Equilibrium Configurations of Distribution Channels in Bilaterally Oligopolistic Industries

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Abstract

The paper develops a model in which two manufacturers bid for representation by each of two available retailers who then choose noncooperatively which manufacturer’s bid, if any, to accept. This framework allows for interlocking relationships: each manufacturer can employ both retailers and conversely each retailer can represent both manufacturers. In contrast to the extant literature, which does not provide a classificatory characterization of equilibria in such a setting, the present paper establishes necessary and sufficient conditions for every distribution configuration to arise in equilibrium. The analysis is performed for different cases, namely, when bids are fully contingent and completely unconstrained and when they are subject to various constraints. In each case the paper identifies the conditions under which the most efficient configuration can be implemented as an equilibrium.

Keywords: Bilaterally Oligopolistic Industry, Market Structure, Retail Power.

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1 Introduction

The paper studies an equilibrium configuration of distribution channels in an environment where two manufacturers seek to distribute their products through two available retailers which are capable to exert some market power. The work is basically an extension of the Bernheim and Whinston (1998) approach where the authors explore similar issues but in a setting where manufacturers are locked into relationship with a single retailer. As the authors themselves acknowledge, the assumption that only one retailer is available is often unrealistic because manufacturers can usually find an alternative retailer to distribute their products. Although there are incidences where a single retailer acts as a ‘gatekeeper’ in a market for final consumers,\(^1\) in many industries retail markets are better described as oligopolies. For example, in their excellent study of retailing activity Dobson and Waterson (1999) find that “in an increasing number of markets, retailing is better viewed as competition between small number of strategic players”\(^2\).

The present paper differs from Bernheim and Whinston (1998) in that it treats the case when there are two competing (but possibly differentiated) retailers. In particular, it studies the game in which two manufacturers bid for representation by each of two available retailers who then choose non-cooperatively which manufacturer’s bid, if any, to accept. The environment which closely resembles to such a model is perhaps the Norwegian grocery sector. As reported by Gabrielsen and Sorgard (1999), large grocery retailers first decide how many brands to carry and then invite manufacturers to participate in an auction to compete for the distribution of these brands. After having received manufacturers’ bids, retailers finally choose manufacturers to establish trading relationships. In another motivating case, observed by Foros and Kind (2006), large retail chains usually form procurement alliances or buyer groups. Rather than allow manufacturers to negotiate with each sub-chain separately, the headquarters of each buyer group usually act as ‘gatekeepers’ to their own networks. Typically, each headquarter runs a procurement auction and grants access in all-or-nothing manner to whomever offers the best deal. As the European Commission (1996) states, the leading Finish retailer groups, Kesko and Tuko, are organized in this way.

Building on the approach of Bernheim and Whinston, I allow bids to be contingent on a particular distribution configuration. The motivation is that firms will probably renegotiate their contract terms in case some expected trading links have not been actually established. Letting each manufacturer submit a bid for every possible configuration can be viewed as a “short-cut” of such “reactive renegotiation”.

In such a setting, I first derive necessary and sufficient conditions for every possible distribution configuration to arise in equilibrium. I then use these

\(^1\)Specifically, Bernheim and Whinston (1997) apply their analysis to the case, Standard Fashion Company v. Magrane-Houstoun Company (1922), where Magrane-Houstoun was a single distributor of dress patterns.

\(^2\)Dobson and Waterson (1999), page 138.
conditions to examine whether the most efficient configuration, defined as the one which generates the greatest total surplus, can be an equilibrium outcome. The analysis is performed for the case when manufacturers submit any bids (including negative ones) and for the case when bids are subject to various constraints.

When bids are unconstrained, I find that any distribution configuration, except monopoly and foreclosure ones,\(^3\) can always be implemented as an equilibrium. Monopoly can never be an equilibrium while foreclosure can be equilibrium only if it generates the total surplus which is greater than the one generated by exclusive dealing.\(^4\) Moreover, in the most preferred equilibria, except foreclosure ones, manufacturers jointly earn all the surplus from the trade while in all foreclosure equilibria they earn zero.

Thus, if one believes that only Pareto-undominated equilibria (from the viewpoint of manufacturers) are likely to arise, then foreclosure will probably never be observed in such an environment. For its occurrence in this case must be due to a coordination failure among manufacturers. This implies a subtle difference between my results and those of Bernheim and Whinston. In particular, they demonstrate that the form of distribution configuration that maximize total surplus is always a Pareto-undominated equilibrium for manufacturers. Consequently, in their setup the market outcome is always efficient. Here, in contrast, the market outcome is inefficient whenever monopoly or foreclosure generates the greatest total surplus. This is because monopoly is never an equilibrium while foreclosure, whenever it can arise, is always a Pareto-dominated equilibrium for manufacturers.

The intuition for these results is as follows. A particular distribution configuration can be sustained in equilibrium only if no manufacturers finds it profitable to deviate by inducing a different distribution configuration. When bids are fully contingent, manufacturers can easily deter all deviations to all configurations, in which at least one retailer acts as a common agent. For it suffices that they demand sufficiently high payments from this retailer, so that it will never accept to distribute the products of both manufacturers. Next, when negative bids are allowed, all deviations to all (remaining) configurations, in which each retailer carries one product only, can also be deterred. For it suffices that one manufacturer actually promise to pay each retailer for the exclusive distribution of its product so that the other manufacturer will never find it profitable to outbid these offers. The fact that all deviations can be deterred in this way in turn implies that any distribution configuration (but foreclosure and monopoly) can be implemented as an equilibrium. Moreover, in any such equilibrium (again except foreclosure and monopoly) retailers earn zero. This may seem somewhat surprising, given that they eventually face positive offers (for at least exclusive representation). The reason is that manufacturers now

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\(^3\)A monopoly configuration is the one in which trading relationship is established between only one manufacturer and one retailer. A foreclosure configuration is the one in which one manufacturer is excluded from the market.

\(^4\)An exclusive dealing configuration is the one in which each manufacturer distributes its product through one retailer only.
exploit mis-coordination among retailers. In particular, they induce them to play the game that has as a pure-strategy equilibrium only the outcome where both retailers are worse off.

In contrast, under foreclosure manufacturers compete in a Bertrand-type fashion to sign up both retailers into exclusive relationship. Consequently, in order to win both auctions, each manufacturer has to leave all their bilateral surplus to each retailer. In addition, since in any such equilibrium a manufacturer always contemplates the possibility to employ one retailer only, it can exist only if the bilateral surplus is greater under foreclosure than under exclusive dealing.

Given the assumptions that bids are fully contingent and totally unconstrained play a critical role for the sustainability of equilibria, it is natural to ask how the results would alter if they were subject to some vertical restraints. Guided by this intuition, I consider two types of restraints. In the first case, I assume that manufacturers face limited liability and therefore cannot submit negative bids. Not surprisingly, limited liability is found to further impede the implementation of the most efficient configuration as an equilibrium. In particular, any configuration, in which at least one retailer acts as a common agent, will not necessarily arise in equilibrium even if it generates the greatest total surplus while the foreclosure configuration can be the unique equilibrium even if it is inefficient. Intuitively, when manufacturers cannot make payments to retailers, it becomes more difficult to deter foreclosure deviations. As a result, in order that a given configuration be an equilibrium, it must generate total surplus which sufficiently exceeds the one under foreclosure.

In the second case, I consider the situation where manufacturers make less contingent offers. This can happen because fully contingent contracts may be found uncompetitive or because the parties involved in a contract may not know the nature of the relationship between the other parties. Intuitively, less contingent offers reduce manufacturers’ ability to deter deviations. As a result, manufacturers may obtain lower equilibrium payoffs and in some circumstances may not even be able to sustain the most efficient configuration as an equilibrium. Furthermore, with less contingent offers manufacturers have less instruments to induce retailers to make the decisions in favor of manufacturers. This may give rise to the situation where even the most efficient configuration cannot be implemented as the unique continuation equilibrium.

The rest of the paper is organized as follows. Section 2 reviews the relevant economic literature. Section 3 introduces the framework and considers as a benchmark the case when only one retailer is available and manufacturers are constrained by limited liability. Section 4 treats the case of two retailers and derives necessary and sufficient conditions for every distribution configuration to arise in equilibrium subject to manufacturers’ limited liability constraint. Building on this analysis, it also derives similar conditions in the case when negative bids are allowed. Using the results obtained, it identifies the circumstances under which the most efficient configuration can be an equilibrium. Section 5 discusses similar issues but in a context of less contingent offers. Section 6 asks how the allocation of the bargaining power between manufacturers and retail-
ers affect the implementation of the most efficient configuration in equilibrium. Section 7 concludes.

2 Review of the literature

The economic literature on modeling an environment where both the upstream and downstream markets are concentrated is still in its infancy stage. A notable exception is the case where one of these markets is effectively monopolized. In this context the literature typically addresses the questions of whether exclusion is possible and whether an equilibrium outcome is efficient from the viewpoint of firms. It turns out that even in this admittedly simplified setting no general conclusion can be drawn and the answer usually depends on allocation of bargaining power between the parties as well as on the nature of the contracts they sign. For example, Mathewson and Winter (1987) study the situation where two rival manufacturers distribute their products through a monopolistic retailer. Assuming that bargaining power is entirely upstream and restricting the set of feasible contracts to linear wholesale prices, they find that exclusion can arise as the unique equilibrium outcome. In contrast, by allowing for the efficient contracts such as (non-contingent) two-part tariffs, in a similar setting Bernheim and Whinston (1998) show that exclusion does not occur and firms can achieve the profits of the vertically integrated structure.6

Likewise, in the other extreme case where a single manufacturer sells its product to two possibly differentiated retailers, downstream foreclosure may or may not arise. In this setup, Marx and Shaffer (2006) assume that retailers have all the bargaining power and offer non-contingent “three-part tariffs”.7 Their main result is that in all equilibria only one retailer purchases from the manufacturer. The exclusion is inefficient because the overall joint profit of the manufacturer-retailer pair is less than the joint profit that potentially could have been obtained if both retailers had purchased from the manufacturer.8 Building on their analysis but instead allowing for contingent three-part tariffs, Rey et al. (2005) reach strikingly different conclusions. In particular, they show that there exist equilibria in which both retailers purchase from the manufacturer and in which firms can achieve the industry-wide monopoly profit.9

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5 To be more precise, although exclusive equilibria exist, they are Pareto-dominated by common agency equilibria.
6 Note, finally, that if the retailer had instead the initiative to make offers, it would prefer to contract with both manufacturers. In which case it would set the transfer price equal to the costs of production and thus sustain the industry monopoly outcome.
7 A three-part tariffs contract includes an upfront payment (slotting allowance), which is paid by the manufacturer regardless of whether the retailer will buy a product afterwards, and a conditional fixed fee which is paid by the retailer only if it actually buys a product.
8 The authors also consider a situation where retailers offer non-contingent two-part tariffs. In which case they find that if the offers are made simultaneously then no pure-strategy equilibrium exists. However, if the they are made sequentially then in any equilibrium both retailers purchase from the manufacturer.
9 As the authors notice, if instead the manufacturer had all the bargaining power then efficiency could also be restored even without slotting allowances or conditional fees. In which
While most of the existing literature focuses on the case where monopoly is either upstream or downstream, relatively few papers treat the case where competition exists at both levels and, in addition, retailers are capable to exert some market power vis-à-vis manufacturers.\textsuperscript{10} Dobson and Waterson (2001) are perhaps the first to explore this issue. They develop a model where two differentiated manufacturers distribute their products through two differentiated retailers. The market power of retailers stems from the fact that they are allowed to choose whether to enter into exclusive relationship with a manufacturer.\textsuperscript{11} In the absence of exclusive dealing, wholesale contracts are restricted to linear transfer prices while exclusive dealing contracts instead involve two-part tariffs. This clearly implies that the benefit from establishing exclusive dealing relationship is that it allows the parties to avoid the double marginalization problem while the cost is that both the retailer and the manufacturer have to limit their product ranges. Not surprisingly, an exclusive dealing equilibrium arises only if manufacturers’ products and retailers’ services are slightly differentiated. Otherwise, an equilibrium involves both retailers distributing the products of both manufacturers.

Using a similar framework but focusing on non-contingent two-part tariffs, Rey and Verge (2004) study the possibility of double common agency to arise in equilibrium.\textsuperscript{12} Although in their model manufacturers make take-or-leave-it offers, still retailers are capable to exert some market power by refusing to carry the manufacturer’s brand. The authors find that the full analysis in such a setup is technically complicated and neither derive prices nor establish conditions for the existence of a double common agency equilibrium.\textsuperscript{13} Instead they show that such an equilibrium does not always exist despite the fact that there is a positive case, it could charge wholesale prices above marginal costs, so as to ensure that consumer prices be equal to monopoly levels, and recover any remaining retail profit through fixed fees. It is important, however, that in all these papers manufacturers are assumed to have all the bargaining power and thus effectively impose various restraints on retailers.\textsuperscript{14} The situation where there exists both upstream and downstream competition has been extensively studied in the literature of vertical restraints. Representative examples include Bonanno and Vicker (1988), Rey and Stiglitz (1988, 1995), Lin (1990), O’Brien and Shaffer (1993), Besanko and Perry (1994), Moner et al (2004). It is important, however, that in all these papers manufacturers are assumed to have all the bargaining power and thus effectively impose various restraints on retailers.

\textsuperscript{10}In their paper, exclusive trading implies that a retailer purchases the product only from one manufacturer and this manufacturer in turn distributes its product only through this retailer.

\textsuperscript{11}In a double common agency situation each manufacturer uses both retailers, and conversely each retailer carries both brands.

\textsuperscript{12}Intuitively, the complexity of the analysis stems from the following fact. When offers are non-negotiated and products are substitutable, a retailer can gain more by distributing one product only. This implies that manufacturers must leave a rent to retailers to convince them to carry their products. Since in any double common agency equilibrium manufacturers seek to minimize these rents then under some conditions it can be quite easy for a deviating manufacturer to sign up one or two retailers into exclusive dealing arrangements. This allows a manufacturer to deviate in many different ways and, as a result of the multiplicity of possible deviations, such an equilibrium there does not always exist.
demand for each manufacturer’s product at each store.

More recently Moner et al. (2005) slightly modify the approach of Dobson and Waterson (2001). In their model retailers are identical and, after manufacturers have publicly announced their linear wholesale prices, retailers non-cooperatively choose to carry none, one or two brands. Assuming that in the market for final consumers retailers compete à la Cournot, the authors find that in equilibrium both retailers choose to be multi-brand sellers for all degrees of product differentiation.\footnote{In the paper the authors also study the case when the profitability of brands differ. In particular, when it diverges significantly, an equilibrium may involve both retailer distributing the most profitable brand only.}

The present paper differs from the extant literature mainly in two respects. First, it treats the joint surplus earned by manufacturer(s) and retailer(s) as exogenously given. The advantage of such an approach is that it can be applied to a variety of economic environments. For example, the results do not depend on a particular type of contracts signed by manufacturers and retailers, on whether retailers compete in prices or quantities, or whether there are economies of scale in production or distribution. Consequently, I can make predictions about the structure of an industry under quite general conditions. However, the cost of this approach is that it does not allow us to fully evaluate the welfare consequences of a particular distribution configuration arising in equilibrium.

The second difference from the aforementioned papers is that I allow contracts to be contingent on a particular distribution configuration. Besides the fact that this assumption seems more realistic in an environment where firms can renegotiate their contract terms, in addition, it allows manufacturers to sharply limit the scope for possible deviations. Thus, in contrast to Rey and Verge (2004) where the main difficulty stems from multiplicity of deviations, here this problem is considerably simplified. As a result, I am capable to obtain necessary and sufficient conditions for every possible distribution configuration to arise in equilibrium.

3 Framework

Consider an environment where two upstream firms (manufacturers \(h, k \in \{A, B\} \)) compete for representation by two downstream firms (retailers \(i, j = 1, 2\)).

