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Political Economy of Mechanisms∗

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Abstract

We study the optimal Mirrlees taxation problem in a dynamic economy. In contrast to the standard approach where the taxation mechanism is operated by a benevolent planner with full commitment power, we focus on economies in which policy decisions are made by self-interested politicians, who cannot commit to policies. The society controls politicians using elections. We show that the provision of incentives to politicians can be partly separated from redistribution across agents and that political economy constraints can be modeled as introducing additional aggregate distortions in the dynamic Mirrlees problem. We provide conditions under which the political economy distortions persist or disappear in the long run. If the politicians are as patient as the agents, the “best sustainable mechanism” leads to an asymptotic allocation where the aggregate distortions arising from political economy disappear. In contrast, when politicians are less patient than the citizens, positive aggregate labor and capital taxes remain even asymptotically. We conclude by providing a brief comparison of centralized mechanisms operated by self-interested politicians to anonymous markets.

Keywords: dynamic incentive problems, mechanism design, optimal taxation, political economy, public finance.

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1 Introduction

The major insight of the optimal taxation literature pioneered by Mirrlees (1971) is that the tax structure ought to provide incentives to individuals to work, exert effort and invest, while also providing insurance. This insight is also central to the recent optimal dynamic taxation literature.¹ This literature characterizes the structure of optimal taxes assuming that policies are decided by a benevolent government with full commitment power. In practice, however, tax structures are designed by politicians, who care about reelection, self-enrichment or their own individual biases and cannot commit to future policies or to dynamic mechanisms.²

In this paper we investigate how political economy affects the structure of dynamic taxation and politics. The main challenge in this exercise is to design (equilibrium) political and economic systems that provide incentives both to individuals and to politicians. As a first step in the investigation of the political economy of (tax) mechanisms, we combine the baseline dynamic Mirrlees economy with the canonical model of political agency. In particular, we use the electoral accountability model originally developed by Barro (1973) and Ferejohn (1986), and widely used in the political economy literature (see Persson and Tabellini, 2000, Chapter 4, for a modern exposition and references). In this class of models, politicians decide a range of policies, and citizens can vote them out of office and replace them by other politicians.

In Section 2, we start with an analysis of dynamic taxation in a neoclassical growth model with identical agents and self-interested politicians subject to electoral accountability. The main innovation of this part of the paper is to explicitly model the economic decisions of citizens and the tax decisions of politicians without restricting attention to specific classes of tax policies (such as linear taxes) and to focus on subgame perfect equilibria that maximize citizens’ ex ante utility. In preparation for our main results, we refer to such an equilibrium as the best sustainable mechanism. Our focus on the best sustainable mechanism is motivated by our interest in understanding how the society might best avoid the distortions created by the presence of self-interested politicians and lack of commitment.³ While the previous literature typically focuses on stationary voting rules, we show that the best equilibrium is non-stationary and has qualitatively different implications than stationary equilibria.

¹See, for example, Golosov, Kocherlakota, and Tsyvinski (2003), Werning (2002), Sleet and Yeltekin (2004), Kocherlakota (2005), Albanesi and Sleet (2005), Battaglini and Coate (2005), Farhi and Werning (2006), and Golosov, Tsyvinski and Werning (2007), among others.

²Since in the literature following Mirrlees the optimal tax-transfer program is a solution to a mechanism design problem, we use the terms “optimal tax-transfer program” and “mechanism” interchangeably. The results in this paper can be generalized to other, non-tax, dynamic mechanism design problems.

³Other equilibria will involve more distortions and will not necessarily answer the question of what the best feasible tax structure is in the presence of political economy distortions. We discuss the implications of different equilibrium concepts, including, Markov perfect equilibria, stationary equilibria and renegotiation-proof equilibria below.
What makes the (generalized) electoral accountability model described above particularly interesting is its relationship to our main focus, the dynamic Mirrlees economy with self-interested politicians, which is presented in Section 3. In this economy, a large number of individuals face idiosyncratic risks and their histories of idiosyncratic shocks are private information. The tax structure must again balance individual incentives and insurance, but is now decided by self-interested politicians facing elections.\footnote{Strictly speaking, we are referring to the structure of implicit taxes or “wedges” that represent marginal distortions. Throughout the paper we refer to these as “taxes” to clarify the context and simplify the terminology. It should be borne in mind that there may be more than one tax structure corresponding to the same set of implicit taxes, especially when we refer to aggregate capital distortions.} We show that the provision of incentives to individuals and politicians can be partly separated, so that our dynamic Mirrlees economy, when expressed in terms of aggregate variables, is mathematically equivalent to the generalized electoral accountability model of Section 2. Consequently, similar results apply to our dynamic Mirrlees economy. In particular, we show that when politicians are as patient as the citizens, additional distortions created by political economy disappear in the long run and the allocation of resources converges to that of a dynamic Mirrlees economy with an exogenous level of public good spending. In this limiting equilibrium, there are no additional taxes on labor beyond those implied by the optimal Mirrleesian taxation and no aggregate taxes on capital.\footnote{This result is therefore similar to that of zero limiting taxes on capital in the Ramsey-type models, e.g., Chamley (1986) or Judd (1985), but is derived here without any exogenous restriction on tax instruments (see Kocherlakota, 2005, for the zero capital tax result using the Mirrlees approach).} In contrast, when politicians are (strictly) less patient than the citizens, the structure of taxes never converges to that of a dynamic Mirrlees economy and features additional labor and capital taxes even asymptotically.

