Mathematics for Economists

Lecturer: Kirill A. Bukin
Class teachers: Ivan O. Kachkovsky, Anatoly S. Arlashin, Milan Martinovič

Course description

Mathematics for Economists is a two-semester course for the second year students studying at ICEF.

This course is an important part of the bachelor stage in education of the future economists. It has give students skills for implementation of the mathematical knowledge and expertise to the problems of economics. Its prerequisites are both the knowledge of the single variable calculus and the foundations of linear algebra including operations on matrices and the general theory of systems of simultaneous linear equations.

The assessment of the students will be by the University of London (UoL) examinations at the end of the fourth semester. During the fourth semester students are specially prepared to the University of London examination. The course covers several variable calculus, optimization theory and the selected topics drawn from the theory of differential and difference equations. That course is aimed at teaching students to master comparative statics problems, optimization problems and dynamic models using the acquired mathematical tools. The course is taught in English. The students are also studying for Russian degree in Economics, and knowing Russian terminology through reading in Russian is also a must.

The latter means the thinking over the theoretical material, working on the home assignments given by the lecturer. During each semester there will be a midterm examination. Since the major part of the students has the final UoL exam "Mathematics–I" while the students specializing in Economics have the two UoL exams "Mathematics–I" and "Mathematics–II", midterm exams resemble UoL examinations to a great extent.

Teaching objectives

The objectives of the course are:

- to acquire the students' knowledge in the field of mathematics and to make them ready to analyze simulated as well as real economic situations;

- to develop the students' ability to apply the knowledge of the differential and difference equations which will enable them to analyze dynamics of the processes.

Teaching methods

The course program consists of:

- lectures,
- classes,
- regular self-study.

Assessment

Control takes the following forms:

- written home assignments (25);
- midterm exam in the middle of fall semester (120 min.),
- mock exam (180 min) in the University of London examination format in the middle of the spring semester;
- written exam (120 min) at the end of the fall semester,
- University of London exam by the end of the spring semester (180 min), Mathematics–I and Mathematics–II

Grade determination

The fall semester will be determined from the following activities:

- average grade for the home assignments (20%);
- fall semester midterm exam (20%);
- fall exam (60%).

Course grade for those students who specialize in Economics is determined by:

- University of London exam grade for "Mathematics–I" (20%),
- University of London exam grade for "Mathematics–II" (50
- fall term grade (20%),
- mock exam grade (10%).

Course grade for the rest of the students is determined by:

- University of London exam grade for "Mathematics–I" (40%),
- fall term grade (40%),
- mock exam grade (20%).

Main reading

1. Carl P. Simon and Lawrence Blume. Mathematics for Economists, W. W. Norton & Company, 1994.
2. A. C. Chiang. Fundamental Methods of Mathematical Economics, 3rd edition, McGrow-Hill, 1984.

Additional reading

1. Б. П. Демидович. Сборник задач и упражнений по математическому анализу, М., Наука, 1966.
2. А. Ф. Филиппов. Сборник задач по дифференциальным уравнениям. М., Наука, 1973.
3. Anthony M. and Biggs N., Mathematics for Economics and Finance, Cambridge University Press, Cambridge, UK, 1996.
4. Anthony M., Reader in Mathematics, LSE, University of London; Mathematics for Economists, Study Guide, University of London.

Internet resources and databases

- University of London Exam papers and Examiners reports for the last three years:
  `http://www.londonexternal.ac.uk/current_students/programme_resources/lse/index.shtml`.
- Current course materials are post at the ICEF information system
  `http://mief.hse.ru`

Course outline

Multi-dimensional calculus

1. Main concepts of set theory

Operations on sets. Direct product of sets. Relations and functions. Level sets and level curves.

(SL Sections 2.1–2.2; C Sections 1.1–2.7)

2. Space $\mathbb{R}^n$

Metric in n-dimensional space. The triangle inequality. Euclidean spaces. Neighborhoods and open sets in $\mathbb{R}^n$, Sequences and their limits. Close sets. The closure and the boundary of a set.

(SL Sections 10.1–10.4; C Sections 12.1–12.6)

3. Functions of several variables

Limits of functions. Continuity of functions.

(SL Sections 13.1–13.5; C Sections 6.4–6.7)

4. Partial differentiation

Economic interpretation, marginal products and elasticities. Chain rule for partial differentiation.

(SL Sections 14.1–14.3; C Section 7.4)

5. Total differential

Geometric interpretation of partial derivatives and the differential. Linear approximation. Differentiability. Smooth functions. Directional derivatives and gradient.

(SL Sections 14.4–14.6; C Sections 8.1–8.7)

6. Higher-order derivatives

Young's theorem. Hessian matrix. Economic applications.

(SL Sections 14.8–14.9; C Sections 7.6, 9.3)

7. Implicit functions. Implicit function theorem

(SL Sections 15.1–15.2; C Section 8.5)

8. Vector-valued functions. Jacobian

(SL Section 14.7; C Section 8.5)

9. Implicit function theorem for the vector-valued functions

(SL Sections 15.3, 15.5; C Section 8.5)

10. Economic applications of the IFT for the comparative statics problems

(SL Section 15.4; C Section 8.6)

Optimization

11. Unconstrained optimization of the multi-dimensional functions

Stationary points. First-order conditions.

(SL Sections 17.1–17.2; C Sections 11.1–11.2)

12. Second-order conditions for extrema

Second differential. Quadratic forms and the associated matrices. Definiteness and semi-definiteness of the quadratic forms. Sylvester criterion. Second-order conditions for extrema.