Let \(S = \{\emptyset, MA, MB, MA\&MB\} \) denote the set of possible representations available to retailer \(i\) and \(s_i\) denote an element from \(S\). If \(s_i = \emptyset\) then retailer \(i\) represents neither of the manufacturers, if \(s_i = MA\) then it represents manufacturer \(h\) and if \(s_i = MA\&MB\) then it represents both. Any pair \((s_1, s_2)\) defines a distribution configuration. Then sixteen different distribution configurations arise. To simplify the matters, I assume that manufacturers and retailers are equally efficient. Therefore, by symmetry, only six of them need to be distinguished which are shown on figure 1 and characterized below.
- **Common agency:** one retailer is inactive while the other retailer represents both manufacturers and thus acts as a multi-product monopolist on the downstream market. This refers to the \((M_A&M_B, \emptyset)\) and \((\emptyset, M_A&M_B)\) configurations.

- **Monopoly:** one manufacturer and one retailer are inactive while the other manufacturer and the other retailer enter in a trading relationship. This refers to the \((M_h, \emptyset)\) and \((\emptyset, M_h)\) configurations for \(h = A, B\).

- **Exclusive dealing:** each retailer represents one manufacturer only. This refers to the \((M_A, M_B)\) and \((M_B, M_A)\) configurations.

- **Mixed configuration:** one retailer represents one manufacturer while the other retailer represents both. This refers to the \((M_A&M_B, M_A)\), \((M_A&M_B, M_B)\), \((M_A, M_A&M_B)\) and \((M_B, M_A&M_B)\) configurations.

- **Full competition or double common agency:** each retailer represents both manufacturers and thus acts as a multi-brand dealer. This refers to the \((M_A&M_B, M_A&M_B)\) configuration.

- **Foreclosure:** one manufacturer is inactive while the other one is represented by both retailers. This refers to the \((M_A, M_A)\) and \((M_B, M_B)\) configurations.

Building on the approach of Bernheim and Whinston (1998), the interaction between manufacturers and retailers is modeled as the three-stage game \(G\).

**Stage 1.** Each manufacturer \(h \in \{A, B\}\) announces a menu \(\Sigma_h\). All menus are simultaneous and public and each menu consists of “required payments” which each retailer \(i = 1, 2\) pays to manufacturer \(h\) in the event it accepts to represent manufacturer \(h\). Each payment corresponds to a particular distribution configuration and therefore \(\Sigma_h\) consists of:

- common agency payments \(U_{CA}^{i,h}\) for \(i = 1, 2\) in the event retailer \(i\) chooses to represent both manufacturers while retailer \(j \neq i\) chooses to represent none,

- monopoly payments \(U_{M}^{i,h}\) for \(i = 1, 2\) in the event retailer \(i\) chooses to represent manufacturer \(h\) only while retailer \(j \neq i\) chooses to represent none,

- exclusive dealing payments \(U_{ED}^{i,h}\) for \(i = 1, 2\) in the event retailer \(i\) chooses to represent manufacturer \(h\) only while retailer \(j \neq i\) chooses to represent manufacturer \(k\) only,

- mixed payments: \(U_{MX}^{i,h}\) in the event retailer \(i = 1, 2\) chooses to represent manufacturer \(h\) only while retailer \(j \neq i\) chooses to represent both, \(u_{MX}^{i,h}\) (or \(v_{MX}^{i,h}\)) in the event retailer \(i = 1, 2\) chooses to represent both manufacturers and retailer \(j \neq i\) chooses to represent manufacturer \(h\) (or \(k\)),
• competitive payments $U_{hi}^C$ for $i = 1, 2$ in the event both retailers choose to represent both manufacturers,

• foreclosure payments $U_{hi}^F$ for $i = 1, 2$ in the event both retailers choose to represent manufacturer $h$ only.

One way to think about these payments is that they are demanded for the establishment of trading relationship with a manufacturer before the parties sign any contract. Throughout, I shall assume that manufacturers are constrained by limited liability and thus cannot offer negative payments. As it will be clear later, if negative payments were allowed the analysis would be slightly modified. This, however, would lead to different conclusions and therefore I shall discuss this case as well.

Stage 2. Each retailer $i = 1, 2$ chooses an element $s_i \in S$. All retailers’ decisions are simultaneous and public and after they have been made a distribution configuration, $(s_1, s_2)$, is realized.

Stage 3. Each retailer signs a contract with each manufacturer that it has chosen to represent. Similarly to Bernheim and Whinston, I treat the process of this stage as a “black box” and assume that the joint surplus that the concerned parties earn through the sales in a given outlet is $\Pi^{CA}$ under common agency, $\Pi^M$ under monopoly, $\Pi^{ED}$ under exclusive dealing, $\Pi^C$ under full competition, $\Pi^F$ under foreclosure, $\Pi^{MX}$ if a retailer contracts with both manufacturers and $\Pi^{MX}$ if a retailer contracts with one manufacturer in either mixed configuration. Finally, each retailer $i$ pays the required payment(s) that correspond to the realized distribution configuration, $(s_1, s_2)$. If it has chosen $s_i = \emptyset$ then it pays nothing and earns zero.

An important element of the model is that in the second stage retailers make their decisions of which, if any, manufacturer to represent non-cooperatively. This is a simple way to capture the fact that there is downstream competition for manufacturer’s representation. To formalize the matters, I denote by $g(\Sigma_A, \Sigma_B)$ the second stage continuation game between retailers, after manufacturers have proposed the menus $\Sigma_A$ and $\Sigma_B$ in stage one. Taking into account the outcomes of stage three, Table 1 represents $g(\Sigma_A, \Sigma_B)$ where each cell of the matrix gives the retailers’ payoffs (the upper line in each cell shows retailer 1’s payoff) in the corresponding distribution configuration.

A key assumption underlying the model is that manufacturers are allowed to condition their payments upon the distribution configuration. This is a simple way to capture the fact that trading parties will likely renegotiate their contract terms in response to a change of the market environment. Letting each manufacturer to propose a payment for every possible distribution configuration is a way to model this “reactive renegotiation”.

Another crucial assumption is that the manufacturers’ offers are public. It greatly simplifies the equilibrium analysis of $g(\Sigma_A, \Sigma_B)$ and allows us to avoid technical difficulties related to the definition of reasonable conjectures in the event of unexpected offers. On the other hand, the observability of retailers’ acceptance decisions is not important because in any equilibrium each retailer
(and therefore each manufacturer) can correctly anticipate the behavior of its rival in the second stage and therefore the parties will expect to obtain the same payoffs in the subsequent contracting stage.

Before proceeding to solve the game $G$, consider as a benchmark the case of a single retailer.

### 3.1 Benchmark: a single retailer

When a single retailer is available, only four market configurations are possible, i.e., common, monopoly (or exclusive) and null representations. Therefore the contingent menu $\Sigma_h$ boils down to the contingent pair $(U^M_h, U^{CA}_h)$ for $h \in \{A, B\}$.

**ASSUMPTION 1.** $\Pi^{CA} < 2\Pi^M$.

This assumption implies that when products are substitutes, a marginal contribution to total surplus from a sale of a product is larger when this product is sold alone than when both products are sold.

Though the contingent offers are now constrained to be non-negative, the equilibrium analysis is similar to the one performed in Bernheim and Whinston (1998) (i.e., where negative offers are permitted). Hence, I omit the details and state the final result.

**Proposition 1.**

(i) There always exist two exclusive dealing equilibria where either manufacturer is active. In both cases the retailer earns $\Pi^M$ while manufacturers earn zero.

(ii) Common agency equilibria exist if and only if $\Pi^{CA} \geq \Pi^M$. In which case both manufacturers prefer the equilibrium in which each of them earns its marginal contribution to total surplus, $\Pi^{CA} - \Pi^M$, while the retailer earns $2\Pi^M - \Pi^{CA}$.

From the proposition it follows that, even when manufacturers are bound by limited liability, the form of representation is chosen to maximize the joint surplus of the manufacturers and the retailer. In particular, common agency will necessarily arise whenever it generates the greatest joint surplus. To understand this result, recall that when manufacturers seek to implement common agency as an equilibrium they pursue two goals. The first is to extract more rent from a common agent and the second is to enhance the stability of the equilibrium. When one retailer is available, these tasks are interrelated: by raising monopoly offers, manufacturers reduce the rent attributed to the retailer, however, by doing so, they also increase the scope for monopoly deviations. In the most preferred equilibrium manufacturers resolve this trade-off by choosing both monopoly and common agency offers to be non-negative and equal to $\Pi^{CA} - \Pi^M$. Therefore, the limited liability constraint has no impact on the equilibrium offers and, as a consequence, on the condition governing the existence of common agency equilibria.

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15 The proof is available upon request.
4 Solving the model

I use subgame-perfect Nash equilibrium as an equilibrium concept and therefore solve the game $G$ in a recursive manner. In stage two retailers choose which, if any, manufacturer to represent while anticipating the consequences of their choices in stage three. Since they make their decisions non-cooperatively, the configuration that arises in stage two must be an equilibrium of the corresponding retailer game. In stage one, manufacturers choose their contingent offers to maximize their payoffs while anticipating the effects of their actions on the subsequent play of the game.

To formalize the matters, the pair of menus $(\Sigma_A, \Sigma_B)$ gives rise to the $(s_1, s_2)$ configuration and constitutes an equilibrium of $G$ if it satisfies the following conditions:

(i) $(s_1, s_2)$ is the unique pure-strategy equilibrium in $g(\Sigma_A, \Sigma_B)$.\(^{16}\)

(ii) No manufacturer $h \in \{A, B\}$ can gain by offering a menu $\tilde{\Sigma}_h \neq \Sigma_h$. Any such deviation implies that manufacturer $h$ induces the game $g(\tilde{\Sigma}_h, \Sigma_k)$ which has some configuration, $(\tilde{s}_1, \tilde{s}_2)$, as a pure-strategy equilibrium.\(^{17}\)

The present model can have multiple equilibria. Following Bernheim and Whinston (1998), I restrict attention on the equilibria that are most preferred by manufacturers. Similarly to their work, I establish conditions, under which each of the aforementioned distribution configurations arises in equilibrium of $G$, and thus make conclusions regarding the occurrence of the efficient form of equilibrium representation.

4.1 Common agency equilibria

Since retailers are identical, then for the sake of concreteness suppose that manufacturers wish to implement the $(M_A&M_B, \emptyset)$ configuration as a continuation equilibrium. Denote by $\Sigma^{CA}_h$ a contingent menu offered by manufacturer $h$ and let us start with the conditions that ensure that $(M_A&M_B, \emptyset)$ is an equilibrium in the corresponding retailer game $g(\Sigma^{CA}_A, \Sigma^{CA}_B)$.

Since the second retailer is available, then it must be that if retailer 1 chooses common representation, retailer 2 chooses to represent no manufacturer, i.e.,

$$U^{MX}_{A2}, U^{MX}_{B2} > \Pi^{MX} \text{ and } U^{CA}_{A2} + U^{CA}_{B2} > \Pi^{CA}_{inact}.$$  

Conversely, if retailer 2 chooses to be inactive, retailer 1 must represent both manufacturers, i.e.,

$$\Pi^{CA} - U^{CA}_{A1} - U^{CA}_{B1} \geq \max\{0, \Pi^{M} - U^{M}_{A1}, \Pi^{M} - U^{M}_{B1}\}.$$  

\(^{16}\)The uniqueness is required in order to avoid the ambiguity related to the selection of a particular equilibrium in case of multiple continuation equilibria in $g(\Sigma_A, \Sigma_B)$.

\(^{17}\)Deviations which lead to multiplicity of continuation equilibria can be omitted. As I will show below, this is because a deviating manufacturer can always ensure that the continuation equilibrium which yields the highest payoff is unique.
Note that in any equilibrium the above condition cannot hold with strict inequality because manufacturer \( h \) could then profitably deviate by slightly increasing \( U^{CA}_h \). In other words, retailer 1 must be indifferent between accepting both manufacturers’ offers, only one or none, i.e.,

\[
\Pi^{CA} - U^{CA}_{A1} - U^{CA}_{B1} = \max\{0, \Pi^M - U^M_{A1}, \Pi^M - U^M_{B1}\}. \quad \text{CA}^{\text{act}}
\]

The next step is to consider possible deviations by manufacturers. For this purpose I assume (and show in the Appendix that this is indeed the case) that \( \Sigma^{CA}_A \) and \( \Sigma^{CA}_B \) induce the retailer game \( g(\Sigma^{CA}_A, \Sigma^{CA}_B) \) that has \( (M_A&M_B, \emptyset) \) as the unique pure-strategy equilibrium.

**Foreclosure deviations.** Suppose that manufacturer \( h \) wishes to destroy common agency and instead induce both retailers to distribute exclusively its product. In which case it announces the menu \( \tilde{\Sigma}^F_h \) such that the corresponding continuation game \( g(\tilde{\Sigma}^F_h, \Sigma^{CA}_k) \) has the \( (M_h, M_h) \) configuration as an equilibrium. The latter is possible only if the following conditions hold:

\[
\Pi^F - \tilde{U}^F_{hi} \geq \max\{0, \Pi^{ED} - U^{ED}_{ki}, \Pi^{MX}_{A1} - \tilde{u}^{MX}_{hi} - \tilde{v}^{MX}_{ki}\}, \quad \text{DEV}^F_{hi}
\]

for \( i = 1, 2 \). These conditions imply that if retailer \( i \) chooses to represent manufacturer \( h \) then so does retailer \( j \).

Note that, by setting \( \tilde{u}^{MX}_{hi} = \infty \), manufacturer \( h \) can only facilitate its foreclosure deviations because, by doing so, it reduces the set of alternative representations available to retailer \( i \). Hence, the necessary conditions governing the possibility of such deviations can be written as follows:

\[
\tilde{U}^F_{hi} \leq \min\{\Pi^F, (\Pi^F - (\Pi^{ED} - U^{ED}_{ki})\}, \quad \text{DEV}^F_{hi}
\]

for \( i = 1, 2 \).

In fact, \( \text{DEV}^F_{h1} \) and \( \text{DEV}^F_{h2} \) are also sufficient conditions. To show this, suppose that manufacturer \( A \) announces the following menu \( \tilde{\Sigma}^F_A \) of contingent payments:18

- \( \tilde{U}^{MX}_{A2} < \Pi^{MX} \) so that \( (M_A&M_B, \emptyset) \) is no longer an equilibrium because an inactive retailer is now willing to accept an offer of manufacturer \( A \).
- \( \tilde{u}^{MX}_{A1} = \infty \) and \( \tilde{U}^F_{Ai} \) satisfies \( \text{DEV}^F_{Ai} \) for \( i = 1, 2 \) so that \( (M_A, M_A) \) becomes instead an equilibrium.
- The other payments of \( \tilde{\Sigma}^F_A \) coincide with their counterparts of \( \Sigma^{CA}_A \).

\(^{18}\)Although there can be other deviation strategies that lead to foreclosure, it is not important because they only differ in the way the common agency equilibrium is destroyed. A common and essential feature of all such strategies is that the \( \text{DEV}^F_i \) constraints must be satisfied for \( i = 1, 2 \).
By construction, \((M_A, M_A)\) is an equilibrium of \(g(\Sigma^F_A, \Sigma^A_A)\). Moreover, it is the unique equilibrium, given that \((M_A, M_B, \emptyset)\) is the unique equilibrium of \(g(\Sigma^A_A, \Sigma^B_B)\).\(^{19}\)

**Exclusive dealing deviations.** Suppose now that manufacturer \(h\) deviates by inducing as a continuation equilibrium the configuration in which retailer \(i\) distributes exclusively its product while retailer \(j\) distributes exclusively the product of its rival. Denote by \(\hat{\Sigma}^ED_h\) the menu announced by manufacturer \(h\) in this case. First of all, in order that such deviations be possible, retailer \(j\) must be willing to accept an exclusive offer of manufacturer \(h\), i.e., it must be \(U^i_{kj} \leq \Pi^ED\). In that case, the configuration under study can be an equilibrium in the retailer game \(g(\hat{\Sigma}^ED_h, \Sigma^C_A)\) only if the following condition holds:

\[
\Pi^ED - \tilde{U}^ED_{hi} \geq \max\{0, \Pi^F - U^F_{ki}, \Pi^MX - \tilde{u}^M_{hi}, \tilde{v}^M_{hi}\}.
\]

That is, retailer \(i\) must find it profitable to represent exclusively manufacturer \(h\). As before, manufacturer \(h\) can only facilitate its exclusive dealing deviations by setting \(\tilde{v}^M_{hi} = \infty\). Therefore, the necessary conditions for such deviations to be possible are as follows:

\[
\tilde{U}^ED_{hi} \leq \min\{\Pi^ED, \Pi^ED - (\Pi^F - U^F_{ki})\}, \quad \text{DEV}^{ED}_{hi}
\]

and

\[
U^ED_{kj} \leq \Pi^ED, \quad \text{DEV}^{ED}_{kj}
\]

where \(i \neq j\).