These results are useful as a first step towards understanding how incentives can be provided both to individuals and to politicians, and what types of dynamic tax systems might be optimal among those that are politically feasible. We also show that the equilibria that support these tax structures are renegotiation-proof. We then prove that equilibria that impose stationarity in the strategies of citizens never feature convergence to optimal allocations of a dynamic Mirrlees economy and can lead to significant additional distortions. Finally, in Section 4 we present a brief comparison of the efficiency of resource allocation under anonymous markets and centralized mechanisms operated by self-interested politicians.

Our results are related to a number of different literatures. The provision of incentives to politicians in our model is closely connected to dynamic principal-agent analyses (see, among others, Harris and Holmstrom, 1982, Lazear, 1981, Ray, 2002). The most general formulation of dynamic principal-agent problems is provided by Ray (2002). Ray shows that, under fairly weak assumptions, the optimal provision of dynamic incentives induces backloading of payments to
the agent. We show that similar backloading may occur in our economy in the sense that politicians that remain in power for a long time are rewarded more (though we also show examples where this is not true). Conceptually, our focus is very different from Ray, since we analyze political equilibria in a game between citizens and politicians, and we characterize equilibrium distortions and derive the conditions under which various different optimal tax structures are politically feasible. In addition, our technical results extend those in Ray (2002) in two significant directions. First, we characterize the equilibrium in environments where discount factors of citizens and politicians differ. When politicians have a lower discount factor, backloading no longer applies and tax distortions remain even in the long run. Second, we analyze a dynamic economy with capital accumulation. The presence of capital introduces an additional state variable and implies that rewards to politicians are not necessarily backloaded even when they have greater discount factors than the citizens. These two differences are not only of technical interest, but also of substantive importance for our focus. Politicians are often argued to be more short-sighted than the agents and the issue of capital taxation is one of the main questions motivating the recent optimal dynamic taxation literature.

Our paper also builds upon the literature on the political economy of public finance, pioneered by the Public Choice school (e.g., Buchanan, 1968, Buchanan and Tullock, 1962, Brennan and Buchanan, 1980). Recent work in this area is more closely related to our paper. For example, Persson, Roland and Tabellini (2000) analyze equilibrium taxation and public good provision in a model of electoral accountability similar to ours. The main difference between our approach and existing work in this literature such as those surveyed in Persson and Tabellini (2000) is that we neither restrict citizens to stationary electoral policies nor impose exogenous restrictions on tax instruments. Our work is also clearly related to the growing literature on dynamic political economy. Our main contribution relative to this work is again our focus on non-Markovian equilibria, general tax structures and endogenous constraints on policies arising from the incentives of politicians and citizens. This generalized setup enables us to provide a tight characterization of the conditions under which political economy distortions persist or disappear in the long run. In contrast, as we show below, when attention is restricted to stationary strategies, these political economy distortions never disappear. Finally, to the best of our knowledge, no other paper in the literature has considered the political economy

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8In this respect, our model is related to the literature on sustainable government policy, such as Chari and Kehoe (1990, 1993), which focuses on subgame perfect equilibria of policy games without commitment, but do not feature political economy interactions.
of dynamic tax structures or mechanisms.

2 A Model of Electoral Accountability

In this section, we present a version of the well-known Barro-Ferejohn model of electoral accountability embedded in a neoclassical growth economy. We characterize the subgame perfect equilibria that maximize the ex ante utility of citizens.

2.1 Preferences and Technology

We consider an infinite horizon economy in discrete time, populated by a continuum of identical individuals (citizens). Individual preferences at time \( t = 0 \) are given by

\[
\sum_{t=0}^{\infty} \beta^t U(c_t, l_t),
\]

where \( c \) denotes consumption and \( l \) is labor supply. We denote the set of citizens by \( I \) and use \( i \) to denote a generic citizen. We impose the standard conditions on \( U \):

**Assumption 1 (utility)** \( U(c, l) \) is real-valued, twice continuously differentiable, strictly increasing in \( c \), strictly decreasing in \( l \) and jointly concave in \( c \) and \( l \). We adopt the normalization \( U(0, 0) = 0 \). Moreover, \( l \in [0, \bar{L}] \).

