System Consequence

Robert E. Kent
Ontologos

"In the same way, the world is not the sum of all the things that are in it. It is the infinitely complex network of connections among them. As in the meaning of words, things take on meaning only in relation to each other."

The Invention of Solitude — Paul Auster

"world"	—	information system
"thing"	—	information resource
"connection"	—	constraint link

Abstract

My paper discusses system consequence, which is a central idea in the project to lift the theory of information flow to the general level of universal logic and the theory of institutions. At the same time, it uses ideas from information flow to extend the theory of institutions.

ICCS 2009, 26–31 July 2009

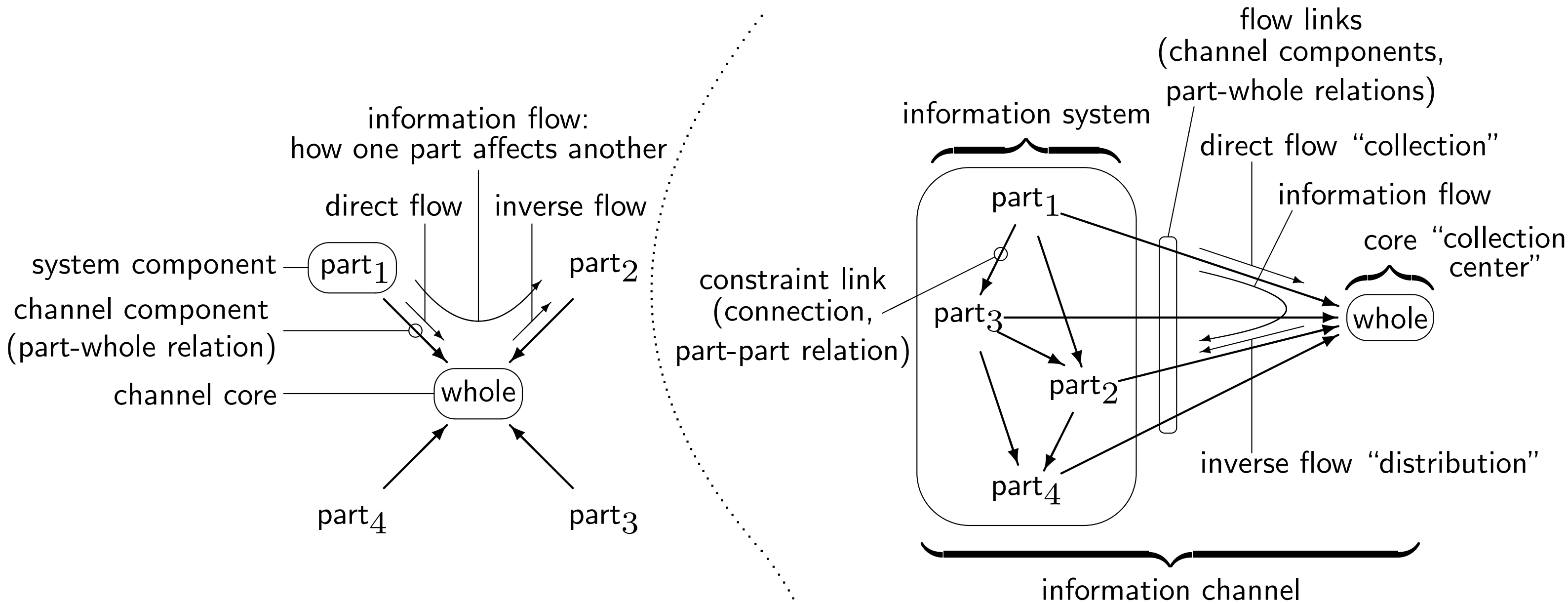
Table of Contents

Slogan: *"Systems of information resources have a consequence!"*

[1] IF and INS are the relevant foreground theories. Relevant background theories include Formal Concept Analysis (FCA), Conceptual Graphs (CG), Semantic Web technology (SW), etc.

[2] ". . . good general theory does not search for the maximum generality, but for the right generality." – Mac Lane

Main Concepts



top view
as in *Information Flow*
by Barwise and Seligman

side view

- information system

  e.g. distributed information resources of a scientific community

- information channel

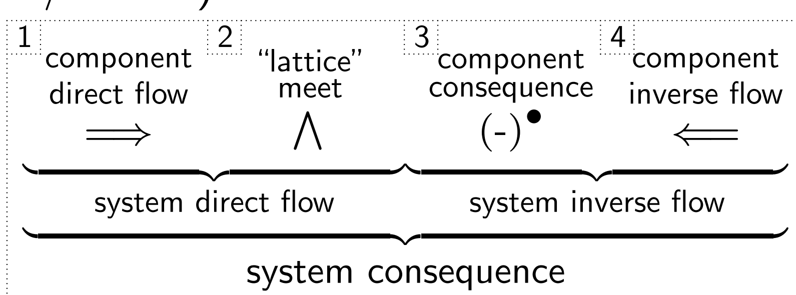
  architecture underlying information flow

- information flow (direct/inverse)

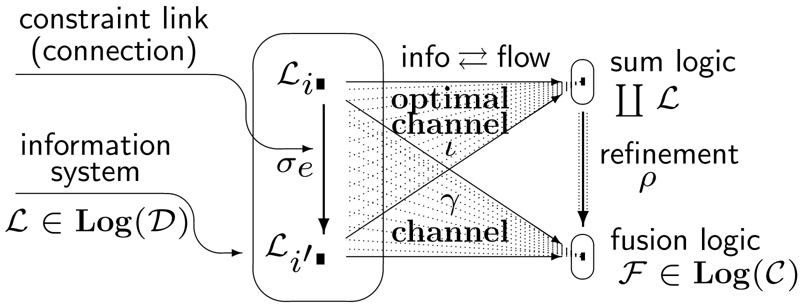
  how different parts interact

- system consequence

  closure of information flow



A Conference Analogy



Consider a conference such as the ICCS 2009.

