(p_1(t),\overline{p}_{i\neq1},\overline{y})dp_1(t)$.

Then we directly derive:

Theorem 1. *Let preference relation* $\succsim$ *satisfies Assumptions 1 and 2. Then area to the left from Walrasian demand curve* $x_1(p_1)$ *for the first good has classical cardinal interpretation: there exists a differentiable indirect money metric utility function* $\tilde{v}^C(\cdot)\in V^C(\succsim)$ *that is monotonous transformation of original function* $v(p,y)$ *such that for any path* $\psi(t)=(p_1(t),\overline{p}_{i\neq1},x(t),\overline{y})$ *we have:* $CS(0,1)=\int_{p_1(0)}^{p_1(1)} x_1(p_1(t))dp_1(t)=\tilde{v}^C(1)-\tilde{v}^C(0)$. *In words: Marshallian surplus - area to the left from demand - is exactly equal to the change in money metric indirect utility.*

Let assume also that $\succsim$ *is* C^r*, then area to the left from demand curve* $x_1(p_1)$ *can be interpreted as a net gain in utility from exchanging money (numeraire) to good 1. Or, in mathematical terms, there exist a* C^r *utility function* $\tilde{u}^C(x)\in U^C(\succsim)$ *such that* $\nabla\tilde{u}^C(x(t))=p(t)$ *at each point t of adjustment path and* $CS(0,1)=\int_{p_1(0)}^{p_1(1)} x_1(p_1(t))dp(t)=\int_0^{x_1(1)}\tilde{p}_1(x_1(t))dx_1(t)-p_1(1)x_1(1)+x_1(0)p_1(0)$ *where* $\tilde{p}_1(x_1(t))$ *is inverse demand function along the path.*

Idea of Proof: we proceed in several simple steps:

Step 1. Let $p_1(0) > p_1(1)$. Observe that there exists every time monotonous path $\psi(t) = (p_1(t), \overline{p}_{i\neq 1}, x(t), \overline{y})$ defined by simple homotopy: $p_1(t) = p_1(0)(1-t) + p_1(1)t$. Really, $\dot{p}_1(t) = p_1(1) - p_1(0) < 0$ for each $t \in [0,1]$. Then, from Roy's identity, applying $\lambda(t) = \partial v(p(t), y(t) / \partial y(t) > 0$ (by Assumption 1 – strict monotonicity plus consumer's FOC along the path) we have: $\partial v(\cdot) / \partial p_1(t) < 0$ for each *t*, and finally $\dot{v}(t) = [\partial v(p(t), y(t)) / \partial p_1(t)]\, \dot{p}_1(t) > 0$ $\forall t \in [0,1]$.

Step 2. Apply Proposition 1 from [Moskalionov, 2008], see also [Zajac, 1979], to show that CS represents preference relation along subset of (piecewise) monotonous parameterizations:

$$\{\text{CS}(0,1) \geq 0 \text{ along subset of MAP(PMAP)}\} \Leftrightarrow\ x(1) \succsim x(0)\ \Leftrightarrow\ u(x(1)) \geq u(x(0))$$

Step 3. Combining Step 2 with Lemma 1, we receive that CS works for *any path* that goes along Walrasian demand curve for the good 1, $\forall t \in [0,1]$, $s = (p(0), x(0), y(0))$ and $s' = (p(1), x(1), y(1))$:

$$\{\text{CS}(\psi(\text{s,s}')) \geq 0 \text{ along any } \psi(t) = (p_1(t), \overline{p}_{i\neq 1}, x(t), \overline{y})\} \Leftrightarrow\ x(1) \succsim x(0)\ \Leftrightarrow\ u(x(1)) \geq u(x(0))$$