The last part of the assumption implies that the maximum labor that an individual can supply is \( \bar{L} \). The normalization \( U(0, 0) = 0 \) is without loss of any generality since \( U \) is real valued.

The production side of the economy is described by the aggregate production function

\[
Y_t = F(K_t, L_t)
\]

where \( K \) denotes capital. Let us denote the derivatives of this function by \( F_K \) and \( F_L \).

**Assumption 2 (production structure)** \( F \) is strictly increasing and continuously differentiable in both of its arguments, exhibits constant returns to scale and satisfies the Inada conditions, \( \lim_{L \to 0} F_L(K, L) = \infty \) for all \( K \geq 0 \) and \( \lim_{K \to \infty} F_K(K, L) < 1 \) for all \( L \in [0, \bar{L}] \). Moreover, capital fully depreciates after use.

The full depreciation assumption is without loss of generality, since we allow (in fact, below we assume) that \( F(0, L) > 0 \) when \( L > 0 \). The condition that \( \lim_{K \to \infty} F_K(K, L) < 1 \), together with the fact that the maximum amount of labor in the economy is bounded, implies that there is a maximum amount of output that can be produced \( \bar{Y} \in (0, \infty) \) uniquely defined by \( \bar{Y} = F(\bar{Y}, \bar{L}) \). The condition that \( \lim_{L \to 0} F_L(K, L) = \infty \), combined with the feature that \( U(c, l) \) is real-valued, implies that in the absence of distortions there will be positive production.
2.2 Political Economy

The allocation of resources in this economy is entrusted to a politician (ruler). This assumption captures the notion that society needs to concentrate the monopoly of violence and the power to tax in a single body. The fundamental political dilemma faced by societies is to ensure that the body to which these powers have been delegated does not use them for its own interests. In the current model, this fundamental dilemma is partly resolved by the control of the politicians via elections.

We assume that there is a large number of potential (and identical) politicians whose utility at time $t$ is given by

$$\sum_{s=0}^{\infty} \delta^s v(x_{t+s}),$$

where $x$ denotes the politician’s consumption (rents), $v : \mathbb{R}_+ \to \mathbb{R}$ is the politician’s instantaneous utility function. Notice also that the politician’s discount factor, $\delta$, is potentially different from that of the citizens, $\beta$. To simplify the analysis, we assume that potential politicians are distinct from the citizens and never engage in production, and that once they are replaced they do not have access to capital markets (see footnote 11).

Assumption 3 (politician utility) $v$ is twice continuously differentiable, concave, and satisfies $v'(x) > 0$ for all $x \in \mathbb{R}_+$ and $v(0) = 0$. Moreover $\delta \in (0, 1)$.

We follow the existing political economy literature and assume that the politician that is in power decides the tax structure and the provision of public goods. However, in contrast to this literature, we do not restrict attention to a specific class of taxes (e.g., linear capital and labor taxes). Given this general structure of taxes, it is equivalent and simpler to think of the politician as directly choosing the allocation of resources in the society. The only restriction we impose is that consumption-labor pairs must be feasible, which we capture with an abstract constraint of the form

$$(c_t, l_t) \in \Lambda \text{ for all } t.$$ (2)

Such a constraint might result, for example, from the requirement that $U(c_t, l_t) \geq 0$ for all $t$. To simplify the analysis we assume that $\Lambda$ is convex. Moreover, throughout we adopt the convention that $c_t \geq 0$, $l_t \in [0, \bar{L}]$ and $U(c_t, l_t) \geq 0$ are also part of the constraint $(c_t, l_t) \in \Lambda$. Therefore, $(c_t, l_t) \in \text{Int}\Lambda$ implies that $c_t > 0$, $l_t < \bar{L}$ and $U(c_t, l_t) > 0$ (where $\text{Int}\Lambda$ denotes the interior of the set $\Lambda$).

We consider the following game form between citizens and politicians. At each time $t$, the economy starts with a stock of capital inherited from the previous period, $K_t$. Then:
1. Individuals make labor supply decisions, denoted by $[l_{i,t}]_{i \in I}$, where $l_{i,t} \geq 0$. Output $F(K_t, L_t)$ is produced, where $L_t = \int_{i \in I} l_{i,t} di$.

2. The politician chooses the consumption function $c_t: \mathbb{R}_+ \rightarrow \mathbb{R}_+$, which assigns a level of consumption for each level of labor supply, and also decides the amount of rents $x_t$. We assume that $x_t$ cannot exceed $\eta F(K_t, L_t)$ for some $\eta \in (0, 1]$. The parameter $\eta$ can, for example, measure the institutional constraints that limit the ability of the politician to appropriate rents. The capital stock left for the next period is determined as

$$K_{t+1} = F(K_t, L_t) - C_t - x_t,$$

where $C_t = \int_{i \in I} c_t(l_{i,t}) di$ is aggregate consumption.