Information System: $\mathcal{L} \in \mathbf{Log}(\mathcal{D})$, for $\mathcal{D} : \mathbf{I} \rightarrow \mathbf{V}$, $\mathbf{V} = \mathbf{Lang}$ or $\mathbf{Struc}$

A community's knowledge is distributed as a system of information resources (the member's knowledge); that is, an information system $\mathcal{L} = \{\mathcal{L}_i = \langle \mathcal{D}_i, T_i \rangle \mid i \in \mathbf{I}\}$. The knowledge implicit $\mathcal{L}_i^{\bullet}$ (*consequence*) in a member's understanding is based upon that person's abilities $\mathcal{L}_i$ with $\mathcal{L}_i \geq \mathcal{L}_i^{\bullet}$.

Information Channel: $\gamma : \mathcal{D} \Rightarrow \mathcal{C}$

A conference serves the needs of a community. It is represented as an information channel $\gamma = \{\gamma_i : \mathcal{D}_i \rightarrow \mathcal{C} \mid i \in \mathbf{I}\}$ with components γ_i and core $\mathcal{C}$. [3]

Direct System Flow: $\boldsymbol{dir}(\gamma) : \mathbf{Log}(\mathcal{D}) \Rightarrow \mathbf{Log}(\mathcal{C})$

People come to discuss topics and increase their knowledge (*direct specification flow*) $\mathcal{L}_i \mapsto \boldsymbol{dir}(\gamma_i)(\mathcal{L}_i)$. Activities at the conference include paper presentations, poster sessions, informal discussions, with disagreements aired and recent advances revealed (*information fusion*) $\mathcal{F} \doteq \bigwedge \boldsymbol{dir}(\gamma)(\mathcal{L}) \in \mathbf{Log}(\mathcal{C})$.

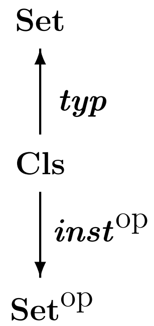
Inverse System Flow: $\boldsymbol{inv}(\gamma) : \mathbf{Log}(\mathcal{D}) \Leftarrow \mathbf{Log}(\mathcal{C})$

An attendee's understanding of the issues potentially includes all the knowledge implicitly revealed by the conference activities (*fusion consequence*) $\mathcal{F}^{\bullet}$. Assuming a good conference, an attendee takes home with themselves this increased understanding (*inverse specification flow*) $\mathcal{F}^{\bullet} \mapsto \boldsymbol{inv}(\gamma_i)(\mathcal{F}^{\bullet})$.

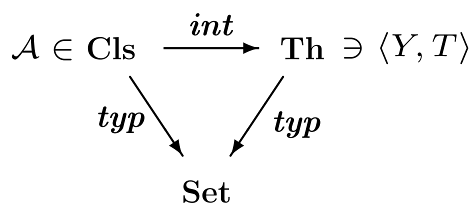
System Consequence: $(\text{-})^{\blacklozenge\gamma} \doteq \boldsymbol{dir}(\gamma) \circ \boldsymbol{inv}(\gamma)$

There has been an increase in knowledge $\mathcal{L} \geq \mathcal{L}^{\blacklozenge}$ distributed over the whole community $\mathcal{L}^{\blacklozenge} = \boldsymbol{inv}(\gamma)(\boldsymbol{dir}(\gamma)(\mathcal{L}))$.

[3] For the ICCS 2009 conference, the (*channel core*) $\mathcal{C}$ is a single object. However, with satellite conferences, etc., the channel core would be distributed $\mathcal{C} = \{\mathcal{C}_j \mid j \in \mathbf{J}\}$.



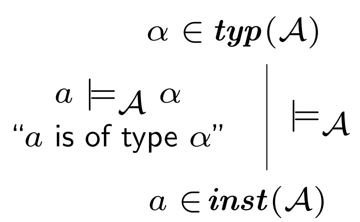
Information Flow



$Y = \boldsymbol{typ}(\mathcal{A})$
$T \subseteq \boldsymbol{seq}(Y)$

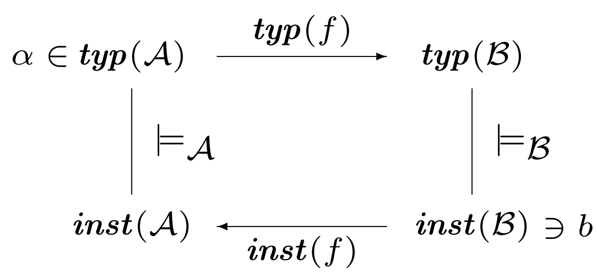
- Summarized: *Information Flow: The Logic of Distributed Systems* by Barwise and Seligman.
- Concepts: classification, infomorphism, theory, (local) logic, information flow, distributed systems, information systems, information channel, and the distributed logic (i.e., consequence) of an information system.
- Core concept and property: $\mathbf{Cls}$

classification



$\mathcal{A} = \langle \boldsymbol{inst}(\mathcal{A}), \boldsymbol{typ}(\mathcal{A}), \models_{\mathcal{A}} \rangle$

infomorphism



$f = \langle \boldsymbol{inst}(f), \boldsymbol{typ}(f) \rangle : \mathcal{A} \rightleftarrows \mathcal{B}$
"classification is invariant under instance-type flow"
$\boldsymbol{inst}(f)(b) \models_{\mathcal{A}} \alpha$ iff $b \models_{\mathcal{B}} \boldsymbol{typ}(f)(\alpha)$

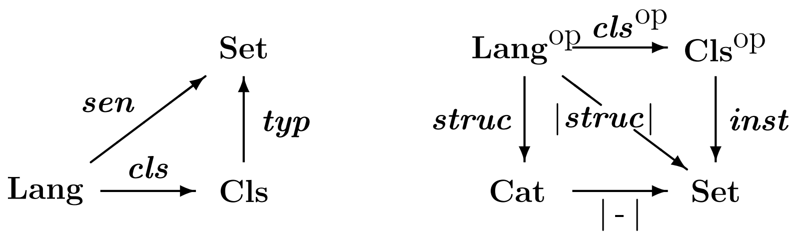
- Examples:

$\mathcal{A} =$ astronomical objects
mars, rigel $\in \boldsymbol{inst}(A)$
mars $\models_A$ planet, mars $\not\models_A$ star

$f = \langle \hat{f}, \check{f} \rangle : \langle A, \leq_A \rangle \rightleftarrows \langle B, \leq_B \rangle$
adjoint pair of monotonic functions

$\mathcal{A} = \langle A, \leq_A \rangle$ preorder, where $\boldsymbol{inst}(\mathcal{A}) = \boldsymbol{typ}(\mathcal{A}) = A$ and $\models_{\mathcal{A}} = \leq_A$

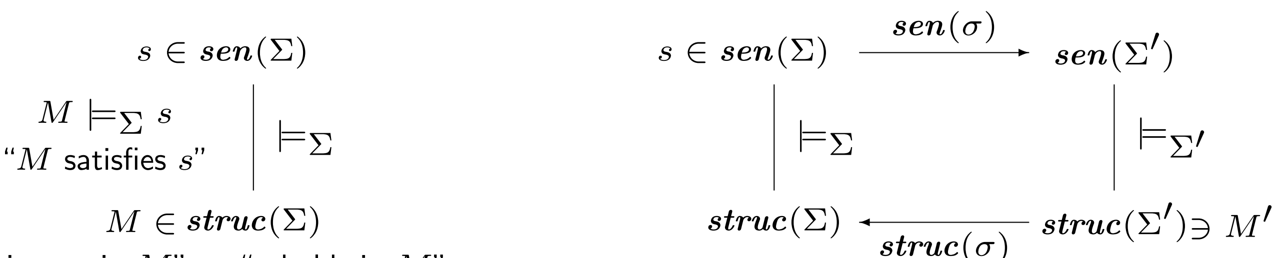
Institutions



- Start: 1992 paper "Institutions: Abstract Model Theory for Specification and Programming" by Goguen and Burstall.
- Heterogeneous elements (atoms):

language (signature)	$\Sigma \in \mathbf{Lang}$
structure (model)	$M \in \boldsymbol{struc}(\Sigma)$
sentence	$s \in \boldsymbol{sen}(\Sigma)$

- Core concept and property: $\boldsymbol{cls} = \langle \boldsymbol{struc}, \boldsymbol{sen} \rangle : \mathbf{Lang} \to \mathbf{Cls}$



"s is true in M" or "s holds in M"
"s is satisfied in M" or "M is a model for s"
language Σ
$\boldsymbol{cls}(\Sigma) = \langle \boldsymbol{struc}(\Sigma), \boldsymbol{sen}(\Sigma), \models_{\Sigma} \rangle$
satisfaction for language Σ

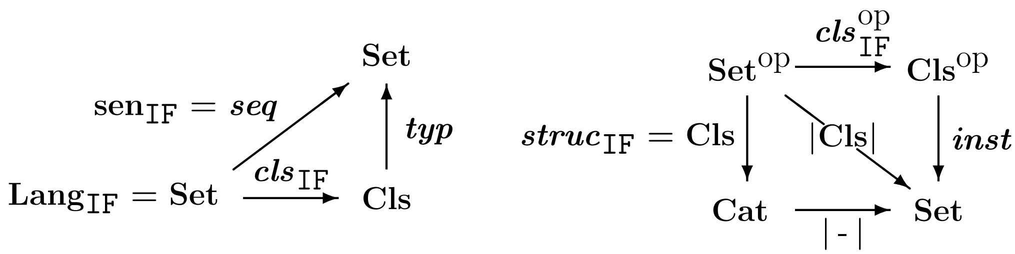
language morphism $\sigma : \Sigma \to \Sigma'$
$\boldsymbol{cls}(\sigma) = \langle \boldsymbol{struc}(\sigma), \boldsymbol{sen}(\sigma) \rangle : \boldsymbol{cls}(\Sigma) \rightleftarrows \boldsymbol{cls}(\Sigma')$
"satisfaction is invariant under sentence-structure flow"
$\boldsymbol{struc}(\sigma)(M') \models_{\Sigma} s$ <u>iff</u> $M' \models_{\Sigma'} \boldsymbol{sen}(\sigma)(s)$

- Examples:

FOL
$\Sigma = \{f \in \boldsymbol{ftn}(\Sigma), r \in \boldsymbol{rel}(\Sigma)\}$
s composed of f, r, $\wedge$, $\vee$, $\neg$, $\exists$, $\forall$
$M = \{F_f : U^n \to U, R_r \subseteq U^m\}$
$M \models_{\Sigma} s$ when s is true in M

FOL interpretation
$\sigma = \{f \mapsto t', r \mapsto \varphi' \mid t' \in \boldsymbol{trm}(\Sigma'), \varphi' \in \boldsymbol{fmla}(\Sigma')\}$
$\boldsymbol{sen}(\sigma)(r(t_1, ..., t_n)) = \varphi'(\sigma^*(t_1), ..., \sigma^*(t_n))$
$\boldsymbol{struc}(\sigma)(M') = M'$ where
$\boldsymbol{struc}(\sigma)(M') \models_{\Sigma} s$ <u>iff</u> $M' \models_{\Sigma'} \boldsymbol{sen}(\sigma)(s)$