I now show that \(\text{DEV}^{ED}_{hi}\) and \(\text{DEV}^{ED}_{kj}\) are also sufficient conditions. For this, suppose \(U^{ED}_{B2} \leq \Pi^{ED}\) and consider the following menu \(\hat{\Sigma}^ED\) announced by manufacturer \(A\):\(^{20}\)

- \(\tilde{U}^CA_{A1} = \infty\) and \(\tilde{U}^M_{A1} \leq \min\{0, U^M_{B1}\}\) so that \((M_A, M_B, \emptyset)\) is no longer an equilibrium because retailer 1, acting before as a common agent, now prefers to represent exclusively manufacturer \(A\).

- \(\tilde{U}^F_{A2} = \tilde{u}^M_{A2} = \infty\) so that retailer 2 now rejects to deal with manufacturer \(A\) and instead chooses to represent manufacturer \(B\) (since by supposition \(U^{ED}_{B2} \leq \Pi^{ED}\)) if it anticipates that retailer 1 will represent exclusively manufacturer \(A\). \(\tilde{v}^M_{A1} = \infty\) and \(\tilde{U}^{ED}_{A1}\) satisfies \(\text{DEV}^{ED}_{A1}\). Taken together, these conditions imply that \((M_A, M_B)\) is now an equilibrium.

- The other payments of \(\hat{\Sigma}^ED\) coincide with their counterparts of \(\Sigma^C_A\).

\(^{19}\)This is because, by construction, \(\hat{\Sigma}^F\) does not alter the retailers’ payoffs in all other configurations but in \((M_A, M_B, M_A)\), \((M_A, M_A, M_B)\) and \((M_A, M_A)\). Consequently, if some configuration (except those three) was not an equilibrium in the former game \(g(\Sigma^C_A, \Sigma^C_B)\) it cannot be an equilibrium in the new game \(g(\hat{\Sigma}^F, \Sigma^C_B)\) either. Finally, neither \((M_A, M_B, M_A)\) nor \((M_A, M_A, M_B)\) can be an equilibrium of \(g(\hat{\Sigma}^F, \Sigma^C_A)\) because \(\tilde{u}^F_{Ai} = \infty\) for \(i = 1, 2\).

\(^{20}\)See footnote for a similar argument.
As in the case of foreclosure, one can check that \((M_A, M_B)\) is the unique equilibrium in \(g(\Sigma^{ED}_A, \Sigma^{CA}_B)\), given that \((M_A & M_B, \emptyset)\) was the unique equilibrium of \(g(\Sigma^{CA}_A, \Sigma^{CA}_B)\).

**Monopoly deviations.** Since any such deviation implies that one retailer must be inactive, it is possible only if this retailer rejects the exclusive dealing offer of a rival manufacturer, i.e., it must be

\[
U^{ED}_{hi} > \Pi^{ED}_{hi}
\]

for at least some \(h\) and some \(i\). If the above condition hold, then manufacturer \(k\) can always undertake monopoly deviations and earn at most \(\min\{\Pi^{M}, U^{M}_{hi}\}\).

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for at least some \(h\) and some \(i\). If the above condition hold, then manufacturer \(k\) can always undertake monopoly deviations and earn at most \(\min\{\Pi^{M}, U^{M}_{hi}\}\).

To see this, suppose, for example, that \(U^{ED}_{B2} > \Pi^{ED}_{B2}\) and consider the following menu \(\tilde{\Sigma}^M_A\) announced by manufacturer \(A\):

- \(\tilde{v}^{CA}_{A1} = \infty\) and \(\tilde{v}^{M}_{A1} \leq \min\{\Pi^{M}, U^{M}_{B1}\}\) so that \((M_A & M_B, \emptyset)\) is no longer an equilibrium because retailer 1, acting before as a common agent, now prefers to represent exclusively manufacturer \(A\).

- \(\tilde{v}^{F}_{A2} = \tilde{w}^{MX}_{A2} = \infty\) so that retailer 2 now rejects to represent either manufacturer (recall that \(U^{ED}_{B2} > \Pi^{ED}\)) if it anticipates that retailer 1 will represent exclusively manufacturer \(A\). Taken together, these conditions imply that \((M_A, \emptyset)\) is now an equilibrium.

- The other payments of \(\tilde{\Sigma}^M_A\) coincide with their counterparts of \(\Sigma^{CA}_A\).

One can check that, by deviating in this way, manufacturer \(A\) induces the game \(g(\tilde{\Sigma}^M_A, \Sigma^{CA}_B)\) that has \((M_A, \emptyset)\) as the unique pure-strategy equilibrium in which it earns \(U^{M}_{A1} \leq \min\{\Pi^{M}, U^{M}_{B1}\}\).

**Deviations to common agency, double common agency and mixed configurations.** Manufacturers can fully deter all such deviations by simply destroying common agency in all these configurations. That is, each manufacturer \(h \in \{A, B\}\) must then set

\[
v^{MX}_{hi} = u^{MX}_{hi} = U^{C}_{hi} = v^{CA}_{hi} = \infty,
\]

for \(i = 1, 2\).

So far I have shown that all deviations but possibly those to foreclosure, exclusive dealing and monopoly could in principle be deterred. The following lemma states the most effective way to deter the remaining deviations.

**Lemma 1.** When manufacturers are constrained by limited liability, the most effective way to deter deviations to foreclosure, exclusive dealing and monopoly is as follows:

\( (i) \) If \(\Pi^{ED} \leq \Pi^{F}\) then each manufacturer \(h \in \{A, B\}\) must set \(U^{ED}_{hi} = 0\), \(U^{F}_{hi} < \Pi^{F} - \Pi^{ED}\) and \(U^{M}_{hi} \geq 0\) for \(i = 1, 2\). In which case monopoly and exclusive dealing deviations are impossible while foreclosure deviations yield at most \(2(\Pi^{F} - \Pi^{ED})\).
(ii) If \( 0 < \Pi^{ED} - \Pi^F \leq 2\Pi^F \) then the lowest payoff that each manufacturer can earn by deviating is \( \Pi^{ED} - \Pi^F \). This would be the case if, for example, each manufacturer \( h \in \{A,B\} \) set \( \hat{U}_{hi}^F = 0, \hat{U}_{hi}^M \geq 0, \hat{U}_{hi}^{ED} < \Pi^{ED} \) for \( i = 1,2 \) and \( \hat{U}_{hi}^{ED} < \Pi^{ED} - \Pi^F \) for at least some \( i \). In that case monopoly and foreclosure deviations are impossible while exclusive dealing deviations yield at most \( \Pi^{ED} - \Pi^F \).

(iii) If \( 2\Pi^F < \Pi^{ED} - \Pi^F \) then each manufacturer \( h \in \{A,B\} \) must set \( \hat{U}_{hi}^F \geq 0, \hat{U}_{hi}^{ED} > \Pi^{ED} \) and \( \hat{U}_{hi}^{M} \leq \min\{\Pi^{M},2\Pi^F\} \) for \( i = 1,2 \). In which case exclusive dealing deviations are impossible while monopoly and foreclosure deviations yield at most \( 2\Pi^F \).

Proof. See the Appendix.

I now proceed to establishing necessary conditions for the existence of common agency equilibria. Suppose that \( \Pi^{ED} \leq \Pi^F \) then, by lemma 1, the most effective way to deter all deviations implies that only foreclosure deviations are possible and in that case a deviating manufacturer earns at most \( 2(\Pi^F - \Pi^{ED}) \).

Since the maximal joint payoff that manufacturers can obtain in any equilibrium is \( \Pi^{CA} \), then such equilibria can exist only if

\[
\Pi^{CA} \geq 4(\Pi^F - \Pi^{ED}) \quad \text{if} \quad \Pi^{ED} \leq \Pi^F.
\]

Suppose now that \( \Pi^{ED} > \Pi^F \). As lemma 1 implies, in seeking to minimize the gain from deviations, manufacturers choose between two alternatives: they can allow either exclusive dealing deviations or foreclosure and monopoly deviations only. Moreover, one should remember that, whenever monopoly deviations are important, manufacturers cannot extract all retailer’s surplus in equilibrium, as it follows from \( \text{CA}_{ac} \). Consequently, in that case monopoly offers must be chosen to resolve the trade-off between the rent extraction and the deterrence of deviations.

Consider, first, the case when \( 2\Pi^F \geq \Pi^{M} \). In this case case monopoly deviations, even if they are possible, effectively play no role because foreclosure deviations are more profitable. By lemma 1, whenever a manufacturer can deviate to foreclosure it can earn \( 2\Pi^F \). On the other hand, whenever it can deviate to exclusive dealing it can earn at most \( (\Pi^{ED} - \Pi^F) \). Hence, in this case the necessary condition for the existence of common agency equilibria can be written as follows:

\[
\Pi^{CA} \geq 2\min\{2\Pi^F,\Pi^{ED} - \Pi^F\}.
\]

Consider now the case \( 2\Pi^F < \Pi^{M} \) which implies that, whenever monopoly deviations are allowed, they may be chosen by manufacturers. In order to reduce the gain from such deviations, manufacturers should lower monopoly offers. However, by doing so, they leave more rent to the common agent and thus reduce their equilibrium payoffs. As proposition 1 states, the most effective way to resolve this trade-off is to set both monopoly and common agency offers equal to \( \Pi^{CA} - \Pi^{M} \). Given that foreclosure deviations are also possible and yield at most \( 2\Pi^F \), no manufacturer will undertake such deviations only if \( \Pi^{CA} - \Pi^{M} \geq 2\Pi^F \).
Finally, since manufacturers can always adopt the strategies which allow only exclusive dealing deviations, in this case such equilibria can exist only if

\[ \Pi^{CA} \geq \min \left\{ 2(\Pi^{ED} - \Pi^F), \Pi^M + 2\Pi^F \right\} . \]

Taken all results together, the necessary condition for the existence of common agency equilibria can be stated as follows:

\[ \Pi^{CA} \geq \min \left\{ 2(\Pi^{ED} - \Pi^F), \max \{ 4\Pi^F, \Pi^M + 2\Pi^F \} \right\} \text{ if } \Pi^{ED} > \Pi^F. \tag{2} \]

The Appendix shows that (1)-(2) are also sufficient conditions and defines the most preferred common agency equilibria for manufacturers. The following proposition summarizes the main results in this section.

**Proposition 2.** When the manufacturers are constrained by limited liability, common agency equilibria exists if and only if (1)-(2) hold. Moreover, if the following conditions hold:

\[ 2(\Pi^{ED} - \Pi^F) > \Pi^{CA} \geq \Pi^M + 2\Pi^F > 4\Pi^F, \tag{3} \]

then both manufacturers prefer the equilibria in which each of them earns \( \Pi^{CA} - \Pi^M \) while the retailer earns \( 2\Pi^M - \Pi^{CA} \). In other cases, whenever such equilibria exist, both manufacturers prefer the ones in which they jointly earn \( \Pi^{CA} \) while the retailer earns zero.

**Proof.** See the Appendix.

Intuitively, condition (3) implies that exclusive dealing deviations are most profitable and, in addition, they destroy common agency equilibria. Hence, they constitute the main issue. The most effective way to deter such deviations is to make it impossible for a deviating manufacturer to induce an exclusive dealing configuration as a continuation equilibrium. That is, to make it that, whenever one retailer is willing to accept an exclusive dealing offer of one manufacturer, the other retailer always rejects a similar offer of the other manufacturer (and thus represents either none or only one manufacturer). Next, the condition \( 4\Pi^F < \Pi^M + 2\Pi^F \) (or \( 2\Pi^F < \Pi^M \)) implies that a single manufacturer prefers to distribute its product through one retailer only. Naturally, in this case the retailer obtains some market power and therefore it must earn some rent in equilibrium.

As the Appendix shows, in other cases manufacturers are capable to extract all surplus of the common agent. This is not a priory obvious in the setting where they have to make the offers which leave some rents to retailers (e.g., in order to maximally deter foreclosure deviations it must be \( U_{kt}^{ED} < \Pi^{ED} \) for at least some \( i \)). The intuition is that manufacturers now exploit the facts that offers are fully contingent and retailers make their decisions non-cooperatively. Taken together, this allows manufacturers to induce retailers to play a mis-coordination game that has as an equilibrium only the outcome (i.e., the common agency configuration) where both retailers are worse off.
The analysis implies that foreclosure and exclusive dealing deviations basically pose the main problem for the sustainability of an equilibrium. Intuitively, the most effective way to deter them is to intensify bidding competition for exclusive representation, i.e., to set all exclusive bids at the lowest level. When manufacturers face limited liability, the lowest bid is zero. Since $\Pi^F$ and $\Pi^{ED}$ generally differ, this, however, is not sufficient to fully deter foreclosure and exclusive dealing deviations. In contrast, if negative bids were allowed, all deviations could be deterred. For it suffices that one manufacturer actually promise to pay each retailer (i.e., submit negative bids) for exclusive distribution of its product so that the other manufacturer will never find it profitable to outbid these offers. As the Appendix shows, in that case a common agency equilibrium can always be sustained. Furthermore, in the best equilibrium manufacturers jointly earn $\Pi^{CA}$.

Taken together, these results lead to the following conclusions. First, unlike the case of a single retailer, in the case of two retailers manufacturers can fully extract all retailer’s surplus. In addition, when manufacturers are allowed to submit negative bids, they can always induce common agency representation. In contrast, limited liability, though it does not impede manufacturers to fully extract retailer’s surplus, may yet cause a difficulty in implementing common agency even if it is most efficient. This is due to manufacturers’ inability to fully prevent all foreclosure deviations. As it follows from (1), even if it generates the greatest total surplus (so that in particular $\Pi^{CA} > 2\Pi^F$), yet it will not arise in equilibrium when $\Pi^F > 2\Pi^{ED}$ and $\Pi^{CA} < 2\Pi^F + 2(\Pi^F - 2\Pi^{ED})$. This observation leads to the second conclusion. When manufacturers are constrained by limited liability, common agency will not necessarily arise even if it is most efficient. Thus, unlike the case of a single retailer where a similar constraint appears to be neutral and only contracting inefficiencies may prevent common agency to arise, here, it may be the principal cause for an inefficient market outcome.

### 4.2 Double common agency equilibria

The analysis is similar to the one conducted for common agency equilibria and therefore I briefly discuss this case while stressing the main points.

The following conditions ensure that $(M_A&M_B, M_A&M_B)$ is an equilibrium of the subsequent retailer game:

$$\Pi^C - U^C_{Ai} - U^C_{Bi} = \max\{0, \Pi^{MX} - U^{MX}_{Ai}, \Pi^{MX} - U^{MX}_{Bi}\},$$

for $i = 1, 2$. These condition implies that each retailer prefers to serve both manufacturers if it anticipates that the other retailer will do the same.

A simple way to fully prevent all deviations to all configurations, in which at least one retailer distributes the products of both manufacturers, is to destroy common agency in all these configurations. That is, each manufacturer $h \in \{A, B\}$ must then set $v^M_{hi} = u^M_{hi} = U^C_{hi} = \infty$ for $i = 1, 2$. Thus, like in a common agency equilibrium, the only issue is to minimize the gains from
monopoly, foreclosure and exclusive dealing deviations. Consequently, one can apply the results of lemma 1 to obtain the most effective way to do it.