3. Elections are held and citizens jointly decide whether to keep the politician or replace him with a new one, denoted by $\rho_t \in \{0, 1\}$, where $\rho_t = 1$ denotes replacement. Replacement of politicians is without any costs.

The important feature here is that even though individuals make their economic decisions independently, they make their political decisions—elections to replace the politician—jointly. This is natural since there is no conflict of interest among the citizens over the replacement decision. Joint political decisions can be achieved by a variety of procedures, including various voting schemes (see, for example, Persson and Tabellini, 2000, Chapter 4). Here we simplify the discussion by assuming that the decision $\rho_t \in \{0, 1\}$ is taken by a randomly chosen citizen.\textsuperscript{9}

2.3 Equilibrium

We use the notion of subgame perfect equilibrium (SPE). In the text, we focus on pure strategy equilibria. Randomizations are discussed in Appendix A. Let

$$h^t = (K_0, [l_{i,0}]_{i \in I}, c_0, x_0, \rho_0, K_1, ..., [l_{i,t}]_{i \in I}, c_t, x_t, \rho_t, K_{t+1})$$

denote the history of the game up to date $t$, and $H^t$ be the set of all such histories. A SPE is given by labor supply decisions $[l_{i,t}^*]_{i \in I}$ at time $t$ given history $h^{t-1}$, policy decisions $c_t^*, x_t^*, K_{t+1}^*$ by the politician in power given $h^{t-1}$ and $[l_{i,t}]_{i \in I}$, and electoral decisions by the citizens, $\rho_t^*$ at time $t$, given history $h^{t-1}$ and $[l_{i,t}]_{i \in I}$, $c_t^*, x_t^*, K_{t+1}^*$ that are best responses to each other for all histories.

We will focus on SPE that maximizes utility of the citizens. We refer to the resulting equilibrium tax structure as the 	extit{best sustainable mechanism}. The reasoning for this terminology

\textsuperscript{9}Exactly the same equilibrium is obtained if there are majoritarian elections over the replacement decision and each individual votes sincerely (which is the weakly dominant strategy for each citizen in the election).
will become clear below. In preparation for the characterization of the best equilibrium (or the best sustainable mechanism), consider the following constrained optimization problem:

$$\max_{\{C_t, L_t, K_t, x_t\}_{t=0}^\infty} \sum_{t=0}^\infty \beta^t U(C_t, L_t)$$

subject to an initial capital stock $K_0$, the resource constraint

$$C_t + K_{t+1} + x_t \leq F(K_t, L_t) \text{ for all } t,$$

the sustainability constraint for the politician,

$$\sum_{s=0}^\infty \delta^s v(x_{t+s}) \geq v(\eta F(K_t, L_t)) \text{ for all } t,$$

and the feasibility constraint (2).\(^{10}\) We have written this program using capital letters, since the consumption and labor supply levels refer both to individual and aggregate quantities.

Intuitively, the sustainability constraint, (5), requires the equilibrium utility of the current politician, the left-hand side, to be such that he does not want to choose the maximum level of rents this period, $x_t = \eta F(K_t, L_t)$, which would give him utility $v(\eta F(K_t, L_t)).$\(^{11}\) We refer to a sequence $\{C_t, L_t, K_t, x_t\}_{t=0}^\infty$ that is a solution to this problem as a social plan, to emphasize that it is an implicitly-agreed allocation of resources. The sustainability constraint, (5), is sufficient to ensure that the politician does not want to deviate from the social plan. The next result shows that this social plan can be sustained as a SPE.

**Proposition 1** The allocation of resources in the best SPE (best sustainable mechanism) is identical to the solution of the maximization problem in (3) and can be supported with no replacement of the initial politician along the equilibrium path.

**Proof.** Let $\{\tilde{C}_t, \tilde{L}_t, \tilde{K}_t, \tilde{x}_t\}_{t=0}^\infty$ be a solution to (3). We first show that $\{\hat{C}_t, \hat{L}_t, \hat{K}_t, \hat{x}_t\}_{t=0}^\infty$ can be supported as a SPE with no politician replacement along the equilibrium path. Introduce the following notation: $h_t = \hat{h}_t$ if $(K(h^s), x(h^s)) = (\tilde{K}_s, \tilde{x}_s)$ and $c[l_{t,s}](h^s) = \hat{C}_s$ for $l_{t,s} = \tilde{L}_s$ and $c[l_{t,s}](h^s) = 0$ for $l_{t,s} \neq \tilde{L}_s$ for all $s \leq t$. Consider the strategy profile $\rho$ for the citizens such that $\rho(h^t) = 0$ if $h^t = \hat{h}_t$ and $\rho(h^t) = 1$ if $h^t \neq \hat{h}_t$. That is, citizens replace the politician unless the politician has always chosen a strategy inducing the allocation

\(^{10}\)There is also an additional constraint $x_t \leq \eta F(K_t, L_t)$, but this can be ignored without any loss of generality in view of (5).