Generalization: The IF Institution



- Extend classification from types to sequents

$\mathcal{A} = \langle \boldsymbol{inst}(\mathcal{A}), \boldsymbol{typ}(\mathcal{A}), \models_{\mathcal{A}} \rangle$ $\quad$ $f = \langle \boldsymbol{inst}(f), \boldsymbol{typ}(f) \rangle : \mathcal{A} \rightleftarrows \mathcal{B}$

$$\begin{array}{ccc} & \langle \Gamma, \Delta \rangle \in \boldsymbol{seq}(\mathcal{A}) & \\ a \models_{\mathcal{A}} \langle \Gamma, \Delta \rangle & \Big| \models_{\mathcal{A}} & \\ & a \in \boldsymbol{inst}(\mathcal{A}) & \end{array} \qquad \begin{array}{ccc} \langle \Gamma, \Delta \rangle \in \boldsymbol{seq}(\mathcal{A}) & \xrightarrow{\boldsymbol{seq}(f)} & \boldsymbol{seq}(\mathcal{B}) \\ \Big| \models_{\mathcal{A}} & & \Big| \models_{\mathcal{B}} \\ \boldsymbol{inst}(\mathcal{A}) & \xleftarrow[\boldsymbol{inst}(f)]{} & \boldsymbol{inst}(\mathcal{B}) \ni b \end{array}$$

$a \models^{\forall}_{\mathcal{A}} \Gamma$ implies $a \models^{\exists}_{\mathcal{A}} \Delta$ $\qquad$ $\boldsymbol{inst}(f)(b) \models_{\mathcal{A}} \langle \Gamma, \Delta \rangle$ <u>iff</u> $b \models_{\mathcal{B}} \boldsymbol{seq}(f)(\Gamma, \Delta)$

- The IF institution [4] $\mathrm{Lang}_{\mathtt{IF}} = \mathrm{Set} \xrightarrow{\boldsymbol{cls}_{\mathtt{IF}}} \mathrm{Cls}$

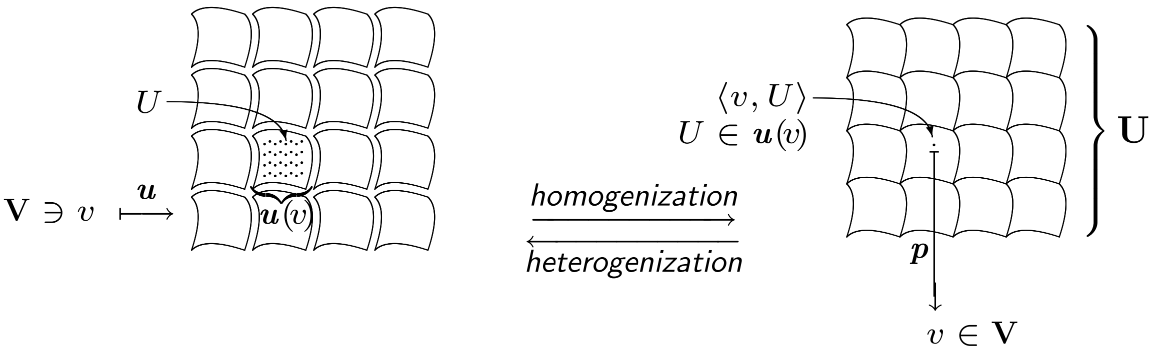
(type) set Y $\qquad$ (type) function $g : Y \rightarrow Y'$

$$\begin{array}{ccc} & \langle \Gamma, \Delta \rangle \in \boldsymbol{seq}(Y) & \\ \mathcal{A} \models_{Y} \langle \Gamma, \Delta \rangle & \Big| \models_{Y} & \\ & \mathcal{A} \in \mathrm{Cls}(Y) & \end{array} \qquad \begin{array}{ccc} \langle \Gamma, \Delta \rangle \in \boldsymbol{seq}(Y) & \xrightarrow{\boldsymbol{seq}(g)} & \boldsymbol{seq}(Y') \\ \Big| \models_{Y} & & \Big| \models_{Y'} \\ \mathrm{Cls}(Y) & \xleftarrow[\mathrm{Cls}(g)]{} & \mathrm{Cls}(Y') \ni \mathcal{A}' \end{array}$$

$a \models_{\mathcal{A}} \langle \Gamma, \Delta \rangle$ all $a \in \boldsymbol{inst}(\mathcal{A})$ $\qquad$ $\mathrm{Cls}(g)(\mathcal{A}') \models_{Y} \langle \Gamma, \Delta \rangle$ <u>iff</u> $\mathcal{A}' \models_{Y'} \boldsymbol{seq}(g)(\Gamma, \Delta)$
where $a' \models_{\mathrm{Cls}(g)(\mathcal{A}')} \alpha$ when $a' \models_{\mathcal{A}'} g(\alpha)$ for all $a' \in \boldsymbol{inst}(\mathcal{A}')$ and $\alpha \in Y$

[4] In IF, languages are (type) sets, structures are classifications and sentences are sequents.

Extension: Heterogeneous versus Homogeneous



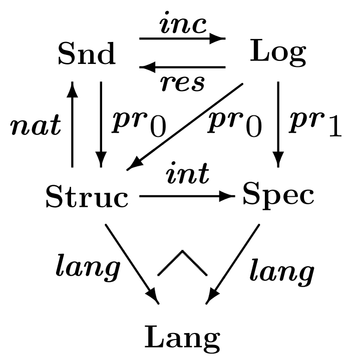
indexed set
(heterogeneous representation)

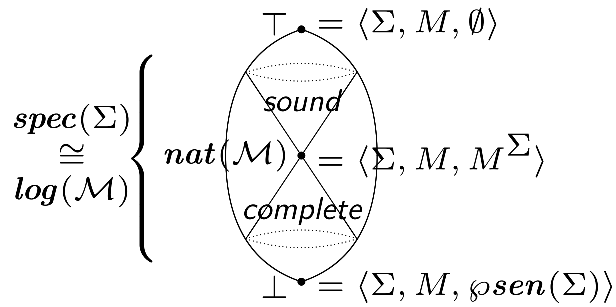
fibered set
(homogeneous representation)