Note now that in a double common agency equilibrium only ‘mixed’ offers can constraint the manufacturers payoffs. However, since deviations to the mixed configurations are no longer possible, manufacturers can set $U^{MX}_{A_i} = U^{MX}_{B_i} = \infty$ for $i = 1, 2$ which allows them to capture all the surplus from the trade $2\Pi^C$. Coupling this observation with the results of lemma 1, one obtains the following condition for the existence of double common agency equilibria:

$$2\Pi^C \geq 2 \max \{ 2(\Pi^F - \Pi^{ED}), \min\{2\Pi^F, \Pi^{ED} - \Pi^F\} \}.$$  \hfill (4)

It is perhaps surprising that such a simple condition is only required for the existence of a double common agency equilibrium, given that Rey and Verge (2004) reach strikingly different conclusions. As they show, in any such equilibrium retailers earn positive profits and it is technically difficult to establish the conditions under which it always exists.

The key difference between my model and that of Rey and Verge is that they consider non-contingent contracts.\(^{21}\) In this case and provided that manufacturer’s products are substitutable, each retailer can earn higher profit by distributing one product only. Consequently, manufacturers must leave some rents to retailers to convince them to carry their products. Since in any equilibrium manufacturers seek to minimize these rents, then under some conditions it can be quite easy for a deviating manufacturer to sign up one or two retailers into exclusive dealing arrangements. This leads to multiplicity of deviations and, as a result of the richness of possible deviations, a double common agency equilibrium does not always exist.

In contrast, when the contracts are fully contingent, manufacturers can sharply reduce the scope for deviations. In particular, the fact that deviations to the mixed configurations can now be ignored allows manufacturers to insist on double common agency by withdrawing their ‘mixed’ offers (i.e., by setting $U^{MX}_{A_i} = U^{MX}_{B_i} = \infty$ for $i = 1, 2$). In that case they leave each retailer with no alternatives but to accept serving non or both manufacturers and, as a consequence, share no rent with them in equilibrium. Furthermore, considering only foreclosure and exclusive dealing deviations gives rise to a simple condition governing the existence of double common agency equilibria.

To complete the analysis, it remains to find the menus that ensure that $(M_A&M_B, M_A&M_B)$ is the unique continuation equilibrium and all foreclosure and exclusive dealing deviations are maximally deterred. This can be done by employing basically the same mechanism as in the case of common agency, i.e., by inducing retailers to play the mis-coordination game which has instead $(M_A&M_B, M_A&M_B)$ as the unique equilibrium.\(^{22}\) To recap, I can state the following proposition.

\(^{21}\)There is also difference in the set of feasible contracts. Rey and Verge focus on two-part tariffs while here only lump-sum payments are considered.

\(^{22}\)See the Appendix for the proof.
**Proposition 3.** When manufacturers are constrained by limited liability, double common agency equilibria exist if and only if (4) holds. In that case they prefer the equilibria in which they jointly earn total surplus $2\Pi^C$ while each retailer earns zero.

As before, under some conditions, namely, $\Pi^F > 2\Pi^{ED}$ and $2\Pi^C < 2\Pi^F + 2(\Pi^F - 2\Pi^{ED})$, double common agency will not necessarily arise even if it is most efficient. Likewise, it is easy to verify that if negative bids were allowed, all deviations could be totally deterred and, consequently, a double common agency equilibrium could always be sustained.

### 4.3 Equilibria in mixed configurations

This section studies the situation where manufacturer $A$ is served by both retailers while manufacturer $B$ is served by retailer 1 only, which corresponds to the $(M_A&M_B, M_A)$ configuration. Note, in particular, that retailer 1, who is supposed to act as a common agent, will choose to represent both manufacturers, only if the following condition holds:

$$\Pi^{MX}_{AB} - u^{MX}_{A1} - v^{MX}_{B1} = \max\{0, \Pi^{ED} - U^{ED}_{B1} \cdot \Pi^F - U^{F}_{A1}\}. \quad (5)$$

Though the formal analysis is slightly different, the main insights remain the same. That is, only foreclosure and exclusive dealing deviations pose the main problem for the sustainability of an equilibrium. The main difference is that now the choice of $U^{ED}_{B1}$ and $U^{F}_{A1}$ affects the manufacturers’ payoffs in equilibrium, as it follows from (5). Thus, for the exception of $U^{ED}_{B1}$ and $U^{F}_{A1}$, the other offers must be chosen to maximally deter foreclosure and exclusive dealing deviations, while $U^{ED}_{B1}$ and $U^{F}_{A1}$ must resolve the trade-off between the rent extraction and the deterrence of deviations. The following proposition states the main results in this section.

**Proposition 4.** When manufacturers are constrained by limited liability, equilibria in mixed configurations exist if and only if the following conditions hold. If $\Pi^{ED} < \Pi^F$ then it must be

$$\Pi^{MX}_{AB} \geq \max\{\Pi^{ED}, 2(\Pi^F - \Pi^{ED})\}, \quad (6)$$

and

$$\Pi^{MX}_{AB} + \Pi^{MX} \geq 4\Pi^F - 3\Pi^{ED}, \quad (7)$$

while if $\Pi^{ED} > \Pi^F$ then it must be

$$\Pi^{MX}_{AB} \geq \min\{2\Pi^F, \Pi^{ED} - \Pi^F\}, \quad (8)$$

and

$$\Pi^{MX} + \Pi^{MX}_{AB} \geq 2\min\{2\Pi^F, \Pi^{ED} - \Pi^F\}. \quad (9)$$
Moreover, when $\Pi_{ED} > 2(\Pi_F - \Pi_{ED}) > 0$ and (6)-(7) hold then both manufacturers prefer the equilibria in which they jointly earn $\Pi_{MX}^{AB} - [\Pi_{ED} - 2(\Pi_F - \Pi_{ED})]$ in outlet 1 while manufacturer A earns $\Pi_{MX}^A$ in outlet 2. Correspondingly, retailer 1 then earns $\Pi_{ED} - 2(\Pi_F - \Pi_{ED})$ and retailer 2 earns zero. In other cases, whenever such equilibria exist, both manufacturers prefer the ones in which they jointly earn $\Pi_{MX}^{AB} + \Pi_{MX}$ while each retailer earns zero.

Proof. Available upon request.

Conditions (6) and (8) in proposition 4 deserve some explanation. Intuitively, manufacturer B can always obtain some payoff by undertaking either foreclosure or exclusive dealing deviations. Hence, in order to sustain an equilibrium, it must be given at least this payoff. This observation, taken together with the facts that manufacturer A is constrained by limited liability and both manufacturers cannot earn more than $\Pi_{MX}^{AB}$ in outlet 1, gives rise to conditions (6) and (8). Conditions (7) and (9) in turn imply that the maximal joint payoff of manufacturers must exceed the sum of their individual payoffs obtained by deviating.

As before, manufacturers induce retailers to play the game that has as an equilibrium only the outcome (i.e., the mixed configuration) where both retailers are worse off. By doing so, in all cases but one they are capable to extract all the surplus from the trade. The fact that, when $0 < 2(\Pi_F - \Pi_{ED}) < \Pi_{ED}$, they have to leave some rent to retailer 1 can be understood as follows.

When manufacturers are constrained by limited liability and $\Pi_F > \Pi_{ED}$, foreclosure deviations are always possible. As $\text{DEV}_{F_{B1}}$ and $\text{DEV}_{F_{B2}}$ imply, the minimal payoff that manufacturer B obtains from such deviations is $2(\Pi_F - \Pi_{ED})$, i.e., when manufacturer A sets $U_{ED}^{A1} = U_{ED}^{A2} = 0$. On the other hand, condition (5), in particular, implies that, in order to extract all surplus from retailer 1 in equilibrium, manufacturer A must set $U_{F}^{A1} = \infty$. However, in that case manufacturer B can also undertake exclusive dealing deviations, namely, to $(M_B, M_A)$ and earn $\Pi_{ED}$, as it follows from $\text{DEV}_{ED_{B1}}$. Clearly, when $2(\Pi_F - \Pi_{ED}) < \Pi_{ED}$ such deviations are more profitable than foreclosure ones. Thus, in order to reduce the gain from exclusive dealing deviations of manufacturer B and, consequently, enhance the sustainability of an equilibrium, manufacturers must attribute some rent to retailer 1.

It is easy to see that when manufacturers are constrained by limited liability, a mixed configuration will not necessarily arise even if it is most efficient. As (7) implies this would be the case if $2\Pi_F < \Pi_{MX}^{AB} + \Pi_{MX} < 2\Pi_F + (2\Pi_F - 3\Pi_{ED})$ and $2\Pi_F > 3\Pi_{ED}$.

Finally, the Appendix also demonstrates that if negative bids were allowed, all deviations could be totally deterred. Consequently, in that case manufacturers could always sustain an equilibrium in mixed configurations.

4.4 Exclusive dealing equilibria

In contrast to the above considered configurations, exclusive dealing will always arise whenever it generates the greatest total surplus. Intuitively, in construct-
ing such an equilibrium, one can restrict attention on the offers aimed to obtain exclusive relationship with each retailer. In that case such an equilibrium can be sustained only if \( \Pi^{ED} > \Pi^F \) because otherwise at least one manufacturer could profitably deviate to foreclosure. As before, manufacturers induce retailers to play the game that has as an equilibrium only the outcome (i.e., the exclusive dealing configuration) where both retailers obtain zero. The following proposition formalizes the result.

**Proposition 5.** When manufacturers are constrained by limited liability, exclusive dealing equilibria exist if and only if \( \Pi^{ED} > \Pi^F \). In that case they prefer the equilibrium in which each of them earns \( \Pi^{ED} \) while each retailer earns zero.

**Proof.** See the Appendix.

The Appendix also shows that if manufacturers were allowed to submit negative bids they could deter all foreclosure deviations and, consequently, sustain an exclusive dealing equilibrium even if \( \Pi^F \geq \Pi^{ED} \).

### 4.5 Foreclosure equilibria

In this case manufacturers compete head-to-head to sign up both retailers into exclusive relationship. Note also that, since in any foreclosure equilibrium a rival manufacturer always contemplates the possibility to mitigate competition and instead deal with one retailer only, it is natural to expect that such an equilibrium can exist only if \( \Pi^F > \Pi^{ED} \).

**Proposition 6.** Foreclosure equilibria exist if and only if \( \Pi^F > \Pi^{ED} \).

When manufacturers are constrained by limited liability, in the most preferred equilibrium manufacturer \( h \) announces the following menu \( \Sigma^F_h \):

- \( U^{CA}_hi = U^{C}_hi = u^{MX}_hi = v^{MX}_hi = U^{MX}_hi = \infty \) for \( i = 1, 2 \).
- \( U^{M}_hi = U^{ED}_hi = 0 \) and \( U^{F}_hi = \Pi^F - \Pi^{ED} - \epsilon \) for \( i = 1, 2 \) where \( \epsilon > 0 \).

The corresponding retailer game \( g(\Sigma^F_A, \Sigma^F_B) \) gives rise to two equilibria, \((M_A, M_B)\) and \((M_B, M_B)\). In case both retailers choose to represent manufacturer \( h \), the latter earns \( 2(\Pi^F - \Pi^{ED} - \epsilon) \) while each retailer earns \( \Pi^{ED} + \epsilon \).

**Proof.** See the Appendix.

As the proposition states, when manufacturers compete for foreclosure, they can still earn positive profits. This is due to the fact that retailers do not cooperate in their choice of a manufacturer. To better understand this result, note that in order to destroy the foreclosure equilibrium of its rival, a manufacturer must make an exclusive dealing offer which yields at least \( \Pi^F \) to a retailer. However, when this manufacturer is constrained by limited liability, it cannot offer more than \( \Pi^{ED} < \Pi^F \). Furthermore, when retailers decide non-cooperatively which manufacturer to represent, it cannot induce them to select the desired foreclosure equilibrium, even if it offers more attractive foreclosure payments.
Hence, no manufacturer $h$ has incentives to deviate from the proposed menu $\Sigma^F_h$.

If instead manufacturers were allowed to submit negative bids then the above strategies $\Sigma^F_A$ and $\Sigma^F_B$ would no longer constitute an equilibrium. This is because manufacturer $h$ could then deviate and ensure that $(M_h, M_h)$ is the unique equilibrium of the retailer game by offering $\tilde{U}^{ED}_{hi} < 0$ and $\tilde{U}^M_{hi} < 0$ for $i = 1, 2$. In which case it could earn $2(\Pi^F - \Pi^{ED} - \epsilon)$ with probability one. Clearly, such deviations will never be profitable only if each manufacturer $h$ earns zero in $(M_h, M_h)$. Correspondingly, in equilibrium each manufacturer $h$ offers $U^F_{hi} = 0$, $U^{ED}_{hi} = -(\Pi^F - \Pi^{ED} - \epsilon)$ for $i = 1, 2$ and thus leaves all rents to retailers.

I complete the analysis by noticing that neither monopoly configuration can ever be an equilibrium of $G$. This is because an inactive manufacturer can always profitably deviate by offering an inactive retailer the payoff which is slightly higher than zero. This observation, taken together with the previous results, leads to the following conclusions. First, monopoly will never arise in equilibrium even if it is most efficient. Second, when bids are unconstrained, any configuration but monopoly and foreclosure can always be sustained in equilibrium. Moreover, whenever such a configuration is most efficient it is the best equilibrium for manufacturers. As for foreclosure, it can be an equilibrium when it is most efficient but it is always a Pareto-dominated equilibrium for manufacturers. Finally, when bids are constrained to be non-negative, any configuration, in which at least one retailer acts as a common agent, will not necessarily arise in equilibrium even if it is most efficient. In contrast, foreclosure can be the unique equilibrium even if some other configuration generates greater total surplus.

5 Less contingent offers

The preceding analysis focused on the sustainability of different equilibria and the maximal profits that manufacturers could achieve, given that they make fully contingent offers. In practice, however, it may be difficult or even prohibited for manufacturers to write a contract contingent on all conceivable situations. Thus, it is natural to ask how the preceding results would differ if the offers were less contingent.

One way to reduce the degree of contingency is to consider an environment where the contract between one manufacturer and one retailer does not depend on the contract signed by the other manufacturer and the other retailer. One reason for such a restraint is that a manufacturer may not know the nature of the relationship in the other manufacturer-retailer pair because, say, it is not directly involved in it. Another reason is that such contracts may be found uncompetitive and consequently terminated by competition authorities.

In order to isolate the effects stemming purely from the reduced degree of contingency, I adapt the competitive game $G$ in which manufacturers' bids can be negative but impose the following constraints: $U^M_{hi} = U^{ED}_{hi}$ and $U^{CA}_{hi} = \psi^{MX}_{hi}$ for $h \in \{A, B\}$ and $i = 1, 2$. The constraint $U^M_{hi} = U^{ED}_{hi}$ implies that, while
trying to sign up retailer $i$ into exclusive relationship, manufacturer $h$ does not observe whether manufacturer $k$ is doing the same with retailer $j$. The constraint $U_{hi}^{CA} = v_{hi}^{MX}$ implies that, while establishing common agency with retailer $i$, manufacturer $h$ does not know whether manufacturer $k$ is offering any contract to retailer $j$. The following proposition states the results of the analysis in this case.

**Proposition 7.** Suppose that manufacturers’ bids can be negative but $U_{hi}^{M} = U_{hi}^{ED}$ and $U_{hi}^{CA} = v_{hi}^{MX}$ for $i = 1, 2$ and $h \in \{A, B\}$. Then, double common agency, exclusive dealing equilibria and equilibria in mixed configurations always exist. Common agency equilibria exist if and only if

$$
\Pi^{CA} \geq \min\{\Pi_{AB}^{MX} + \Pi^{MX}, \max\{0, \Pi_{AB}^{MX} + \Pi^{MX} - \Pi^{ED} + \Pi^{M}\}\}.
$$

Foreclosure equilibria exist if and only if $\Pi^{F} > \Pi^{ED}$ and monopoly can never be an equilibrium. Moreover, in the most preferred double common agency, common agency, exclusive dealing and mixed configuration equilibria manufacturers earn all the surplus from the trade while in all foreclosure equilibria they earn zero.