\(^{11}\)Here we are making use of the assumption that the politician does not have access to capital markets. If he did, he would not consume the entire amount $\eta F(K_t, L_t)$ today, but would invest part of it in the capital market to achieve a smoother consumption profile. When the politician has access to capital markets, deviation from the social plan becomes more attractive and thus (5) becomes more difficult to satisfy, though this does not affect any of our qualitative results.
\[ \left\{ \hat{C}_t, \hat{L}_t, \hat{K}_t, \hat{x}_t \right\}_{t=0}^{\infty} \] in all previous periods. It is a best response for the politician to choose \[ \left\{ \hat{C}_t, \hat{L}_t, \hat{K}_t, \hat{x}_t \right\}_{t=0}^{\infty} \] after history \( h^t \) only if

\[
\mathbb{E} \left[ \sum_{s=0}^{\infty} \delta^s v \left( \hat{x}_{t+s} \left( h^{t+s} \right) \right) \right] \geq \max_{x_t, K_{t+1}, \eta} \mathbb{E} \left[ \left\{ v \left( x_t \right) + \delta v^t \left( K_{t+1}, \eta \right) \right\} \right] | h^t |
\]

where \( v^t \left( K^t_{t+1}, \eta^t \right) \) is the politician’s continuation value following a deviation to \( \left( K^t_{t+1}, \eta^t \right) \). Under the candidate equilibrium strategy, \( v^c = 0 \) following any deviation, thus the best deviation for the politician is \( x_t^c = \eta F(\hat{K}_t, \hat{L}_t) \), which gives \( (5) \). Consequently, \( (5) \) is sufficient for the politician not to deviate from \( \left\{ \hat{C}_t, \hat{L}_t, \hat{K}_t, \hat{x}_t \right\}_{t=0}^{\infty} \). Conversely, if \( (5) \) is violated after any history \( h^t \), \( \left\{ \hat{C}_t, \hat{L}_t, \hat{K}_t, \hat{x}_t \right\}_{t=0}^{\infty} \) cannot be sustained as a SPE. Concavity of \( U \) combined with this observation establishes that no SPE can provide higher utility than \( \left\{ \hat{C}_t, \hat{L}_t, \hat{K}_t, \hat{x}_t \right\}_{t=0}^{\infty} \). To complete the proof, we only need to show that citizens’ strategy (in particular, \( \rho \left( h^t \right) = 1 \) if \( h^t \neq \hat{h}^t \)) is subgame perfect. This follows by considering the following continuation strategy for a politician: if \( h^t \neq \hat{h}^t \), then \( x_t = \eta F(\hat{K}_t, \hat{L}_t) \) and \( c [l] = 0 \) for all \( l \), which is a best response to \( \rho \), while replacement following \( h^t \neq \hat{h}^t \) is a best response for the citizens given this strategy for the politician.

This proposition enables us to focus on the constrained maximization problem given by (3). Moreover, it implies that in the best equilibrium, the initial politician will be kept in power forever. This latter result follows since more effective incentives can be provided to the politician when he has a longer planning horizon (i.e., when he expects to remain in power for longer). Naturally, he is only kept in power along the equilibrium path—if he deviates from the social plan, he will be replaced. It is also important to verify that it is a best response for the citizens to keep the initial politician in power. This can be guaranteed by considering continuation equilibria in which if citizens replace a politician who has not deviated from the social plan, all future politicians will also expect to be replaced immediately and choose \( x = \eta F(K, L) \) and \( c [l] = 0 \) for all \( l \).

### 2.4 The Best Sustainable Mechanism

As a benchmark, let us start with the efficient allocation without political economy constraints, that is we solve problem (3) without the politician sustainability constraint (5). An allocation \( (C_t, L_t, K_t, x_t) \) with \( (C_t, L_t) \in \text{Int} \Lambda \) is undistorted (in the aggregate) if it satisfies

\[
F_L (K_t, L_t) U_C (C_t, L_t) = U_L (C_t, L_t),
\]

\[ (6) \]

\[ \]This continuation equilibrium is subgame perfect, but not renegotiation proof. We will see in Theorem 2 that renegotiation-proof continuation equilibria also support the same behavior.
\[ U_C(C_t, L_t) = \beta F_K (K_{t+1}, L_{t+1}) U_C(C_{t+1}, L_{t+1}), \]  

where \( U_C, U_L, F_K, \) and \( F_L \) denote the partial derivatives of the \( U \) and \( F \) functions.