- heterogeneous $\boldsymbol{u} : \mathbf{V} \rightarrow \mathbf{Set}$;
  i.e. $\boldsymbol{u} = \{\boldsymbol{u}(v) \in \mathbf{Set} \mid v \in \mathbf{V}\}$
- disjoint union $\mathbf{U} \doteq \{\langle v, U \rangle \mid U \in \boldsymbol{u}(v)\}$
- homogeneous $\boldsymbol{p} : \mathbf{U} \rightarrow \mathbf{V} : \langle v, U \rangle \mapsto v$
- for mathematical contexts, disjoint union extended[5] to order ($spec \leq gen$) and morphism ($gen \rightarrow spec$)
- Homogeneous elements (molecules):

	$\mathbf{V}$	$\boldsymbol{u}$	$\mathbf{U}$	$U \in \boldsymbol{u}(v)$	$\langle v, U \rangle$
	Lang	***struc***	**Struc**	$M \in \boldsymbol{struc}(\Sigma)$	$\mathcal{M} = \langle \Sigma, M \rangle$
specification	**Lang**	***spec***	**Spec**	$T \in \boldsymbol{spec}(\Sigma)$	$\mathcal{T} = \langle \Sigma, T \rangle$
logic	**Struc**	***log***	**Log**	$L \in \boldsymbol{log}(\Sigma, M)$	$\mathcal{L} = \langle \Sigma, M, T \rangle$

[5] extension called Grothendieck construction





- fibered context $\mathbf{Struc}$: [6]

structure	*structure morphism*
$\mathcal{M} = \langle \Sigma, M \rangle$	$\sigma : \mathcal{M} \rightarrow \mathcal{M}'$
$\Sigma \in \mathbf{Lang}, M \in \boldsymbol{struc}(\Sigma)$	$\sigma : \Sigma \rightarrow \Sigma', M \geq \boldsymbol{struc}(\sigma)(M')$

- fibered context $\mathbf{Spec}$: [7]

specification	*specification morphism*
$\mathcal{T} = \langle \Sigma, T \rangle$	$\sigma : \mathcal{T} \rightarrow \mathcal{T}'$
$\Sigma \in \mathbf{Lang}, T \in \boldsymbol{th}(\Sigma) = \wp\boldsymbol{sen}(\Sigma)$	$\sigma : \Sigma \rightarrow \Sigma', T \geq \boldsymbol{inv}(\sigma)(T')$

- fibered context $\mathbf{Log}$: [8] [9]

logic	*logic morphism*
$\mathcal{L} = \langle \Sigma, M, T \rangle$	$\sigma : \mathcal{L} \rightarrow \mathcal{L}'$
$\Sigma \in \mathbf{Lang}, M \in \boldsymbol{struc}(\Sigma),$	$\sigma : \Sigma \rightarrow \Sigma', M \geq \boldsymbol{struc}(\sigma)(M'),$
$T \in \boldsymbol{th}(\Sigma)$	$T \geq \boldsymbol{inv}(\sigma)(T')$

[6] Entailment order $\leq$ in the fibers $\boldsymbol{spec}(\Sigma) \cong \boldsymbol{log}(\mathcal{M})$ is concept lattice order "$spec \leq gen$". Morphism $\rightarrow$ in the fibered contexts $\mathbf{Struc}$, $\mathbf{Spec}$ and $\mathbf{Log}$ reduces to reverse entailment order $\geq$ in the fibers; we express this fact as "$gen \rightarrow spec$".

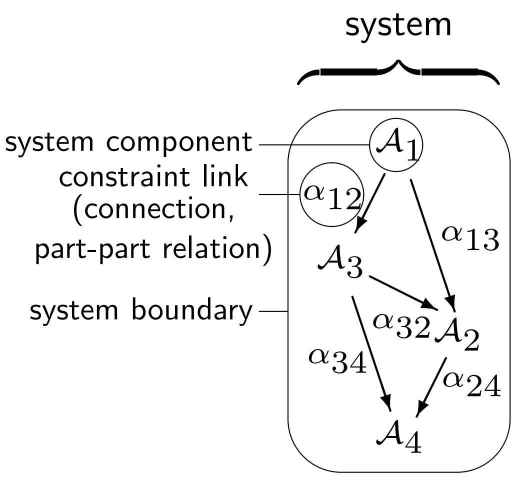
[7] A *theory* in information flow is a pair $\mathcal{T} = \langle Y, T \rangle$, where $Y \in \mathbf{Set}$ is a (type) set and $T \in \wp\boldsymbol{seq}(Y)$ is a subset of Y-sequents.

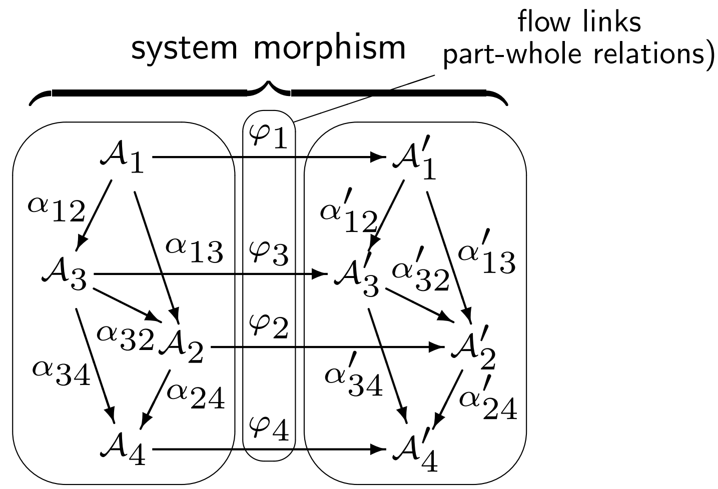
[8] A logic is sound when $T \geq M^{\Sigma}$ and complete when $M^{\Sigma} \geq T$. A (local) logic consists of a general logic $\langle \Sigma, M, T \rangle$ and a sound logic $\langle \Sigma, N, T \rangle$ such that $M^{\Sigma} \wedge T \geq N^{\Sigma}$.