(SL Sections 16.1–16.2, 17.3–17.4; C Sections 11.3–11.7)

13. Constrained optimization

Lagrangian function and multiplier. First-order conditions for constrained optimization.

(SL Sections 18.1–18.2; C Sections 12.1–12.2)

14. Second differential for the function with the dependent variables.

Definiteness of quadratic form under a linear constraint. Bordered Hessian. Second-order conditions for the constrained optimization.

(SL Sections 16.3–16.4, 19.3; C Section 12.3)

15. Envelope theorem

Economic meaning of a multiplier. Applications of the Lagrange approach in economics. Smooth dependence on the parameters. Envelope theorem.

(SL Sections 18.7–19.2, 19.4; C Section 12.5)

Differential and difference equations

16. Simple first-order equations

Dynamics in economics. Simple first-order equations. Separable equations. Concept of stability of the solution of ODE. Exact equations. General solution as a sum of a general solution of homogeneous equation and a particular solution of a nonhomogeneous equation. Bernoulli equation.

(SL Sections 24.1–24.2; C Sections 13.6, 14.1–14.3)

17. Qualitative theory of differential equations

Solow's growth model. Phase diagram.

(Section 24.5; C Sections 14.6–14.7)

18. Second-order linear differential equations with constant coefficients.

(SL Section 24.3; C Section 15.1)

19. Complex numbers and operations on them

Representation of a number. De Moivre and Euler formulae.

(SL Appendix A3; C Section 15.2)

20. Higher-order linear differential equation with constant coefficients

Characteristic equation. Method of undetermined coefficients for the search of a particular solution. Stability of solutions. Routh theorem (without proof).

(SL Section 24.3; C Sections 15.3–15.7)

21. Difference equations

Discrete time economic systems. Difference equations. Method of solving first-order equations. Convergence and oscillations of a solution. Cobweb model. Partial equilibrium model with the inventory.

(SL Section 23.2; C Sections 16.2–16.6)

22. Second-order difference equations.

(C Sections 17.1–17.3)

23. Higher-order difference equations

Characteristic equation. Undetermined coefficients method. Conditions for the stability of solutions.

(C Section 17.4)

Distribution of hours

#	Topic	Total hours	Contact hours		Self study
			Lectures	Seminars	
Multi-dimensional calculus					
1.	Main concepts of set theory. Operations on sets. Direct product of sets. Relations and functions. Level sets and level curves	14	4	4	6
2.	Space $\mathbb{R}^n$. Metric in n-dimensional space. The triangle inequality. Euclidean spaces. Neighborhoods and open sets in $\mathbb{R}^n$, Sequences and their limits. Close sets. The closure and the boundary of a set	14	4	4	6
3.	Functions of several variables. Limits of functions. Continuity of functions	14	4	4	6
4.	Partial differentiation. Economic interpretation, marginal products and elasticities. Chain rule for partial differentiation	12	4	2	6

#	Topic	Total hours	Contact hours		Self study
			Lectures	Seminars	
5.	Total differential. Geometric interpretation of partial derivatives and the differential. Linear approximation. Differentiability. Smooth functions. Directional derivatives and gradient	12	4	2	6
6.	Higher-order derivatives. Young's theorem. Hessian matrix. Economic applications	12	4	2	6
7.	Implicit functions. Implicit function theorem	12	4	2	6
8.	Vector-valued functions. Jacobian	12	4	2	6
9.	Implicit function theorem for the vector-valued functions	12	4	2	6
10.	Economic applications of the IFT for the comparative statics problems	12	4	2	6
Optimization					
11.	Unconstrained optimization of the multi-dimensional functions. Stationary points. First-order conditions	12	4	2	6
12.	Second differential. Quadratic forms and the associated matrices. Definiteness and semi-definiteness of the quadratic forms. Sylvester criterion. Second-order conditions for extrema	10	2	2	6
13.	Constrained optimization. Lagrangian function and multiplier. First-order conditions for constrained optimization	10	2	2	6

#	Topic	Total hours	Contact hours		Self study
			Lectures	Seminars	
14.	Second differential for the function with the dependent variables. Definiteness of quadratic form under a linear constraint. Bordered Hessian. Second-order conditions for the constrained optimization	10	2	2	6
15.	Economic meaning of a multiplier. Applications of the Lagrange approach in economics. Smooth dependence on the parameters. Envelope theorem	10	2	2	6
	Differential and difference equations				
16.	Dynamics in economics. Simple first-order equations. Separable equations. Concept of stability of the solution of ODE. Exact equations. General solution as a sum of a general solution of homogeneous equation and a particular solution of a nonhomogeneous equation. Bernoulli equation	10	2	2	6
17.	Qualitative theory of differential equations. Solow's growth model. Phase diagram	10	2	2	6
18.	Second-order linear differential equations with constant coefficients	10	2	2	6
19.	Complex numbers and operations on them. Representation of a number. De Moivre and Euler formulae	10	2	2	6

#	Topic	Total hours	Contact hours		Self study
			Lectures	Seminars	
20.	Higher-order linear differential equation with constant coefficients. Characteristic equation. Method of undetermined coefficients for the search of a particular solution. Stability of solutions. Routh theorem (without proof)	12	2	2	8
21.	Discrete time economic systems. Difference equations. Method of solving first-order equations. Convergence and oscillations of a solution. Cobweb model. Partial equilibrium model with the inventory	12	2	2	8
22.	Second-order difference equations	14	2	2	10
23.	Higher-order difference equations. Characteristic equation. Undetermined coefficients method. Conditions for the stability of solutions	14	2	2	10
	Total:	270	68	52	150