Standard arguments immediately establish that the efficient allocation will be undistorted in the sense of satisfying (6) and (7) at all \( t \) as long as \((C_t, L_t) \in \text{Int}\Lambda\). We say that an allocation \( \{C_t, L_t, K_t, x_t\}_{t=0}^{\infty} \) features labor distortions at time \( t \) if (6) is not satisfied at \( t \). We refer to these as downward labor distortions if the left-hand side of (6) is strictly greater than the right-hand side. Similarly, if (7) is not satisfied, there are intertemporal distortions at time \( t \), and downward intertemporal distortions if the left-hand side of (7) is strictly less than the right-hand side. Intuitively, downward distortions imply that there is less labor supply and less capital accumulation than in an undistorted allocation.

We next adopt the following sustainability assumption. Both parts of this assumption are used only in part 2 of the next theorem to characterize the equilibrium when the utility provided to a politician reaches the boundary of the set of feasible values, \( \mathbb{W}[K_t] \), at some time \( t \).\(^{13}\) Let us define \( \bar{C} \) and \( \bar{K} \) such that

\[
\bar{C} = \min \{ C : (C, \bar{L}) \in \Lambda \} \quad \text{and} \quad \bar{K} = \arg \max_{K \geq 0} v (F(K, \bar{L}) - K - \bar{C}).
\]

**Assumption 4 (sustainability)**

1. \( \delta v (F(\bar{K}, \bar{L}) - \bar{C} - \bar{K}) / (1 - \delta) > v (\eta F(\bar{K}, \bar{L})) \).
2. \( \bar{C} + \bar{K} \leq F(0, \bar{L}) \).

The first part of Assumption 4 states that there exists a feasible allocation giving sufficient utility to the politician so that the sustainability constraint (5) can be satisfied as a strict inequality. A high discount factor \( \delta \) is sufficient to ensure that this part of the assumption is satisfied. The second part of the assumption is a technical condition, which guarantees that the equilibrium allocation does not get stuck at some arbitrary capital level, and naturally requires that \( F(0, \bar{L}) > 0 \). The proof of Theorem 1 shows that alternative assumptions can be used instead of the second part of Assumption 4, and we will also see that this assumption can be significantly weakened in the case without capital.

**Theorem 1** Suppose that Assumptions 1-4 hold. Then in the best SPE (best sustainable mechanism), the initial politician is never replaced, and the following results hold:

1. there are downward labor distortions at some \( t < \infty \) and downward intertemporal distortions at \( t - 1 \) (provided that \( t \geq 1 \));

\(^{13}\)This set of feasible values is described in greater detail in Appendix A.
2. when $\beta \leq \delta$, the solution to the constrained efficient allocation problem, $\{C_t, K_{t+1}, L_t, x_t\}_{t=0}^{\infty}$ converges to some $(C^*, K^*, L^*, x^*)$. At this allocation, the labor and intertemporal distortions disappear asymptotically, i.e., (6) and (7) hold as $t \to \infty$;

3. when $\beta > \delta$, then there are downward labor and intertemporal distortions, even asymptotically.

Proof. See Appendix B. ■

This theorem is the main result of this section. That the initial politician is never replaced follows from Proposition 1. As long as the initial politician follows the social plan, there is no reason to replace him. Next, part 1 of the theorem illustrates the additional distortion arising from the sustainability constraints. Intuitively, these distortions result from the fact that as output increases, the sustainability constraint (5) implies that more has to be given to the politicians in power, and this increases the effective cost of production. The constrained efficient allocation creates distortions so as to reduce the level of output and thus the amount of rents that have to be paid to the politician. Consequently, the best sustainable mechanism induces aggregate distortions, reducing the levels of aggregate labor and capital, and thus production, below those that would arise without political economy constraints.

The most important results are in parts 2 and 3. Part 2 states that as long as $\beta \leq \delta$, asymptotically the economy converges to an equilibrium where there are no aggregate distortions; even though there will be rents provided to the politician, these will be financed without introducing distortions. This result is important as it implies that in the long run there will be “efficient” provision of rents to politicians, that is, the allocations $(C, L, x, K)$ are undistorted.

Part 3 of the theorem, on the other hand, states that if the politicians are less patient than the citizens, distortions will not disappear. Since in many realistic political economy models politicians are, or act as, more short-sighted than the citizens, this part of the theorem implies that in a number of important cases, political economy considerations will lead to additional distortions that will not disappear even asymptotically. Note also that this result not only implies additional distortions on labor, but also positive aggregate capital taxes, which contrasts with most existing results in the literature on dynamic fiscal policy.