**Proof.** Available upon request.

Thus, the only difference from the case when bids are fully contingent is that now common agency cannot be always implemented as an equilibrium. Intuitively, the constraints $U_{hi}^{CA} = v_{hi}^{MX}$ for $i = 1, 2$ do not allow manufacturer $h$ to fully respond to some deviations of its rival. Namely, when it deviates by employing the second retailer (who is inactive) and thus induces a mixed configuration. As a result, in some circumstances it become more difficult to sustain common agency equilibria.

Using the results of proposition 7, it is easy to check that the most efficient configuration (except the monopoly one) can still be sustained in equilibrium. This, however, might not be true if the degree of contingency were further reduced. Moreover, in this case one might have the situation where even the most efficient configuration cannot be implemented as the unique continuation equilibrium. This is because when offers are less contingent manufacturers have less instruments to affect retailers’ decisions. As a result, they might not be able to get rid of multiple continuation equilibria.

To gain some intuition for this, I consider the extreme case when the offers are completely non-contingent, i.e., when each manufacturer $h$ offers $U_{hi}$ to retailer $i$ in any distribution configuration. In the Appendix, I give the examples of the parameter values under which (i) the most efficient configuration cannot be an equilibrium, and (ii) it cannot be implemented as the unique continuation equilibrium.
6 Bilateral bargaining power

Although I have assumed that manufacturers have all the bargaining power and make take-it-or-leave offers to retailers, the analysis would remain similar if instead retailers had all the bargaining power and made offers to manufacturers. Building on this observation, the purpose of this section is to study the issue of how allocation of the bargaining power affects the possibility to implement the most efficient configuration in equilibrium. Given the multiplicity of equilibria, I restrict attention on the best equilibrium from the viewpoint of the players who act as first-movers.

When bids are unconstrained, the analysis suggests that if either double common agency, exclusive dealing or mixed distribution generates the greatest total surplus then it plays no role of who initiates the offers. In either case the most efficient configuration is the most preferred equilibrium for both sides. In contrast, if either foreclosure or common agency generates the greatest total surplus, then manufacturers and retailers differ in their choice of the most preferred equilibria. In the first case, full efficiency can be achieved only if retailers have all the bargaining power while in the second case - only if manufacturers have the same.

When bids are constrained to be non-negative, the situation is more complicated because one needs to take into account the necessary conditions for every distribution configuration to be an equilibrium. Consequently, three situations are possible. That is, (i) the market outcome is efficient regardless of who has the bargaining power, (ii) efficiency can be achieved only if manufacturers (or retailers) have all the bargaining power, and (iii) efficiency can never be achieved.

To illustrate these points, suppose that double common agency is most efficient, i.e., $2\Pi^C > 2\Pi^F, \Pi^CA$, and, in addition, $2\Pi^F, \Pi^CA > 4\Pi^{ED}$. It can be shown that, when retailers make take-or-leave-it offers, double common agency equilibria exist only if $2\Pi^C \geq 2(\Pi^CA - 2\Pi^{ED})$ and in their most preferred equilibria retailers jointly earn $2\Pi^C$. From this the following conclusions can be drawn: (i) “who has the bargaining power” is unimportant for the efficient outcome to arise only if $\Pi^C \geq 2(\Pi^F - \Pi^{ED})$ and $\Pi^C \geq \Pi^CA - 2\Pi^{ED}$, (ii) if $2(\Pi^F - \Pi^{ED}) < \Pi^C < \Pi^CA - 2\Pi^{ED}$ then the market outcome will be efficient only if manufacturers have all the bargaining power, and (iii) if $\Pi^C < 2(\Pi^F - \Pi^{ED})$ and $\Pi^C < \Pi^CA - 2\Pi^{ED}$ then efficiency can never be achieved.

\[\text{23} \text{ There is a great amount of evidence showing that retailers no longer behave passively but instead exert some bargaining power. For example, there is a belief among industry observers that increasing concentration, introduction of private labels and limited shelf space have contributed to the perceived shift in the balance of power from manufacturers to retailers. Some implications of retail bargaining power have been recently explored in Marx and Shaffer (2006) and Rey et al. (2006).}\]

\[\text{24} \text{ Likewise, if } \Pi^CA - 2\Pi^{ED} < \Pi^C < 2(\Pi^F - \Pi^{ED}) \text{ it will be efficient only if retailers have all the bargaining power.}\]
7 Conclusion

In this paper, I have developed a framework where two rival manufacturers seek to establish trading relationships with two available retailers who in turn decide non-cooperatively which manufacturer’s offer, if any, to accept. Although there are many important issues to explore in this setting, the main focus of the present analysis is on the possibility for the most efficient configuration to be sustained in equilibrium.

The main findings are as follows. In the absence of any constraints on the manufacturers’ offers, the most efficient configuration (except monopoly one) can always be implemented as an equilibrium. Moreover, for the exception of foreclosure it is a Pareto-undominated equilibrium for manufacturers. In contrast, when the offers are subject to various constraints, the efficiency may no longer be achieved. For example, when the offers are constrained to be non-negative or when they are totally non-contingent, the most efficient configuration is not always an equilibrium. In addition, in the first case foreclosure can be the unique equilibrium even if it is inefficient while in the second one it is possible that the most efficient configuration cannot be sustained as the unique continuation equilibrium.

The extant literature on modeling bilaterally oligopolistic industries with retailers being capable to exert some market power vis-à-vis manufacturers is still scarce. Early papers such as Dobson and Waterson (2001) and Moner et al. (2005) obtain equilibria under quite restrictive conditions. In Dobson and Waterson, it is the assumption that under exclusive dealing a manufacturer-retailer pair behaves as a vertically integrated unit while in Moner et al., it is the restriction to inefficient contracts, i.e., linear wholesale prices. In attempt to extend the analysis by including two-part tariffs into the set of wholesale contracts, Rey and Verge (2004), however, find it considerably complicated and only show that some distribution configurations may never arise in equilibrium. The present paper thus contributes to this literature in that it establishes the conditions under which every possible distribution configuration can be sustained in equilibrium and characterizes the most preferred one (for the players who make contract offers). In this respect, it can serve as a building block for a more complex analysis, for example, concerned with vertical arrangements.

However, my model has two notable limitations. First, I have assumed that, after manufacturers have announced their offers, retailers make their decisions simultaneously which effectively implies that all trading links are established at the same time. This reflects reality in many, although not all, settings. For example, some trading links can be established before the others. Moreover, it is also reasonable to expect that the division and the size of the joint surplus is determined by bilateral negotiations. The extension of my analysis to such circumstances is an important avenue for future research. Recent papers that make a start in this direction include de Fontenay and Gans (2005) (who study a dynamic game in which bilateral negotiations take place in a pre-specified order and the parties involved in negotiations bargain over an action affecting their joint surplus and a transfer between them) and Bedre (2006) (who applies
the approach of sequential bargaining to a simpler setting with two retailers and one manufacturer but considers more complex contracts such as two- and three-part tariffs). Second, and perhaps more important, I have assumed that the joint surplus earned in a given outlet is exogenous. This admittedly ad hoc (and perhaps less satisfactory) structure has been adopted to avoid technical difficulties arising when manufacturers offer potentially complex contracts. For example, when they offer (contingent) two-part tariffs then the main difficulty stems from the fact that even if a retailer has accepted to contract with a manufacturer it may find it more profitable to buy zero quantity of its product afterwards. As a result, determination of equilibrium contracts (as well as the joint surplus) becomes somewhat complicated. On the other hand, opening this “black box” is undoubtedly an important issue since it would allow us to evaluate prices and welfare effects and thus fully characterize equilibria in each case. This is again the subject for future research.

Appendix

A The proof of lemma 1

For the sake of concreteness, suppose that manufacturer $A$ deviates. Consider the ability of manufacturer $B$ to deter its deviations by choosing $U_{ED}^B$, $U_{ED}^F$, and $U_{M}^i$ for $i = 1, 2$ while keeping all other payments unchanged.

Case I. $\Pi^F > \Pi^{ED}$.

As it follows from $\text{DEV}_{A_1}^F$, in this case foreclosure deviations are always possible and, in order to minimize the gain from such deviations, manufacturer $B$ must set $U_{ED}^{B_1} = U_{ED}^{B_2} = 0$. Note also that the conditions $U_{ED}^{B_1} = U_{ED}^{B_2} = 0$ eliminates any possibility for monopoly deviations and therefore $U_{M}^{B_1}, U_{M}^{B_2}$ can be chosen in an arbitrary way. As for exclusive dealing deviations, the $\text{DEV}_{A_i}^{ED}$ condition implies that manufacturer $B$ can fully deter them by setting $U_{F}^{B_i} < \Pi^F - \Pi^{ED}$ for $i = 1, 2$.

Case 2. $\Pi^F < \Pi^{ED}$.

In this case manufacturer $A$ can equally deviate to exclusive dealing, monopoly and foreclosure depending on whether $U_{ED}^{B_1}, U_{ED}^{B_2}$ are lower or higher than $\Pi^{ED}$. Therefore, in order to find the most effective way to deter these deviations, four types of strategies need to be distinguished.

Suppose, first, that manufacturer $B$ sets $U_{ED}^{B_1}, U_{ED}^{B_2} < \Pi^{ED}$. By doing so, it eliminates the possibility for manufacturer $A$ to undertake monopoly deviations and therefore $U_{M}^{B_1}, U_{M}^{B_2}$ can take on any values. In contrast, manufacturer $A$ can then undertake exclusive dealing deviations. As it follows from $\text{DEV}_{A_i}^{ED}$, the gain from such deviations will be minimal only if $U_{F}^{B_1} = U_{F}^{B_2} = 0$. Finally, taking into account limited liability and the $\text{DEV}_{A_1}^{F}, \text{DEV}_{A_2}^{F}$ constraints, manufacturer $B$ can deter all manufacturer $A$’s foreclosure deviations by setting $U_{ED}^{B_i} < \Pi^{ED} - \Pi^F$ for at least some $i$.

\footnote{25Because of limited liability manufacturer $A$ cannot offer a negative $\tilde{U}_{A_1}^{F}$ and thus cannot deviate to foreclosure even if $\tilde{U}_{A_1}^{F} + \tilde{U}_{A_2}^{F} > 0$.}
Suppose now that $U_{B1}^{ED} > \Pi^{ED}$ and $U_{B2}^{ED} < \Pi^{ED}$ which implies that manufacturer A can undertake some monopoly deviations, namely, to $(M_A, \emptyset)$. In order to fully deter such deviations, manufacturer B must set $U_{B1}^M = 0$. Likewise, manufacturer A can undertake some exclusive dealing deviations, namely, to $(M_B, M_A)$. In that case, in order to minimize the gain from such deviations, manufacturer B must set $U_{B2}^F = 0$. As for manufacturer A’s foreclosure deviations, they will be impossible if, in addition, $U_{B2}^{ED} < \Pi^{ED} - \Pi^F$. To recap, the strategy comprising $U_{B1}^{ED} > \Pi^{ED}$ and $U_{B2}^{ED} < \Pi^{ED}$ does not lower the gain from exclusive dealing deviations but instead gives rise to the possibility for monopoly deviations. Note that the same reasoning applies to the case when $U_{B1}^{ED} < \Pi^{ED}$ and $U_{B2}^{ED} > \Pi^{ED}$.

Suppose, finally, that $U_{B1}^{ED}, U_{B2}^{ED} > \Pi^{ED}$. In that case, manufacturer B rules out the possibility for exclusive dealing deviations and therefore can set $U_{B1}^F$ and $U_{B2}^F$ in an arbitrary way. Next, as it follows from DEV$_A$, by undertaking foreclosure deviations, manufacturer A can earn all the surplus from the trade $2\Pi_F$. Note that monopoly deviations are also possible. However, given that manufacturer A can always earn $2\Pi_F$, the best what manufacturer B can do in this case is to choose $U_{B1}^M$ and $U_{B2}^M$ in a way that manufacturer A could not earn more than $2\Pi_F$ from monopoly deviations, i.e., to set $U_{B1}^M \leq \max \{\Pi^M - 2\Pi_F, 0\}$ for $i = 1, 2$.

To recap, when $2\Pi_F > \Pi^{ED} - \Pi^F$ then manufacturer A can earn at most $\Pi^{ED} - \Pi^F$ by deviating. This can be done, for example, as follows: manufacturer B sets $U_{B1}^F = 0$, $U_{B1}^M \geq 0$, $U_{B1}^{ED} < \Pi^{ED}$ for $i = 1, 2$ and $U_{B2}^{ED} < \Pi^{ED} - \Pi^F$ for at least some $i$. When $2\Pi_F \leq \Pi^{ED} - \Pi^F$ the most effective way to deter manufacturer A’s deviations is to set $U_{B1}^F \geq 0$, $U_{B1}^{ED} > \Pi^{ED}$ and $U_{B1}^M \leq \max \{\Pi^M - 2\Pi_F, 0\}$ for $i = 1, 2$. Since a similar analysis applies to manufacturer B’s deviations, lemma 1 is established.

**B The proof of proposition 2**

Since it has been shown in the text that (1)-(2) are necessary conditions, it remains only to show that they are also sufficient. For this, four cases need to be distinguished.

**Case 1.** $\Pi^{ED} \leq \Pi^F$.

In this case common agency equilibria exist if $\Pi^{CA} \geq 4 (\Pi^F - \Pi^{ED})$. Consequently, in order to prove the ‘only if’ part of the proposition, I construct the menus $\Sigma^{CA}$ and $\Sigma^{BA}$ such that (i) the corresponding retailer game $g(\Sigma^{CA}, \Sigma^{BA})$ has only $(M_A&M_B, \emptyset)$ as a pure-strategy equilibrium in which manufacturers jointly earn $\Pi^{CA}$ and (ii) by deviating, each manufacturer can earn at most $2 (\Pi^F - \Pi^{ED})$. This can be done as follows. Suppose that each manufacturer $h$ announces the menu $\Sigma^{CA}_h$:

- $U^{CA}_{h2} = \frac{\Pi^{CA}}{2}$ while $U^{M}_{h2} = 0$ and $U^{M}_h = U^{CA}_h = U^{MX}_h = U^{C}_h = u^{MX}_{h1} = \infty$ for $i = 1, 2$. 

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neither exclusive dealing nor foreclosure configurations can be equilibria in the game in which each manufacturer earns deviation it can earn since and can undertake both foreclosure and exclusive dealing deviations. Since offer a negative gain from deviations to and can undertake monopoly deviations because in that case all common agency configurations because its rival sets are never profitable because in that case manufacturer can gain by deviating. To begin, no manufacturer can gain by deviating to the mixed and double common agency configurations because its rival sets for and can undertake monopoly deviations because for i = 1, 2. Consider now manufacturer k’s foreclosure deviations. By plugging into DEV, one finds that in the most profitable foreclosure deviation manufacturer k earns next, manufacturer k’s exclusive dealing deviations to (Mk, Mk) and (Mk, Mk). Note that both type of deviations are equally possible because deviations to (Mk, Mk) are never profitable because in that case manufacturer k has to offer a negative which follows from DEV, deviations to (Mk, Mk) cannot gain from deviations to (Mk, Mk) either. Indeed, by plugging the condition into DEV, one obtains that in any such deviation it can earn

Let us turn to the case when . In this case each manufacturer k can undertake both foreclosure and exclusive dealing deviations. Since and the most profitable foreclosure deviation yields the most profitable exclusive dealing deviation also yields . The condition implies and therefore neither of these deviations is profitable. Note that in this case all deviations can be deterred and consequently a common agency equilibrium can always be sustained. 

Case 2. 0 < 2(ΠED − ΠF) ≤ max{4ΠF, ΠM + 2ΠF}. In this case common agency equilibria exist if ΠCA ≥ 2(ΠED − ΠF). The proof is similar to the one conducted in the previous case. The only difference is that now the menus ΣA and ΣB must be such that, by deviating, each
manufacturer can earn at most $\Pi^{ED} - \Pi^F$. This can be done as follows. Suppose that each manufacturer $h$ announces the menu $\Sigma^C_h$:

- $U^A_h = \frac{1}{2}\Pi^C$ while $U^M_{h1} = 0$ and $U^M_{h2} = U^C_{h1} = U^M_{h1} = U^C_{h1} = \infty$ for $i = 1, 2$.
- $U^D_{h1} = 0$, $U^D_{h2} = \Pi^{ED} - \lambda\Pi^F$ where $\lambda \in (0, 1)$ and $U^D_i = 0$ for $i = 1, 2$.