[9] A *(general) logic* in information flow is a triple $\mathcal{L} = \langle Y, A, T \rangle$, where $Y \in \mathbf{Set}$ is a (type) set, $A \in \mathbf{Cls}(Y)$ is a Y-classification, and $T \subseteq \wp\boldsymbol{seq}(Y)$ is a Y-theory.

General Systems (Gr: $\sigma\upsilon\sigma\tau\eta\mu\alpha$)

"a regularly interacting or interdependent group of entities forming a integrated whole"

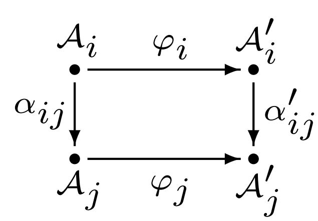




$$\mathcal{A} : \mathbf{I} \to \mathbf{V}$$
$$\mathbf{I} = \{1, 2, 3, 4\}, \mathcal{A}_i \in \mathbf{V}$$

$$\mathcal{A} \stackrel{\varphi}{\Longrightarrow} \mathcal{A}'$$
$$\mathbf{I} = \{1, 2, 3, 4\}, \varphi_i : \mathcal{A}_i \to \mathcal{A}'_i$$

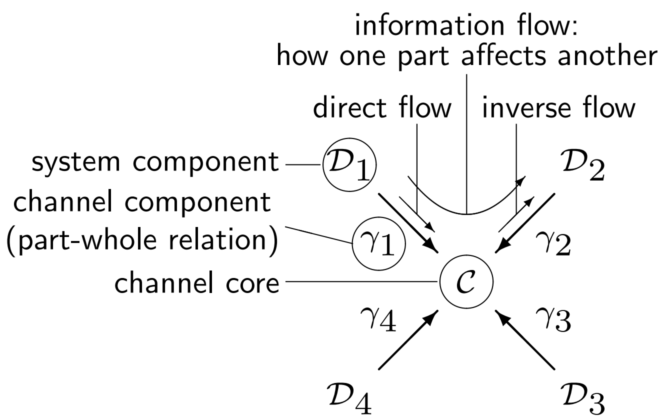
- A system is a collection of elements (or parts) connected together by constraint links. [10]
- A system morphism, linking one system (source) to another (target), is a collection of flow links between source and target components. [11]
- Link types:
  Constraint links are orthogonal to flow links.
  Flow links naturally translate constraint links:



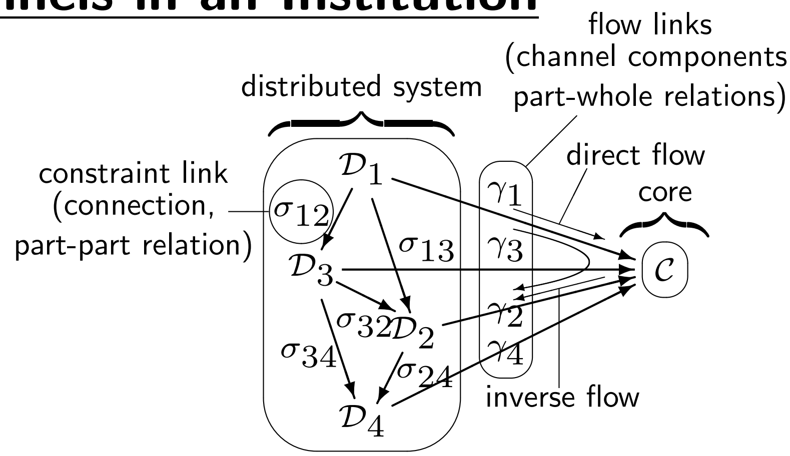
[10] In information systems, constraint links represent "ontology alignment".

[11] In information systems, system morphisms are information channels. These facilitate "ontology unification" (fusion) and system consequence.

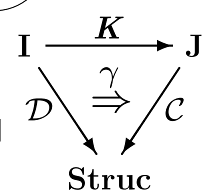
Information Channels in an Institution



information system $\mathcal{D} : \mathbf{I} \to \mathbf{Struc}$



information channel



System Principle: *Information flow results from regularities in a distributed system.* [12]

Structure Principle: *Information flow crucially involves structures of the world* (generalized from classifications to structures). [13]

Connection Principle: *It is by virtue of regularities among connections that information about some components of a distributed system carries information about other components.* [14]

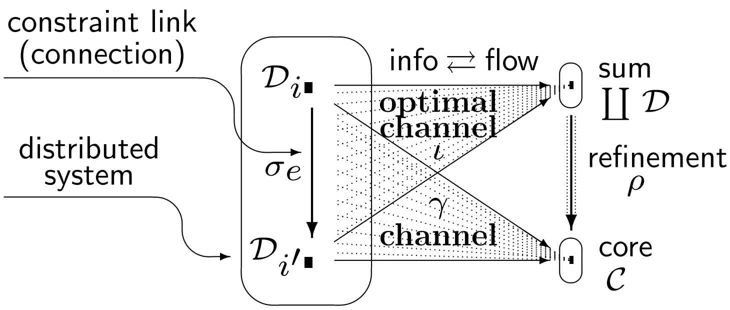
Channel Principle: *The regularities of a given distributed system are relative to its analysis in terms of information channels.*

[12] Motivates the representation of distributed systems by diagrams of objects (specifications or logics) that can incorporate regularities.

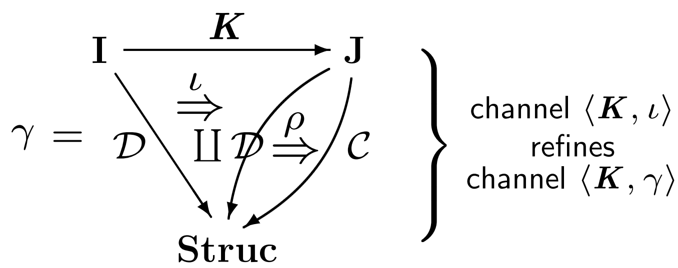
[13] Motivates the use of structures to underpin logics and structure morphisms to facilitate the flow of logics.