We now provide a heuristic proof of this theorem, with the technical details provided in Appendix B. Let the discounted utility of the politician be denoted by $w$. Following Thomas and Worrall (1990), we formulate the maximization problem (3) recursively with $V(K, w)$ corresponding to the value of a representative citizen when a lifetime utility of $w$ has been promised to the politician in power and the current level of capital stock is $K$. Then:

$$V(K, w) = \max_{C, L, K^+, x, w^+} \{U(C, L) + \beta V(K^+, w^+)\}$$

(9)
subject to

\[ C + x + K^+ \leq F(K, L), \tag{10} \]
\[ w = v(x) + \delta w^+, \tag{11} \]
\[ v(x) + \delta w^+ \geq v(\eta F(K, L)), \tag{12} \]
\[(C, L) \in \Lambda \text{ and } w^+ \in \mathbb{W}[K^+], \tag{13} \]

where we use the notation \( w^+ \) for next period’s \( w \) (and similarly for \( C, L, K \) and the multipliers), and \( \mathbb{W}[K^+] \) denotes the set of feasible values that can be provided to the politician starting with capital stock \( K^+ \). In this formulation (10) is the equivalent of (4), (12) is identical to (5), except that it is written recursively. Equation (11) is the promise-keeping constraint which ensures that \( w \) will indeed be the utility of the politician. This equation also incorporates the fact that the politician will not be replaced along the equilibrium path (cf. Proposition 1).

In Appendix A we prove that, once we allow randomizations between two points at each date, \( V(K, w) \) is differentiable in \((K, w)\) and concave in \( w \). Here, we ignore the issue of randomization and denote the partial derivatives of \( V \) by \( V_w \) and \( V_K \). Assigning multipliers \( \lambda \geq 0, \gamma \) and \( \psi \geq 0 \) to the three constraints above and assuming that we are in the interior of both the sets \( \Lambda \) and \( \mathbb{W} \), we obtain the following first-order conditions:

\[ U_C (C, L) = \lambda, \tag{14} \]
\[ U_L (C, L) = (-\lambda + \psi \eta v' (\eta F(K, L))) F_L (K, L), \tag{15} \]
\[ \beta V_K (K^+, w^+) = \lambda, \tag{16} \]
\[ \beta V_w (K^+, w^+) = -\gamma \delta - \psi \delta. \tag{17} \]

We also have the following two envelope conditions

\[ V_w (K, w) = -\gamma, \tag{18} \]
\[ V_K (K, w) = (\lambda - \psi \eta v' (\eta F(K, L))) F_K (K, L). \tag{19} \]

Now combining (14) and (15) yields:

\[ U_L (C, L) + U_C (C, L) F_L (K, L) = \psi \eta v' (\eta F(K, L)) F_L (K, L). \tag{20} \]

A comparison of this condition with (6) shows that, since \( v' > 0 \) and \( F_L > 0 \), there will be downward labor distortions as long as \( \psi > 0 \), that is, as long as the sustainability constraint (5) is binding. Similarly, combining (16) with (14) and (19), we obtain an expression for the intertemporal distortion:

\[ \beta F_K (K^+, L^+) U_C (C^+, L^+) - U_C (C, L) = \psi^+ \eta v' (\eta F(K^+, L^+) ), \]
which shows that there will be downward intertemporal distortions when $\psi^+ > 0$ (which explains why in Part 1 of Theorem 1 labor distortions at time $t$ are coupled with intertemporal distortions at time $t - 1$). Conversely, when the sustainability constraint (5) is slack, so that $\psi = \psi^+ = 0$, we will have achieved an undistorted allocation.

Next let us focus on the case where the politician has the same discount factor as the citizens, i.e., $\beta = \delta$. Then, combining (17) with the first envelope condition, (19), we obtain:

$$V_w(K^+,w^+) = -\gamma - \psi \leq -\gamma = V_w(w,K). \quad (21)$$

This implies that as long as (5) is binding, i.e., as long as $\psi > 0$, we have $V_w(K,w) > V_w(K^+,w^+)$; otherwise, the two derivatives are equal. Therefore, $\beta \leq \delta$, $\{V_w(K_t,w_t)\}_{t=0}^{\infty}$ is a nonincreasing sequence and necessarily converges (possibly to $-\infty$). We prove in Appendix B that $V_w$ cannot converge to $-\infty$ and also discuss what happens if equilibrium allocations reach the boundary of the set $\mathbb{W}[K]$. Ignoring those issues in the text, it is clear that when $\{V_w(K_t,w_t)\}_{t=0}^{\infty}$ converges to a value $V_w^* > -\infty$, (21) implies that $\{\psi_t\}_{t=0}^{\infty}$ must also converge to 0. But as $\psi_t \to 0$, labor and intertemporal distortions disappear. Loosely speaking, the multiplier on the sustainability constraint asymptotes to zero because we can remove the sustainability constraints in the very far future, without influencing the sequence of utilities promised to the politician at the current date (a further economic intuition for the dynamic behavior of $\{V_w(K_t,w_t)\}_{t=0}^{\infty}$ is provided below after Proposition 2). The same result applies when $\beta < \delta$. This shows that when the politicians are as patient as (or more patients than) the citizens, we will ultimately converge to an undistorted allocation. Notice that an undistorted allocation does not correspond to zero taxes. On the contrary, positive taxes need to be imposed so as to finance the payments to the politician. However, these taxes are raised without any distortions; in other words, when $\beta \leq \delta$ and Assumption 4 holds, in the limit there are no marginal taxes on labor and capital.