Step 4. Construct a function: $\tilde{v}^C(p(\tau), y(\tau)) := CS(0,\tau) + \overline{y}(0)$, where $CS(0,\tau)$ is calculated along any admissible $\psi(t) = (p_1(t), \overline{p}_{i\neq 1}, x(t), \overline{y})$, such that $CS(0,\tau) := \int_{p_1(0)}^{p_1(\tau)} x_1(p_1(t), \overline{p}_{i\neq 1}, \overline{y}) dp_1(t)$, $t \in [0,\tau]$, and show that along this path: $\tilde{v}^C(1) - \tilde{v}^C(0) = CS(0,1) > 0 \Leftrightarrow x(1) \succ x(0) \Leftrightarrow u(x(1)) > u(x(0))$, see Step 3 above[13]. Recalling theory of the curve integrals [Никольский, 1983] or line integrals [Apostol, 1957], convince yourself that $\tilde{v}^C(\tau)$ is a potential function for a subset of adjustment paths $\{\psi(t) = (p_1(t), \overline{p}_{i\neq 1}, x(t), \overline{y})\}$.

Step 5. Verify that potential function $\tilde{v}^C(p(\tau), y(\tau))$ generates new indirect money metric utility function and respectively new utility function $\tilde{u}^C(x(\tau))$ for our consumer[14]. See Zajac (1979) and our forthcoming paper at www.eerc.ru for the proof that CS generates a specific quasilinear in-

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[13] Here again $\tilde{v}^C(\tau) = \tilde{v}^C(p(\tau), y(\tau))$ with slight abuse of notation.

[14] Complete proof require construction of a grand adjustment path which would work as an extension of original path $\psi(t) = (p_1(t), \overline{p}_{i\neq 1}, x(t), \overline{y})$ such that new path $\tilde{\psi}(t) = (p_1(\gamma), \overline{p}_{i\neq 1}, x(\gamma), y(\gamma))$, $\gamma \in [0,\infty)$ will contain original path; say, new path starts at the origin, then follows linear segment to the starting point of $\psi(t)$, then coincides with $\psi(t)$ up to the final point of $\psi(t)$, then again follows a ray with linear increase in each good' consumption. From Assumption 1 it will follow that such grand path intersects each indifference manifold and is monotonous, so respectively defined function $\tilde{v}^C(p(\gamma), y(\gamma))$ will work as indirect utility function with related $\tilde{u}^C(x(\gamma))$ working as a relevant utility function; restriction of these functions on $\psi(\tau)$ are exactly $\tilde{v}^C(p(\tau), y(\tau))$ and $\tilde{u}^C(x(\tau))$.

direct utility function along any MAP or PMAP. Take derivative of $\tilde{v}^C(p(\tau), y(\tau)) := CS(0,\tau) + y(0)$ along general adjustment path (not just when only one price is changing) to check that $\partial\tilde{v}^C(\cdot)/\partial y(\tau) = 1$ (we use here the fact that $CS(0,\tau) = y(\tau) - y(0) - \sum_{i=1}^{N}\int_0^\tau x_i(t)\left[\partial p_i(t)/\partial t\right]dt$): money metric *marginal utility of income or money is unity along any MAP (PMAP).*