It is easy to check that $(MA&M_B, \emptyset)$ is the unique continuation equilibrium in which each manufacturer earns $\frac{1}{2}\Pi^C$. It remains to verify that no manufacturer can gain by deviating. As before, only foreclosure and exclusive dealing deviations need to be considered.

In this case no manufacturer $k$ can undertake foreclosure deviations because of its rival’s exclusive dealing offer $U^D_{k1} = 0$ and the limited liability constraint: as $\text{DEV}^F_k$ implies, in order to induce retailer 1 to carry its product, manufacturer $k$ must then offer a negative payment $\tilde{U}^F_{k1} \leq -(\Pi^{ED} - \Pi^F)$.

In contrast, since $U^D_{k1}, U^D_{k2} < \Pi^{ED}$ then each manufacturer $k$ can always undertake exclusive dealing deviations. By plugging $U^D_{k1} = U^D_{k2} = 0$ into $\text{DEV}^C_{h1}$, one obtains that in any such deviation manufacturer $k$ earns

$$\tilde{U}^D_{k1} \leq (\Pi^{ED} - \Pi^F) \leq \frac{1}{2}\Pi^C$$

Thus, no manufacturer can gain by deviating.

**Case 3.** $\Pi^M + 2\Pi^F < 4\Pi^F < 2(\Pi^{ED} - \Pi^F)$.

In this case common agency equilibria exist if $\Pi^C \geq 4\Pi^F$ and one needs to ensure that only foreclosure deviations are possible. Suppose that each manufacturer $h$ offers the following menu $\Sigma^C_h$: $U^C_{h1} = \frac{1}{2}\Pi^C$ and

$$U^A_{h2} = U^M_{h1} = U^D_{h1} = U^F_{h2} = U^M_{h2} = U^C_{h1} = v^M_{h1} = u^M_{h1} = \infty$$

for $i = 1, 2$.

It is easy to check that $(MA&M_B, \emptyset)$ is the unique equilibrium in which each manufacturer earns $\frac{1}{2}\Pi^C$. It is straightforward to check that there are no profitable deviations either.

Indeed, the conditions $U^C_{k1} = U^C_{k2} = \infty$ imply that exclusive dealing deviations are impossible while in the best foreclosure deviation manufacturer $h$ earns $2\Pi^F \leq \frac{1}{2}\Pi^C$. Finally, the condition $2\Pi^F \geq \Pi^M$ implies that a manufacturer always prefers the best foreclosure deviation to any monopoly one.

**Case 4.** $4\Pi^F < \Pi^M + 2\Pi^F < 2(\Pi^{ED} - \Pi^F)$.

In this case common agency equilibria exist if $\Pi^C \geq \Pi^M + 2\Pi^F$. Thus, the main issue is to ensure that only foreclosure and monopoly deviations be possible. Consider the following menu $\Sigma^C_h$ announced by manufacturer $h$:

- $U^C_{h1} = U^D_{h1} = \Pi^C - \Pi^M$ for $i = 1, 2$.
- $U^C_{h2} = U^D_{h2} = U^F_{h1} = U^M_{h1} = U^C_{h1} = v^M_{h1} = u^M_{h1} = \infty$ for $i = 1, 2$ while $U^D_{h1} = 0$. 

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The condition $U^F_{h_1} = 0$ is added in order to destroy the possibility for $(\emptyset, M_A)$ and $(\emptyset, M_B)$ to be equilibria in the game $g(\Sigma^C_A, \Sigma^C_B)$. Furthermore, it is easy to check that $(M_A & M_B, \emptyset)$ is the unique equilibrium in which each manufacturer earns $\Pi^C_A - \Pi_M$.

As before, because of $U^F_{k_1} = U^F_{k_2} = \infty$ no exclusive dealing deviation is possible while in the best foreclosure deviation manufacturer $h$ earns $2\Pi^F$. The condition $U^M_{k_i} = \Pi^C_A - \Pi_M$ implies that in the best monopoly deviation manufacturer $h$ earns $\Pi^C_A - \Pi_M \geq 2\Pi^F$. This proves the proposition.

**Remark.** I now show that if negative bids were allowed, a common agency equilibrium could always be sustained. To see this, suppose that each manufacturer $h$ announces the following menu $\Sigma^C_h$:

- $U^C_{k_{h_1}} = \Pi^C_A - U^C_{k_{h_1}} \geq 0$ while $U^M_{k_{h_2}} = 0$ and $U^M_{h_1} = U^C_{h_1} = U^M_{h_1} = v_{h_1}$. \[ \frac{\lambda}{2} \]

- If $\Pi^F > \Pi^C$ then $U^F_{h_i} = -U^F_{h_i} = \lambda \Pi^F$ where $\lambda \in (0, 1)$ and $U^F_{h_1} = -(\Pi^F - \Pi^F)$ for $i = 1, 2$. If $\Pi^F > \Pi^E$ then $U^F_{h_1} = \Pi^F - \Pi^E$, $U^F_{h_2} < 0$ and $U^F_{h_i} = -(\Pi^F - \Pi^E)$ for $i = 1, 2$.

When $\Pi^F > \Pi^F$ manufacturers offer negative foreclosure payments, $\Pi^F - \Pi^E$, in order to fully deter exclusive dealing deviations, while when $\Pi^F > \Pi^E$ they offer negative exclusive dealing payments, $\Pi^E - \Pi^F$, in order to fully deter foreclosure deviations. The conditions $U^F_{h_1} = -U^F_{h_2} = \lambda \Pi^E$ where $\lambda \in (0, 1)$ and $U^F_{h_1} = \Pi^F - \Pi^E$, $U^F_{h_2} < 0$ ensure that neither exclusive dealing nor foreclosure configurations can arise as continuation equilibria in the game $g(\Sigma^C_A, \Sigma^C_B)$. Finally, the conditions $U^M_{A_2} = U^M_{B_2} = 0$ ensure that the $(\emptyset, \emptyset)$ representation cannot be an equilibrium.

Thus, by construction, $(M_A & M_B, \emptyset)$ is the unique (pure-strategy) continuation equilibrium in which manufacturers jointly earn $\Pi^C_A$. It can be also verified that no manufacturer can gain by deviating. Consequently, the above menus $\Sigma^C_A$ and $\Sigma^C_B$ indeed constitute an equilibrium.

### C The proof of proposition 3

Since it has been discussed in the text that (3) is a necessary condition, it remains only to show that it is also sufficient. For this, two cases need to be distinguished.

**Case 1.** $\Pi^E \leq 3\Pi^F$.

In this case condition (4) boils down to

$$\Pi^C \geq \max \left\{ (\Pi^E - \Pi^F), 2(\Pi^F - \Pi^E) \right\}.$$ \[ C1 \]

In what follows, I construct the menus $\Sigma^C_A$ and $\Sigma^C_B$ such that (i) $g(\Sigma^C_A, \Sigma^C_B)$ has only $(M_A & M_B, M_A & M_B)$ as a pure-strategy equilibrium in which each manufacturer earns $\Pi^C$ and (ii) by deviating, each manufacturer can earn at most $\max \left\{ (\Pi^E - \Pi^F), 2(\Pi^F - \Pi^E) \right\}$. To show this, suppose that manufacturer $h$ offers the following menu $\Sigma^C_h$.

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otherwise it would be impossible to implement on manufacturer
retailer 2 is inactive retailer 1 switches on manufacturer
After that retailers do the same with manufacturer
for retailer 2 only if retailer 1 were inactive. However, since
equilibrium strategies can be chosen as follows.
unique continuation equilibrium. Indeed, if
would be also a continuation equilibrium. On the other hand,
chooses to represent manufacturer
retailer tries to ‘catch’ the other. To illustrate this, suppose that retailer 2
By making such offers, manufacturers induce the mechanism under which one
of them is profitable if C1 holds.

Case 2. Π^{ED} > 3Π^F.
In this case condition (4) boils down to Π^C ≥ 2Π^F and the corresponding equilibrium strategies can be chosen as follows. Σ^{C}_h consists of

• \( U^{C}_{hi} = \frac{1}{2} \Pi^C \) and \( U^{MX}_{hi} = \infty \) for \( i = 1, 2 \) so that \((M_A&M_B, M_A&M_B)\) is indeed an equilibrium.

• \( v^{MX}_{hi} = U^{CA}_{hi} = u^{MX}_{hi} = \infty \) for \( i = 1, 2 \) so that deviations to common agency and mixed configurations are never profitable.

• \( U^{M}_{hi} = 0 \) for \( i = 1, 2 \) so that the \((0,0)\) representation cannot be an equilib-

• If \( \Pi^{ED} > \Pi^F \) then \( U^{ED}_{h1} = 0, U^{ED}_{h2} = \Pi^F - \lambda \Pi^F \) where \( \lambda \in (0,1) \) and \( U^{F}_{hi} = 0 \) for \( i = 1, 2 \). If \( \Pi^F > \Pi^{ED} \) then \( U^{F}_{h1} = \min\{\Pi^F, 3(\Pi^F - \Pi^{ED})\}, U^{F}_{h2} = 0 \) and \( U^{ED}_{hi} = 0 \) for \( i = 1, 2 \). If \( \Pi^F = \Pi^{ED} \) then \( U^{F}_{h1} = U^{ED}_{h2} = \lambda \Pi^F \) and \( U^{ED}_{h1} = U^{F}_{h2} = 0 \) where \( 0 < \lambda < \min\{1, \frac{\Pi^{CA}}{2\Pi^F}\} \).

It is easy to check that \((M_A&M_B, M_A&M_B)\) is the unique equilibrium of 
\( g(\Sigma^{C}_A, \Sigma^{C}_B) \) in which each manufacturer earns \( \Pi^C \). Note also that foreclosure and exclusive dealing deviations are possible but neither of them is profitable if C1 holds.

The choice of monopoly and foreclosure offers deserves some explanation. By making such offers, manufacturers induce the mechanism under which one retailer tries to ‘catch’ the other. To illustrate this, suppose that retailer 2 chooses to represent manufacturer A. Since \( U^{M}_{A2} = 0 \) this would be profitable for retailer 2 only if retailer 1 were inactive. However, since \( U^{F}_{A1} = 0 \) retailer 1 instead prefers to serve manufacturer A as well. In this case, because of \( U^{F}_{A2} = \infty \) retailer 2 chooses to represent neither manufacturer. However, when retailer 2 is inactive retailer 1 switches on manufacturer B because \( U^{F}_{B1} = 0 \). After that retailers do the same with manufacturer B until retailer 2 switches on manufacturer A and the cycle restarts.

The choice of \( \lambda \) is motivated by the following. \( \lambda \) should be positive because otherwise it would be impossible to implement \((M_A&M_B, M_A&M_B)\) as the unique continuation equilibrium. Indeed, if \( \lambda = 0 \) then, for example, \((0, M_A)\) would be also a continuation equilibrium. On the other hand, \( \lambda \) should not
be too large because otherwise manufacturers could earn more than $2\Pi^F$ by undertaking monopoly deviations. By construction, \((M_A&M_B, M_A&M_B)\) is the unique equilibrium of \(g(\Sigma_A^C, \Sigma_B^C)\) in which each manufacturer earns \(\Pi^C\). It remains to verify that no manufacturer can earn more than $2\Pi^F$ by deviating. Since each manufacturer \(h\) sets \(U_{hi}^{ED} = \infty\) for \(i = 1, 2\) then this is straightforward because only foreclosure and monopoly deviations need to be considered.

First of all, by plugging \(U_{hi}^{ED} = \infty\) into \(\text{DEV}^F_h\), one obtains that in the best foreclosure deviation manufacturer \(h\) earns \(2\Pi^F \leq \Pi^C\). I now show that, by undertaking monopoly deviations, no manufacturer can earn more than $2\Pi^F$. Suppose manufacturer \(A\) wishes to deviate to \((M_A, \emptyset)\). Since manufacturer \(B\) offers \(U_{M_B1} = 0\) and thus leaves all surplus to retailer 2, manufacturer \(A\) has to do the same in order to induce this retailer to carry exclusively its product. This in turn implies that such deviations are never profitable. If instead manufacturer \(A\) wishes to deviate to \((\emptyset, M_A)\) then it can earn no more than \(\lambda\Pi^M\) because manufacturer \(B\) sets \(U_{M_B2} = \lambda\Pi^M\). By construction, \(\lambda \leq \min\{1, \frac{2\Pi^F}{\Pi^M}\}\) which implies that \(\lambda\Pi^M \leq 2\Pi^F\). Since a similar reasoning applies to manufacturer \(B\)’s monopoly deviations, proposition 3 is established.

D Equilibria in mixed configurations

In this section I consider the case when manufacturers are allowed to submit negative bids.\(^{26}\) Let \(\Sigma_h^{MX}\) denote a contingent menu announced by manufacturer \(h\) and let us start with the conditions that ensure that \((M_A&M_B, M_A)\) is an equilibrium in \(g(\Sigma_A^{MX}, \Sigma_B^{MX})\). First, it must be that if retailer 1 chooses to represent both manufacturers, retailer 2 prefers to represent manufacturer \(A\) only, i.e.,

\[
\Pi^{MX} - U^{MX}_{A2} = \max\{0, \Pi^{MX} - U^{MX}_{B2}, \Pi^C - U^{C}_{A2} - U^{C}_{B2}\}.
\]

Conversely, if retailer 2 chooses to represent manufacturer \(A\), retailer 1 must choose to represent both manufacturers, i.e.,

\[
\Pi^{MX}_{AB} - u^{MX}_{A1} - u^{MX}_{B1} = \max\{0, \Pi^{ED} - U^{ED}_{B1}, \Pi^F - U^{F}_{A1}\}.
\]

As before, the remaining payments must be chosen in a way that (i) \((M_A&M_B, M_A)\) is the unique pure-strategy equilibrium of \(g(\Sigma_A^{MX}, \Sigma_B^{MX})\) and (ii) all deviations are maximally deterred.

Similarly to the analysis of common agency equilibria, here it can be shown that manufacturer \(h\) can undertake foreclosure deviations if and only if \(\text{DEV}^F_{hi}\) and \(\text{DEV}^F_{i2}\) hold, exclusive dealing deviations if and only if \(\text{DEV}^{ED}\) and \(\text{DEV}^{ED}_{hi}\) hold for at least some \(i\) and monopoly deviations if and only if \(U_{hi}^{ED} > \Pi^{ED}\) for at least some \(i\). The only difference from the previous analysis is that the mixed configuration must now be destroyed. Likewise, all deviations to

\(^{26}\) The analysis of the case when bids are constrained to be non-negative is available upon request.
common agency, double common agency and the other mixed configurations can also be fully deterred.

I now show that with negative bids manufacturers can deter all deviations and fully extract each retailer’s surplus. To see this, suppose that each manufacturer $h$ offers the following menu $\Sigma_h^{MX}$:

- For $h \in \{A, B\}$: $U_h^{CA} = U_h^{C} = \infty$ for $i = 1, 2$ and $U_h^{MX} = v_h^{MX} = v_h^{\infty} = \infty$.
- $u_{A1}^{MX}$ and $v_{B1}^{MX}$ are such that $u_{A1}^{MX} + v_{B1}^{MX} = \Pi^{MX}_{AB}$.
- For $h = A$: $U_A^{MX} = \Pi^{MX}, U_A^{F} = \Pi^{F} - (1 + \lambda')2\Pi^{F}, v_{A1}^{MX} = U_A^{F} = u_{A2}^{\infty} = \infty, U_{ED}^{A1} = \Pi^{ED} - 2\Pi^{F}; U_{A1}^{M} = 0$ and $U_{A1}^{M} = \lambda\Pi^{M}$.
- For $h = B$: $U_B^{ED} = \Pi^{ED} - 2\Pi^{F}, U_B^{F} = \Pi^{F} - (1 + \lambda')2\Pi^{F}, U_B^{MX} = v_{B1}^{MX} = u_{B2}^{\infty} = U_{ED}^{B2} = \infty, U_{B2}^{M} = \lambda\Pi^{M}$ and $U_{B2}^{M} = 0$ where $\lambda \in (0, 1)$ and $\lambda' > 0$ and sufficiently large.