[14] Motivates the use of logics-over-structures or specifications-over-languages to represent information flow over the channels of a distributed system.

Information Flow in an Institution



concrete representation



abstract representation

- Atomic flow

  structures: $\boldsymbol{struc}(\sigma) : \boldsymbol{struc}(\Sigma) \leftarrow \boldsymbol{struc}(\Sigma')$

  sentences: $\boldsymbol{sen}(\sigma) : \boldsymbol{sen}(\Sigma) \rightarrow \boldsymbol{sen}(\Sigma')$ along $\sigma : \Sigma \rightarrow \Sigma'$

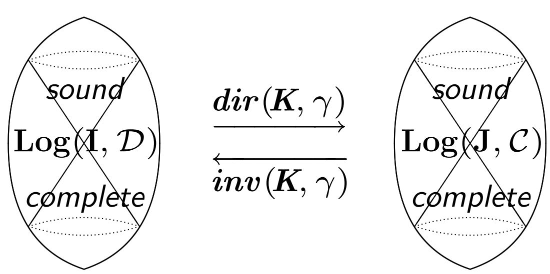
- Molecular flow [15]

  specifications: $\langle \boldsymbol{dir}(\sigma) \dashv \boldsymbol{inv}(\sigma) \rangle : \boldsymbol{spec}(\Sigma) \rightleftarrows \boldsymbol{spec}(\Sigma')$ along $\Sigma \xrightarrow{\sigma} \Sigma'$

  logics: $\langle \boldsymbol{dir}(\sigma) \dashv \boldsymbol{inv}(\sigma) \rangle : \boldsymbol{log}(\mathcal{M}) \rightleftarrows \boldsymbol{log}(\mathcal{M}')$ along $\mathcal{M} \xrightarrow{\sigma} \mathcal{M}'$

- Systemic flow [16]

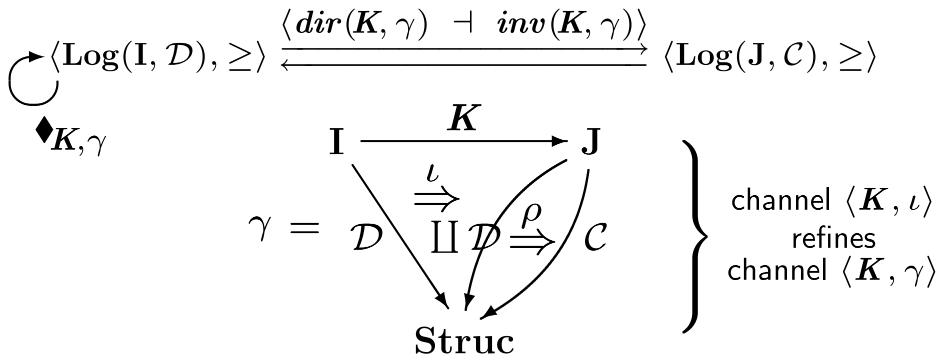
  systems: $\langle \boldsymbol{dir}(\boldsymbol{K}, \gamma) \dashv \boldsymbol{inv}(\boldsymbol{K}, \gamma) \rangle : \mathbf{Log}(\mathbf{I}, \mathcal{D}) \rightleftarrows \mathbf{Log}(\mathbf{J}, \mathcal{C})$
  along information channel $\langle \boldsymbol{K}, \gamma \rangle : \langle \mathbf{I}, \mathcal{D} \rangle \rightarrow \langle \mathbf{J}, \mathcal{C} \rangle$



15 $\boldsymbol{spec}(\Sigma) \doteq \langle \wp\boldsymbol{sen}(\Sigma), \geq \rangle$, where $\leq$ is entailment (concept lattice) order
$\boldsymbol{dir}(\sigma) \doteq \wp\boldsymbol{sen}(\sigma)$ and $\boldsymbol{inv}(\sigma) \doteq \boldsymbol{sen}^{-1}(\sigma)$

16 formal $\mathcal{L} : \mathbf{I} \rightarrow \mathbf{Spec}$ $\mathcal{D} : \mathbf{I} \rightarrow \mathbf{Lang}$
semantic $\mathcal{L} : \mathbf{I} \rightarrow \mathbf{Log}$ $\mathcal{D} : \mathbf{I} \rightarrow \mathbf{Struc}$

System Consequence in an Institution



system consequence $\mathcal{L}^{\blacklozenge} = \boldsymbol{inv}(\boldsymbol{K}, \gamma)(\boldsymbol{dir}(\boldsymbol{K}, \gamma)((\mathcal{L})))$

Generalities of consequence: Along [17]

information channel $\langle \boldsymbol{K}, \gamma \rangle : \langle \mathbf{I}, \mathcal{D} \rangle \rightarrow \langle \mathbf{J}, \mathcal{C} \rangle$

optimal channel $\langle \boldsymbol{K}, \iota \rangle : \langle \mathbf{I}, \mathcal{D} \rangle \rightarrow \langle \mathbf{J}, \coprod \mathcal{D} \rangle$

absolute channel $\langle !_{\mathbf{I}}, \iota \rangle : \langle \mathbf{I}, \mathcal{D} \rangle \rightarrow \langle \mathbf{1}, \coprod \mathcal{D} \rangle$

Properties of consequence:

- closure operator: monotonic, increasing and idempotent
- composite is more specialized: $\blacklozenge_{\langle \boldsymbol{K},\gamma \rangle} \geq \blacklozenge_{\langle \boldsymbol{K},\gamma \rangle \circ \langle \boldsymbol{K}',\gamma' \rangle}$
- absolute is most specialized: $\blacklozenge_{\langle \boldsymbol{K},\iota \rangle} \geq \blacklozenge_{\langle !_{\mathbf{I}},\iota \rangle}$
- $\mathcal{L}^{\blacklozenge} = \boldsymbol{res}\,(\boldsymbol{inc}\mathcal{L})^{\blacklozenge}$ for sound info system $\mathcal{L} : \mathbf{I} \longrightarrow \mathbf{Log}$
- $\boldsymbol{res}(\mathcal{L}^{\blacklozenge}) \leq (\boldsymbol{res}\mathcal{L})^{\blacklozenge}$ for general info system $\mathcal{L} : \mathbf{I} \longrightarrow \mathbf{Log}$

Open Questions:

- $\boldsymbol{res}(\mathcal{L}^{\blacklozenge}) < (\boldsymbol{res}\mathcal{L})^{\blacklozenge}$?
- ????

[17] Absolute channels use shape $\mathbf{I}$ to commence direct specification flow. Optimal channels use subshape $(\boldsymbol{K} \downarrow j) \subseteq \mathbf{I}$, consisting of all $\mathbf{I}$-indexing objects relatively more general than indexing object $j \in \mathbf{J}$.

Examples

Parallel Flow: [18] Discrete system $\mathcal{T} : I \rightarrow \mathbf{Spec}$ where $\mathcal{T} = \{\langle \Sigma_i, T_i \rangle \mid i \in I\}$.

- optimal channel is disjoint union (non-interacting)
- direct system flow is disjoint specification union
- inverse system flow is disjoint specification consequence and selection
- system consequence is $\mathcal{T} = \{\langle \Sigma_i, T_i^{\bullet} \rangle \mid i \in I\}$.

Identity Flow: [19] Constant system $\Delta(\Sigma) : \mathbf{I} \rightarrow \mathbf{Lang}$

- optimal channel is identity
- direct system flow is meet (specification union)
- inverse system flow is specification consequence
- system consequence is $\mathcal{T}_i^{\blacklozenge} = \left(\bigcup_{i \in I} \mathcal{T}_i\right)^{\bullet}$ for all $i \in I$.

Equivalencing: Consider two ontologies (FOL specifications) $\mathcal{T}_0 = \langle \Sigma_0, T_0 \rangle$ and $\mathcal{T}_1 = \langle \Sigma_1, T_1 \rangle$ with reference ontology $\widehat{\mathcal{T}} = \langle \widehat{\Sigma}, \widehat{T} \rangle$, where $\iota_i : \widehat{\mathcal{T}} \rightarrow \mathcal{T}_i$ are injective language morphisms $\iota_i : \widehat{\Sigma} \hookrightarrow \Sigma_i$ for $i = 0, 1$ pictured as $\Sigma_0 \widehat{\Sigma} \Sigma_1$. Then system consequence merges the two ontologies by equivalencing the linked symbols $\iota_0(r) \equiv \iota_1(r)$ for $r \in \widehat{\Sigma}$.

---

[18] or better, parallel non-interacting disjoint flow

[19] Specifically, consider any FOL specification $\mathcal{T} = \langle \Sigma, T \rangle$ with $T = \{s_i \mid i \in I\}$. Totally distribute the axioms over I getting the formal information system $\mathcal{T}^{1,I} : \mathcal{B}(1, I) \rightarrow \mathbf{Spec}$ over the constant distributed system $\Sigma^{1,I} = \Delta(\Sigma) : \mathcal{B}(1, I) \rightarrow \mathbf{Lang}$, where $\mathcal{B}(1, I)$ is the complete bipartite directed graph on $(1, I)$, $\mathcal{T}_i^{1,I} = \langle \Sigma, \{s_i\} \rangle$ and $\mathcal{T}_*^{1,I} = \langle \Sigma, \emptyset \rangle$. Then $\mathcal{T}_i^{\blacklozenge} = \langle \Sigma_i, T \rangle^{\bullet}$ for $i \in I$.

Summary

- The theory of information flow and the theory of institutions are both *abstract*.
- The theory of information flow and the theory of institutions are compatible.
  - The theory of institutions *generalizes* the theory of information flow from one specific logical system IF to an arbitrary logical system. [20] [21]
  - The theory of information flow *extends* the theory of institutions with the concepts of logics, information systems, information flow, system consequence, etc. [22]
- Information flow involves the flow of information systems along information channels. This flow is adjoint between direct and inverse flow. The closure of this adjoint information flow is system consequence, which generalizes specification consequence to systems.

[20] Examples of logical systems include: first order, equational, Horn clause, intuitionistic, modal, linear, higher-order, polymorphic, temporal, process, behavioral, coalgebraic and object-oriented logics.

[21] The paper "System Consequence" describes four important logical systems: unsorted equational logic EQ, information flow IF, unsorted first-order logic with equality FOL, which extends EQ and IF, and the sketch institution Sk.

[22] From hindsight, the fact that the theory of information flow can be used to extend the theory of institutions is clear from **Example 4.11 (Interpretations in First-Order Logic)** in the textbook *Information Flow: The Logic of Distributed Systems* by Barwise and Seligman.

References

[1] Barwise, J., Seligman, J.: Information Flow: The Logic of Distributed Systems. Cambridge University Press, Cambridge (1997)

[2] Goguen, J., Burstall, R.: Institutions: Abstract Model Theory for Specification and Programming. J. Assoc. Comp. Mach. vol. 39, pp. 95–146, (1992)

[3] Kent, R.E.: Semantic Integration in the Information Flow Framework. In: Kalfoglou, Y., Schorlemmer, M., Sheth, A., Staab, S., Uschold, M. (eds.) Semantic Interoperability and Integration. Dagstuhl Seminar Proceedings, vol. 04391, Dagstuhl Research Online Publication Server (2005).

[4] The Information Flow Framework (IFF), [http://suo.ieee.org/IFF/].