Step 6. Finally, assume that $\succsim$ is C^r. Then, original $u(x)$ is also C^r, under our assumptions. Then, due to regularity of demand assumption, Walrasian demand function $x(p(t), y(t))$ is C^{r-1} at each *t*, as was stated above. By Inverse Function Theorem, there exists inverse demand function along the path $\psi(t) = (p_1(t), \overline{p}_{i\neq 1}, x(t), \overline{y})$ which (again with some abuse of notation) is defined by: $p_1(t) = f_1^{-1}(x_1(t))$, where $x_1(t) = f_1(p_1(t), \overline{p}_{i\neq 1}, \overline{y})$ is direct demand function for the first good. This function is again C^{r-1}. Then, using change of variables, rewrite expression for surplus as $CS(0,\tau) = \sum_{i=1}^{N}\int_0^\tau p_i(t)\left[\partial x_i(t)/\partial t\right]dt$, so in our unidimensional case we will have $CS(0,\tau) = \int_0^\tau p_1(t)\left[\partial x_1(t)/\partial t\right]dt + \sum_{l=2}^{N}\overline{p}_l(x_l(\tau) - x_l(0)) = \int_{x_1(0)}^{x_1(\tau)} f_1^{-1}(x_1(t))dx_1(t) + p_1(0)x_1(0)$ $-p_1(\tau)x_1(\tau)$. Hence, our indirect utility function along the path will transform into analogue of direct utility function along this path, which will look like this: $\tilde{u}^C(x(\tau)) := \int_{x_1(0)}^{x_1(\tau)} f_1^{-1}(x_1(t))dx_1(t) + \sum_{l=2}^{N}\overline{p}_l x_l(\tau) + x_1(0)p_1(0)$. Using rules for differentiation of integrals with variable upper limit, take derivative of $\tilde{u}^C(x(t))$ with respect to $x_1(t)$ and evaluate at $t = \tau$. As a result, we have: $\partial\tilde{u}^C(x(\tau))/\partial x_1(\tau) = p_1(\tau)$: marginal utility of the first good (for new transformed utility function) is identically equal to the market price of this good. The same result can be derived if we recall that along our MAP new FOC for consumer with new direct utility function $\tilde{u}^C(x(\tau))$ will look as (along interior optima, evaluated at each τ): $\nabla\tilde{u}^C(x(\tau)) = \tilde{\lambda}(\tau)p(\tau)$, where $\tilde{\lambda}(\tau) = \partial\tilde{v}^C(\cdot)/\partial y(t) = 1$ is new Lagrange multiplier for the new indirect utility function, so we have absolutely the same result. Finally, integrating inverse demand function $\partial\tilde{u}^C(x(\tau))/\partial x_1(\tau) = p_1(\tau) = f_1^{-1}(x_1(\tau))$ from 0 to $x_1(1)$, or taking $p_1(0) = \infty$, and subtracting monetary spending for the first good $p_1(1)x_1(1)$ at final point, we have expression for the Area Variation Measure = Marshallian surplus: this is a total gain in new utility from consumption of the good 1 or gross surplus from the first good minus (additional) monetary spending for this good:

$$CS(0,1) = \int_{p_1(0)}^{p_1(1)} x_1(p_1(t))dp(t) = \int_0^{x_1(1)} \tilde{p}_1(x_1(t))dx_1(t) - p_1(1)x_1(1) + x_1(0)p_1(0)$$

$$= \tilde{u}^C(x_1(1), x_{i\neq 1}(1)) - \tilde{u}^C(o, x_{i\neq 1}(0)).$$

Q.E.D.

4. Conclusion

Old classical criticism against Dupuit' surplus concept, developed by [Marshall, 1890, 1920], [Hicks, 1943], [Samuelson, 1942] and other great theorists, was much more relevant in multi-dimensional, than in one-dimensional partial equilibrium setting, when only market price of the given good is changing, keeping other supply and demand determinants fixed. Applying a simple technique or trick, called *quasilinearization of preferences*, we can every time apply Marshallian surplus concept directly, without involving any unnecessary approximations. Even if consumer preferences are essentially non-quasilinear, in the general setting there exists every time monotonous transformation of original utility function, that generates an indirect utility that has *unitary marginal utility of numeraire, along given Walrasian demand curve*. And this new indirect utility function is *Marshallian surplus by itself*, as simple, as it was originally discovered by [Dupuit, 1844]. There is no need neither in construction of Hicksian compensated or equivalent variation[15] nor in developing more sophisticated techniques [Hausman, 1981] to compute true change in money metric utility[16]. Taking directly integral of observable market demand, we receive in moment what we need. Simplistic introductory micro textbooks appear really smarter than complicated discussions in advanced micro literature!

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[15] [Chipman and Moore, 1980] claimed that compensating variation does not work as correct money metric utility. A sum of compensating or equivalent variations suffers from famous Boadway' paradox. An aggregate surplus concept which we developed [Moskalionov, 2008] does not suffer from Boadway' paradox, but, for sure, is also controversial due to famous impossibility results. The partial equilibrium computation of individual surplus in simple case when one price is changing is the same as in intro textbooks but there are a lot of examples when partial equilibrium intuition will contradict with general equilibrium computations of the social surplus, see extensive discussion in our EERC paper, to be posted at www.eerc.ru.

[16] We don't want to produce impression that approximations to surplus are useless. On the opposite, Walrasian demand is the same difficult function to derive from observable data as compensated or Hicksian demand, so very often meaningful approximations work very well as an alternative empirical welfare estimations. See more [Slesnick, 1998].

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