Intuitively, the negative exclusive dealing offers serve to fully deter foreclosure deviations while the negative foreclosure offers serve to ensure the uniqueness of the $(M_A & M_B, M_A)$ continuation equilibrium in the game $g(\Sigma^{MX}_A, \Sigma^{MX}_B)$ which is represented by Table 2.

[Insert Table 2 here]

Using Table 2, one can easily check that $(M_A & M_B, M_A)$ is the unique pure-strategy equilibrium in which manufacturers jointly earn $\Pi^{MX}_{AB} + \Pi^{MX}$. I now show that there are no profitable deviations for either manufacturer. As before, one needs to consider monopoly, exclusive dealing and foreclosure deviations only.

Let us start with manufacturer $A$. Consider, first, its monopoly deviations to $(M_A, \emptyset)$ and $(\emptyset, M_A)$. Since manufacturer $B$ sets $U_{B2}^{ED} < 0$, deviations to $(M_A, \emptyset)$ are impossible. In contrast, since $U_{B2}^{ED} = \infty$, manufacturer $A$ can deviate to $(\emptyset, M_A)$ in which case it earns no more than $\min\{U_{B2}^{M}, \Pi^{M}\}$. However, given that manufacturer $B$ sets $U_{B2}^{M} = 0$, such deviations are never profitable.

Consider now manufacturer $A$’s exclusive dealing deviations to $(M_B, M_A)$ and $(M_B, M_A)$. Since $U_{B2}^{ED} = \infty$, deviations to $(M_B, M_A)$ are impossible. In contrast, since $U_{B2}^{ED} < 0$, manufacturer $A$ can deviate to $(M_A, M_B)$. However, because manufacturer $B$ sets $U_{B1}^{ED} = \Pi^{F} - (1 + \lambda')2\Pi^{F}$ where $\lambda' > 0$ and large it can only loose from such deviations. Indeed, as $\text{DEV}_{A1}^{ED}$ implies, in that case manufacturer $A$ would earn

$$\tilde{\text{DEV}}_{A1}^{ED} = \min\{\Pi^{ED}, \Pi^{ED} - (\Pi^{F} - U_{B1}^{F})\} = \Pi^{ED} - (1 + \lambda')2\Pi^{F} < 0 \text{ for } \lambda' > 0 \text{ and large.}$$

Consider, finally, manufacturer $A$’s foreclosure deviations to $(M_A, M_A)$. By plugging the conditions $U_{B1}^{ED} = \infty$ and $U_{B2}^{ED} = \Pi^{ED} - 2\Pi^{F}$ into $\text{DEV}_{A1}^{F}$ and $\text{DEV}_{A2}^{F}$, one finds that in any such deviation manufacturer $A$ earns
\[ \tilde{U}_{A_1}^F + \tilde{U}_{A_2}^F \leq \min\{\Pi^F, \Pi^F - (\Pi^{ED} - U_{B_1}^{ED})\} + \min\{\Pi^F, \Pi^F - (\Pi^{ED} - U_{B_2}^{ED})\} = \Pi^F - \Pi^F = 0. \]

This shows that there are no profitable deviations for manufacturer A. One can verify that a similar reasoning applies to manufacturer B. Thus, the proposed menus \( \Sigma_{M}^{MX} \) and \( \Sigma_{B}^{MX} \) indeed constitute an equilibrium of \( G \).

E Exclusive dealing equilibria

Let \( \Sigma_{h}^{ED} \) denote a contingent menu announced by manufacturer \( h \). First of all, notice that if manufacturer \( h \) sets \( U_{h_i}^{CA} = U_{h_i}^{C} = v_{h_i}^{MX} = v_{h_i}^{M} = \infty \) for \( i = 1, 2 \) then manufacturer \( k \) cannot gain by making the offers which allow common representation. In which case, bidding between manufactures is reduced to competition to obtain exclusive relationship with each retailer.

**Lemma 3.** When manufacturers are constrained by limited liability, exclusive dealing equilibria exist only if \( \Pi^{ED} \geq \Pi^F \).

**Proof.** Suppose that the proposed menus \( \Sigma_{A}^{ED} \) and \( \Sigma_{B}^{ED} \) induce the retailer game \( g(\Sigma_{A}^{ED}, \Sigma_{B}^{ED}) \) that has \( (M_A, M_B) \) as an equilibrium which in turn implies that the following conditions must hold:

\[ \Pi^{ED} - U_{A_1}^{ED} \geq \max\{0, \Pi^F - U_{B_1}^{F}\} \text{ and } \Pi^{ED} - U_{B_2}^{ED} \geq \max\{0, \Pi^F - U_{A_2}^{F}\}. \]

That is, if retailer \( 2 \) represents manufacturer \( B \) then retailer \( 1 \) must choose to represent manufacturer \( A \) and vice versa.

Consider now foreclosure deviations of manufacturer \( h \). In which case it offers the following menu \( \tilde{\Sigma}_{h}^{F} \):

- \( \tilde{U}_{hi}^{F} \leq \Pi^F - \max\{0, \Pi^{ED} - U_{hi}^{ED}\} \text{ for } i = 1, 2. \)
- The other payments of \( \tilde{\Sigma}_{h}^{F} \) coincide with those of \( \Sigma_{h}^{ED} \).

Since in any equilibrium the retailers must be willing to accept the exclusive offers then it must be \( U_{A_1}^{ED}, U_{B_2}^{ED} \leq \Pi^{ED} \). Hence, in any foreclosure deviation manufacturer A earns at most

\[ \tilde{U}_{A_1}^F + \tilde{U}_{A_2}^F \leq 2\Pi^F - (\Pi^{ED} - U_{B_2}^{ED}) - \max\{0, \Pi^{ED} - U_{B_1}^{ED}\}, \]

while manufacturer B earns at most

\[ \tilde{U}_{B_1}^F + \tilde{U}_{B_2}^F \leq 2\Pi^F - (\Pi^{ED} - U_{A_1}^{ED}) - \max\{0, \Pi^{ED} - U_{A_2}^{ED}\}. \]

Since in any equilibrium all deviations must be deterred, the following conditions must hold:

\[ 2\Pi^F - (\Pi^{ED} - U_{B_2}^{ED}) - \max\{0, \Pi^{ED} - U_{B_1}^{ED}\} \leq U_{A_1}^{ED}, \quad E1 \]
\[ 2\Pi^F - (\Pi^{ED} - U^F_{A1}) - \max\{0, \Pi^{ED} - U^F_{A2}\} \leq U^{ED}_{B2} \cdot \] E2

Taken together, E1 and E2 imply

\[ 2(2\Pi^F - \Pi^{ED}) - \max\{0, \Pi^{ED} - U^F_{B1}\} - \max\{0, \Pi^{ED} - U^F_{A2}\} \leq 0. \] E3

Suppose that in equilibrium \( U^{ED}_{B1}, U^{ED}_{A2} \geq \Pi^{ED} \). In which case E3 boils down to \( \Pi^{ED} \geq 2\Pi^F \). Suppose now that \( U^{ED}_{B1} > \Pi^{ED} \) and \( U^{ED}_{A2} < \Pi^{ED} \) (a similar arguments applies if \( U^{ED}_{B1} < \Pi^{ED} \) and \( U^{ED}_{A2} \geq \Pi^{ED} \)). In which case E3 boils down to \( 4(\Pi^{ED} - \Pi^{ED}) + U^{ED}_{A2} \leq 0 \). When the manufacturers are constrained by limited liability, then \( U^{ED}_{A2} \geq 0 \) and therefore the above condition can be satisfied only if \( 3\Pi^{ED} \geq 4\Pi^F \). Finally, suppose that \( U^{ED}_{B1}, U^{ED}_{A2} < \Pi^{ED} \). In this case E3 boils down to \( 4(\Pi^F - \Pi^{ED}) + U^{ED}_{B1} + U^{ED}_{A2} \leq 0 \). Again, because of limited liability the latter condition can be satisfied only if \( \Pi^{ED} \geq \Pi^F \). Taken together, these results imply that in any exclusive dealing equilibrium at least the condition \( \Pi^{ED} \geq \Pi^F \) must be satisfied. This proves lemma 4.

I now show that \( \Pi^{ED} > \Pi^F \) is also a sufficient condition (the case \( \Pi^{ED} = \Pi^F \) is discussed below). In fact, in this case the game gives rise to multiplicity of exclusive dealing equilibria. It turns out that it is possible to construct the equilibria in which manufacturers extract each retailer’s surplus. To establish this point, I actually find such an equilibrium. Consider the following menu \( \Sigma^{ED}_h \) offered by manufacturer \( h \):

- \( U_h^{CA} = U_h^{C} = u_h^{MX} = u_h^{MX} = U_h^{MX} = \infty \) for \( i = 1, 2 \).
- For \( h = A \): \( U_{A1}^{ED} = \Pi^{ED}, U_{A2}^{ED} = 2(\Pi^{ED} - \Pi^F), U_{A1}^{F} = 0, U_{A2}^{F} = \infty, U_{A1}^{M} = \lambda \Pi^{M} \) and \( U_{A2}^{M} = 0 \). For \( h = B \): \( U_{B1}^{ED} = 2(\Pi^{ED} - \Pi^F), U_{B2}^{ED} \geq \Pi^{ED} \), \( U_{B1}^{F} = \infty, U_{B2}^{F} = 0, U_{B1}^{M} = 0 \) and \( U_{B2}^{M} = \lambda \Pi^{M} \) where \( \lambda \in (0, \bar{\lambda}) \). \( \bar{\lambda} = 1 \) if \( \Pi^{ED} < 2\Pi^F \) and \( \bar{\lambda} = \min\{1, \frac{\Pi^{ED}}{2\Pi^F}\} \) otherwise.

When manufacturers announce such \( \Sigma^{ED}_A \) and \( \Sigma^{ED}_B \), the number of possible configurations to consider is reduced from sixteen to nine. Table 3 represents the payoff matrix obtained from \( g(\Sigma^{ED}_A, \Sigma^{ED}_B) \) by deletion of the (dominated) strategies which involve common representation, i.e., \( s_i = M_A \& M_B \) for \( i = 1, 2 \).

[Insert Table 3 here]

As Table 3 shows, \( (M_A, M_B) \) is the unique equilibrium in the game \( g(\Sigma^{ED}_A, \Sigma^{ED}_B) \) in which each manufacturer earns \( \Pi^{ED} \). It remains to verify that no manufacturer can gain by deviating. This is straightforward because only foreclosure and monopoly deviations need to be considered.

To begin, consider foreclosure deviations of manufacturer \( A \). Since manufacturer \( B \) sets \( \Pi^{ED}_{B1} = 2(\Pi^{ED} - \Pi^F) \) and \( \Pi^{ED}_{B2} = \Pi^{ED} \), then by plugging these conditions into \( \text{DEV}^F_{A1} \) and \( \text{DEV}^F_{A2} \), one obtains that manufacturer \( A \) then earns

\[
\tilde{U}^F_{A1} + \tilde{U}^F_{A2} \leq [\Pi^F - \max\{0, 2\Pi^F - \Pi^{ED}\}] + \Pi^F = \min\{\Pi^{ED}, 2\Pi^F\} \leq \Pi^{ED}.
\]

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Hence, it cannot gain from such deviations.

Consider now manufacturer $A$’s monopoly deviations to $(M_A, \emptyset)$ and $(\emptyset, M_A)$. Note that, since manufacturer $B$ sets $U_{B2}^{ED} = \Pi_{B2}^{ED}$, manufacturer $A$ to deviate to $(M_A, \emptyset)$ and earn at most $U_{B1}^{M}$. Given that $U_{B1}^{M} = 0$, such deviations are never profitable.

While considering manufacturer $A$’s deviations to $(\emptyset, M_A)$, I distinguish two cases which follow from the sign of $\Pi_{B1}^{ED} - U_{B1}^{ED} = \Pi_{B1}^{ED} - 2\Pi_{F}$. If $\Pi_{B1}^{ED} < 2\Pi_{F}$ then deviations to $(\emptyset, M_A)$ are impossible. In contrast, if $\Pi_{B1}^{ED} \geq 2\Pi_{F}$ such deviations are possible and in the best deviation of such a kind manufacturer $A$ earns $U_{B2}^{M} = \lambda \Pi_{M}$. By construction, $\lambda \leq \lambda = \min\{1, \frac{\Pi_{B1}^{ED}}{2\Pi_{F}}\}$ which implies that $\lambda \Pi_{M} \leq \Pi_{B1}^{ED}$. Taken together, these results imply that manufacturer $A$ cannot gain from such deviations either.

By applying a similar reasoning, one can verify that manufacturer $B$ cannot gain from monopoly and foreclosure deviations either. Thus, the proposed strategies $\Sigma_{A}^{ED}$ and $\Sigma_{B}^{ED}$ indeed constitute an equilibrium.

To complete the analysis, consider the case when $\Pi_{B1}^{ED} = \Pi_{F}$. This condition is not sufficient to implement the exclusive dealing configuration, $(M_A, M_B)$, as the unique equilibrium. The main problem in this case is that it is impossible to ensure that all foreclosure deviations be deterred and, at the same time, $(M_B, M_A)$ do not be an equilibrium. Indeed, all foreclosure deviations can be deterred only if $E_{3}$ can be satisfied. When $\Pi_{B1}^{ED} = \Pi_{F}$ $E_{3}$ boils down to $\min\{\Pi_{B1}^{ED} U_{B1}^{ED} \} + \min\{\Pi_{B1}^{ED} U_{A2}^{ED} \} \leq 0$ which is possible only if $U_{B1}^{ED} = U_{A2}^{ED} = 2\Pi_{F} = 0$. In that case, in order to destroy the $(M_B, M_A)$ equilibrium, manufacturers must offer negative foreclosure offers, i.e., $U_{A1}^{F}, U_{B2}^{F} < 0$, which they cannot do because of limited liability. Alternatively, manufacturers could allow a retailer to accept both offers. However, this could help destroy the $(M_B, M_A)$ equilibrium only if $\Pi_{B2}^{M} > \Pi_{B1}^{ED}$.

**Remark.** It has been show that because of limited liability manufacturers cannot fully deter all foreclosure deviations. Consequently, exclusive dealing equilibria cannot be sustained when $F \geq \Pi_{B1}^{ED}$. However, if this assumption were relaxed, such equilibria could be sustained even if $F \geq \Pi_{B1}^{ED}$. To see this, suppose that each manufacturer $h$ announces the following menu $\Sigma_{h}^{ED}$:

- $U_{b_{hi}}^{CA} = U_{b_{hi}}^{C} = U_{b_{hi}}^{MX} = U_{b_{hi}}^{MX} = \infty$ for $i = 1, 2$.
- For $h = A$: $U_{A1}^{ED} = \Pi_{A1}^{ED}$, $U_{A2}^{ED} = 2(\Pi_{A1}^{ED} - \Pi_{F})$, $U_{A1}^{F} = \Pi_{F} - (1 + \lambda')2(\Pi_{F} - \Pi_{B1}^{ED})$, $U_{A2}^{F} = \infty$, $U_{M1}^{A1} = \lambda \Pi_{M}$ and $U_{M1}^{A2} = 0$. For $h = B$: $U_{B1}^{ED} = 2(\Pi_{B1}^{ED} - \Pi_{F})$, $U_{B2}^{ED} = \Pi_{B1}^{ED}$, $U_{B1}^{F} = \infty$, $U_{B2}^{F} = \Pi_{F} - (1 + \lambda')2(\Pi_{F} - \Pi_{B1}^{ED})$, $U_{B1}^{M} = 0$ and $U_{B2}^{M} = \lambda \Pi_{M}$ where $\lambda, \lambda' \in (0, 1)$.

The difference between these menus and those offered when $\Pi_{B1}^{ED} > \Pi_{F}$ is that now $U_{A2}^{F}, U_{B1}^{ED} < 0$ and $U_{A1}^{F}, U_{B2}^{ED} < 0$. Manufacturers should offer negative exclusive dealing payments in order to fully deter foreclosure deviations while

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27As Table 4 shows, when $\Pi_{B1}^{ED} = \Pi_{F}$ the game $g(\Sigma_{A}^{ED}, \Sigma_{B}^{ED})$ has two exclusive dealing equilibria $(M_A, M_B)$ and $(M_B, M_A)$. In the former one, both manufacturers earn $\Pi_{B1}^{ED}$ while in the latter one they earn zero.
negative foreclosure payments serve to ensure the uniqueness of the \((M_A, M_B)\) continuation equilibrium. As before, one can easily verify that no manufacturer can gain by deviating which in turn implies that \(\Sigma_{A}^{ED} \) and \(\Sigma_{B}^{ED} \) are equilibrium strategies.

F Foreclosure equilibria

In this case manufacturers compete to sign up both retailers into exclusive relationship. Given that retailers make their decisions non-cooperatively, this basically implies competition for having a particular foreclosure configuration to be the unique continuation equilibrium.

Let \(\Sigma_{h}^{F} \) denote a contingent menu offered by manufacturer \(h\). Following the analysis of exclusive dealing equilibria, in constructing foreclosure equilibria, one can restrict attention on the strategies under which each manufacturer \(h \in \{A, B\} \) sets \(U_{hi}^{CA} = U_{hi}^{C} = \nu_{hi}^{M} = \nu_{hi}^{X} = \infty \) for \(i = 1, 2\).

Next, in any foreclosure equilibrium the following conditions must hold:

\[
\Pi^{F} - U_{hi}^{F} = \max \{0, \Pi^{ED} - U_{hi}^{ED}\} \quad \text{where } h \neq k,
\]

for \(i = 1, 2\). In words, if retailer \(j\) represents manufacturer \(h\) then it must be that retailer \(i\) chooses to represent manufacturer \(h\) as well.

**Lemma 4.** Foreclosure equilibria exist only if \(\Pi^{F} \geq \Pi^{ED}\).

**Proof.** Suppose that \(\Pi^{ED} > \Pi^{F}\) and \(U_{hi}^{F}, U_{hi}^{ED}\) satisfy F1 for \(i = 1, 2\) and \(h \in \{A, B\}\). It is easy to check that the corresponding retailer game then has two equilibria \((M_A, M_A)\) and \((M_B, M_B)\). In what follows I distinguish two cases.

In the first case, manufacturer \(h\) obtains a strictly positive payoff, i.e., \(U_{hi}^{F} + U_{ki}^{F} > 0\), when retailers choose to represent manufacturer \(h\) only, i.e., when the \((M_{h}, M_{h})\) continuation equilibrium is realized.\(^{28}\) Suppose now that it deviates and sets \(\tilde{U}_{hi}^{ED} = \Pi^{ED} - (\Pi^{F} - U_{hi}^{F})\) for \(i = 1, 2\) (while keeping all other payments unchanged).\(^{29}\) Such deviation are at least weakly profitable because, by doing so, manufacturer \(h\) destroys the \((M_{k}, M_{k})\) continuation equilibrium of its rival and instead induces the \((M_{h}, M_{h})\) continuation equilibrium to be unique.

In the second case, manufacturer \(h\) obtains zero even when the \((M_{k}, M_{h})\) continuation equilibrium is realized, i.e., \(U_{hi}^{F} = U_{ki}^{F} = 0\). As F1 implies, when \(\Pi^{ED} > \Pi^{F}\) this is possible only if \(U_{hi}^{ED} = \Pi^{ED} - \Pi^{F}\) for \(i = 1, 2\). In this case, manufacturer \(h\) can always profitably deviate to exclusive dealing by setting \(\tilde{U}_{hi}^{ED} < \Pi^{ED} - \Pi^{F}\) and \(\tilde{U}_{hi}^{F} = \infty\) for \(i = 1, 2\). By doing so, it destroys the foreclosure continuation equilibrium, i.e., \((M_{h}, M_{h})\) and \((M_{h}, M_{k})\), and instead induces exclusive dealing continuation equilibria, i.e., \((M_{A}, M_{A})\) and \((M_{B}, M_{A})\), in either of which it obtains strictly positive payoff \(\tilde{U}_{hi}^{ED}\).

Suppose now that \(\Pi^{F} > \Pi^{ED}\) and manufacturers face the limited liability constraint. In order to construct a foreclosure equilibrium, one needs to take

\(^{28}\)Clearly, manufacturer \(h\) obtains zero when retailers choose to represent its rival only, i.e., when the \((M_{k}, M_{k})\) continuation equilibrium is realized.

\(^{29}\)Note that when \(\Pi^{ED} > \Pi^{F}\) such deviation is possible even if manufacturer \(h\) is constrained by limited liability.
into account bidding competition between manufacturers. Namely, given that manufacturer \( k \) always seeks to destroy the continuation equilibrium of its rival, i.e., \((M_R, M_h)\), by offering \( U_{h}^{ED} = U_{k}^{ED} = 0 \), manufacturer \( h \) must set \( U_{hi}^F \leq \Pi^F - \Pi^{ED} \) for \( i = 1, 2 \). Only in that case, both retailers then have incentives to serve exclusively manufacturer \( h \).

Next, when retailers decide non-cooperatively which manufacturer to represent, manufacturer \( h \) cannot induce them to select the desired foreclosure equilibrium, \((M_R, M_h)\), even if it lowers \( U_{hi}^F \) for at least some \( i \). This observation, coupling with the fact that \( U_{hi}^F \leq \Pi^F - \Pi^{ED} \) for \( i = 1, 2 \), implies that in any foreclosure equilibrium each manufacturer \( h \) optimally sets \( U_{hi}^F = \Pi^F - \Pi^{ED} - \epsilon \) for \( i = 1, 2 \) where \( \epsilon > 0 \) and small.

To complete the proof, one needs to show that such an equilibrium indeed exists. To see this, suppose that each manufacturer \( h \) offers the following menu \( \Sigma_h^F \):

- \( U_{hi}^{CA} = U_{hi}^{C} = u_{hi}^{MX} = v_{hi}^{MX} = U_{hi}^{MX} = \infty \) for \( i = 1, 2 \).
- \( U_{hi}^{M} = 0, U_{hi}^{ED} = 0 \) and \( U_{hi}^{F} = \Pi^F - \Pi^{ED} - \epsilon \) for \( i = 1, 2 \) where \( \epsilon > 0 \) and small.

By construction, the continuation game has only two equilibria, \((M_A, M_A)\) and \((M_B, M_B)\). Furthermore, in the \((M_h, M_h)\) equilibrium manufacturer \( h \) earns \( 2(\Pi^F - \Pi^{ED} - \epsilon) \) while manufacturer \( k \) earns zero. It remains to check that there is no profitable deviation for either manufacturer.

Deviations to any configuration which involves common representation can never be profitable because each manufacturer insist on exclusivity. Monopoly deviations are impossible because each retailer is always ready to accept the exclusive dealing offer of a rival manufacturer. Exclusive dealing deviations cannot be profitable because a manufacturer then has to offer the payoff which is larger than \( \Pi^{ED} \). Finally, no manufacturer \( h \) can profitably increase its foreclosure payments because in that case the \((M_h, M_h)\) equilibrium will be destroyed and \((M_k, M_k)\) will be instead the unique continuation equilibrium. This proves proposition 6.

**Remark.** When \( \Pi^F > \Pi^{ED} \) but negative bids are allowed the menus \( \Sigma_A^F \) and \( \Sigma_B^F \) no longer constitute an equilibrium. Recall that \( \Sigma_A^F \) and \( \Sigma_B^F \) give rise to two equilibria, \((M_A, M_A)\) and \((M_B, M_B)\), and only in the \((M_h, M_h)\) equilibrium manufacturer \( h \) earns a positive profit. When bids are unconstrained manufacturer \( h \) could deviate from \( \Sigma_h^F \) by offering \( \tilde{U}_{hi}^{ED} < 0 \) and \( \tilde{U}_{hi}^{M} < 0 \) for \( i = 1, 2 \) (while keeping all other payments unchanged). It is easy to verify that in this case \( s_i = M_h \) would be a strictly dominant strategy for each retailer \( i = 1, 2 \) and, as a result, manufacturer \( h \) would earn \( 2(\Pi^F - \Pi^{ED} - \epsilon) \) with probability one. Clearly, such deviations will never be profitable only if each manufacturer earns zero even in the event both retailers choose to represent this manufacturer. As a consequence, in equilibrium each manufacturer \( h \) offers the following payments:

- \( U_{hi}^{CA} = U_{hi}^{C} = u_{hi}^{MX} = v_{hi}^{MX} = U_{hi}^{MX} = \infty \) for \( i = 1, 2 \).
\[ U_{hi}^M = U_{hi}^F = 0 \text{ and } U_{hi}^{ED} = - (\Pi^F - \Pi^{ED} - \epsilon) \text{ for } i = 1, 2. \]

**G Non-contingent offers**

Suppose, for example, that manufacturers wish to implement double common agency as a continuation equilibrium in which case they must make the offers which satisfy the following constraints:

\[ \Pi^C - U_{Ai} - U_{Bi} \geq \max\{0, \Pi^{MX} - U_{Ai}, \Pi^{MX} - U_{Bi}\}, \quad \text{G1} \]

for \( i = 1, 2 \). Let \( \Pi^C < 2\Pi^{MX} \) then in any such equilibrium each retailer must earn some rent. From G1 it then follows that it must be \( U_{Ai}, U_{Bi} \leq \Pi^C - \Pi^{MX} \) for \( i = 1, 2 \) which in turn implies that double common agency equilibria exist only if \( \Pi^C \geq \Pi^{MX} \). This condition, however, does not imply that double common agency can always be implemented as an equilibrium whenever it is most efficient. For example, this is not the case when

\[ \Pi^C - \Pi^{MX}_{AB} > \Pi^{MX} - \Pi^C > 0. \]

To illustrate the difficulties related to the uniqueness of an equilibrium, suppose, for example, that the following conditions hold:

\[ \Pi^{ED} - \Pi^{MX}_{AB} > \Pi^C - \Pi^{MX} > \Pi^{ED} - \Pi^F > 0, \]

and

\[ 2(\Pi^C - \Pi^{MX}) \geq \Pi^{CA} - \Pi^M. \]

Suppose again that manufacturers wish to sustain a double common agency equilibrium which implies that manufacturers’ bids must not exceed \( \Pi^C - \Pi^{MX} \). Note that, since the offers are non-contingent, then in any exclusive dealing configuration each retailer \( i \) earns \( \Pi^{ED} - U_{hi} \geq \Pi^{ED} - (\Pi^C - \Pi^{MX}) > 0 \). This in turn gives rise to the possibility for these configurations to be continuation equilibria as well. Therefore, in order to ensure the uniqueness of the double common agency equilibrium, all other equilibria must be destroyed.

There are two ways to do it. The first is to induce (at least) one retailer to represent both manufacturers whenever the other retailer chooses to represent only one. This is possible only if

\[ \Pi^{MX}_{AB} - U_{hi} - U_{ki} > \Pi^{ED} - U_{hi}, \]

for at least some \( i \). This implies that manufacturer \( k \) must then pay \( U_{ki} < - (\Pi^{ED} - \Pi^{MX}_{AB}) \) to retailer \( i \). However, given that in any double common agency equilibrium it earns at most \( \Pi^C - \Pi^{MX}_{AB} < \Pi^{ED} - \Pi^{MX}_{AB} \) in outlet \( j \), it will never make such offers.

The second way is to induce the game in which one retailer tries to ‘catch’ the other, i.e., whenever, say, retailer 1 chooses to represent manufacturer \( h \)
then so does retailer 2, however, in this case retailer 1 will instead prefer to represent manufacturer $k$. This implies that the following conditions must hold:

$$\Pi^{ED} - U_{h1} > \Pi^i - U_{k1} \text{ and } \Pi^F - U_{h2} > \Pi^{ED} - U_{k2},$$

for $k \neq h$. However, they are impossible to satisfy. Indeed, if $\Pi^{ED} \geq \Pi^F$ then the conditions $\Pi^F - U_{A2} > \Pi^{ED} - U_{B2}$ and $\Pi^F - U_{B2} > \Pi^{ED} - U_{A2}$ are incompatible while if $\Pi^{ED} < \Pi^F$ then the same is true for the conditions $\Pi^{ED} - U_{A1} > \Pi^F - U_{B1}$ and $\Pi^{ED} - U_{B1} > \Pi^F - U_{A1}$. Taken together, these results imply that double common agency cannot be implemented as the unique continuation equilibrium.

It remains to show that such an equilibrium indeed exists. For this, suppose that each manufacturer $h$ offers $U_{hi} = \Pi^C - \Pi^{MX}$ for $i = 1, 2$. Given that $\Pi^{ED} - \Pi^F > 0$, it is easy to check that the corresponding retailer game has three equilibria: one is the double common agency configuration and the other two are the exclusive dealing configurations. Assuming that the retailers always choose the double common agency equilibrium (which means that each manufacturer then earns $2(\Pi^C - \Pi^{MX})$), let us verify that no manufacturer have incentives to deviate.

First, monopoly deviations are impossible because each manufacturer $h$ sets $U_{hi} < \Pi^{ED}$ for $i = 1, 2$.

Second, deviations to mixed configurations are impossible either. Indeed, manufacturer $h$ can undertake such deviations only if it offers $\tilde{U}_{hi}$ which satisfies the following constraint:

$$\Pi^{MX}_{AB} - \tilde{U}_{hi} - U_{ki} \geq \max\{\Pi^{ED} - U_{ki}, \Pi^F - \tilde{U}_{hi}\} > 0, \quad \text{G2}$$

where $U_{ki} = \Pi^C - \Pi^{MX} < \Pi^{ED}$. If $\tilde{U}_{hi} > (\Pi^C - \Pi^{MX}) - (\Pi^{ED} - \Pi^F) > 0$ then G2 boils down to $\Pi^{MX}_{AB} - \tilde{U}_{hi} - U_{ki} \geq \Pi^{ED} - U_{ki}$ or $\tilde{U}_{hi} \leq - (\Pi^{ED} - \Pi^{MX}_{AB})$ which is obviously contradiction. If instead $\tilde{U}_{hi} \leq (\Pi^C - \Pi^{MX}) - (\Pi^{ED} - \Pi^F)$ then G2 down to $\Pi^{MX}_{AB} - \tilde{U}_{hi} - U_{ki} \geq \Pi^F - \tilde{U}_{hi}$ or $\tilde{U}_{hi} \leq \Pi^{MX}_{AB} - \Pi^F$ which is also contradiction because $U_{hi} > 0$ while, by supposition, $\Pi^F > \Pi^{MX}_{AB}$. This implies that no such deviation is possible.

Third, common agency deviations are never profitable because, by supposition, $2(\Pi^C - \Pi^{MX}) \geq \Pi^{CA} - \Pi^M$.

Forth, exclusive dealing deviations are never profitable. Indeed, in any such deviation manufacturer $h$ earns

$$\tilde{U}_{hi} \leq \Pi^{ED} - \Pi^F + U_{ki} = (\Pi^{ED} - \Pi^F) + (\Pi^C - \Pi^{MX}).$$

By supposition, $\Pi^{ED} - \Pi^F < \Pi^C - \Pi^{MX}$ and therefore manufacturer $h$ cannot gain from such deviations.

Finally, since $\Pi^{ED} > \Pi^F$ foreclosure deviations are never profitable since in this case a deviating manufacturer earns at most $2(\Pi^C - \Pi^{MX}) - 2(\Pi^{ED} - \Pi^F)$.

Note that in this example I did not use the condition that double common agency is most efficient. One can easily verify that even if it is, it may not be the unique continuation equilibrium.
REFERENCES


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