Finally, let us consider the case with $\beta > \delta$. Since the politician is now less patient than the citizens, backloading incentives becomes more costly for the citizens. Consequently, (21) no longer holds and even if $\{V_w(K_t,w_t)\}_{t=0}^{\infty}$ converges, $\psi$ remains strictly positive.

Let us next consider the case without capital, so that $F(K,L) = L$. This more specialized economy is useful for a number of reasons. First, it will enable us to illustrate the economic intuition for Theorem 1 more clearly. Second, in this case we can also show that the sequence of utilities for the politician, $\{w_t\}_{t=0}^{\infty}$, is monotonic. Third, the economy without capital will also enable a sharp comparison with the alternative concept of stationary SPE. Finally, in the absence of capital, Assumption 4 can be simplified considerably. In particular, we now impose:

**Assumption 4’ (sustainability without capital)** Let $(\hat{C},\hat{L}) \in \arg\max_{(C,L) \in \Lambda} v(L-C)$. 

12
\[ \frac{v(\bar{L} - \bar{C})}{1 - \delta} > v(\eta \bar{L}). \]

**Proposition 2** Suppose that in Theorem 1, \( F(K, L) = L \) (i.e., there is no capital). Suppose also that Assumptions 1, 3 and 4' hold, \( \beta \leq \delta \), and that \( U_C(0, 0) > U_L(0, 0) \). Then:

1. there are downward labor distortions at \( t = 0 \).
2. \( \{w_t\}_{t=0}^{\infty} \) forms a non-decreasing sequence and the economy again asymptotically converges to a steady state without distortions.

**Proof.** The proof of this proposition is straightforward in view of the proof of Theorem 1. There are three differences from that theorem. First, there are downward distortions at \( t = 0 \) (instead of at some \( t < \infty \)). The reasoning is as follows: since there is no capital, if the sustainability constraint (5) were slack at \( t = 0 \), it would remain so at all future dates, implying that \( x_t = 0 \) for all \( t \), and thus \( w_0 = 0 \). But the assumption that \( U_C(0, 0) > U_L(0, 0) \) implies that in the absence of distortions \( L_0 > 0 \), so that deviating and setting \( x_0 = \eta L_0 > 0 \) would be a profitable deviation for the politician. This yields a contradiction and establishes that (5) binds, and there must be downward labor distortions at \( t = 0 \).

Second, without capital, the result in Theorem 1 implies that the sequence of \( \{V_w(K_t, w_t)\}_{t=0}^{\infty} \) is nonincreasing. This, combined with the concavity of \( V \), implies that \( \{w_t\}_{t=0}^{\infty} \) is nondecreasing.

Finally, it is now sufficient to make sure that the sustainability constraint (5) is slack when \( w_t \) converges to the highest possible level of feasible utility for the politician, i.e., to \( v(\bar{L} - \bar{C})/ (1 - \delta) \) as defined in Assumption 4'. Assumption 4' implies that \( v(\bar{L} - \bar{C})/ (1 - \delta) > v(\eta \bar{L}) \) and ensures this. \( \blacksquare \)

This proposition shows that the result in Theorem 1 that \( \{V_w(K_t, w_t)\}_{t=0}^{\infty} \) is nonincreasing is closely related to the fact that the rewards to the politician, \( \{w_t\}_{t=0}^{\infty} \), is nondecreasing (both in the case where \( \beta \leq \delta \)). Intuitively, current incentives are given by both current payments to the agent (here \( x \) for the politician) and future payments (here in the form of the promised utility \( w^+ \)). Increasing \( w^+ \) relative to \( w \) improves incentives both today and in the future.\(^{14}\) This creates a force towards backloaded incentives for the politician. This backloading result also illustrates the relationship between our findings and Ray’s (2002) result that optimal provision of dynamic incentives involves backloading of payments to the agent. Nevertheless, Theorem 1 (and Proposition 2) is not a special case of Ray’s (2002) results for a number of reasons. First, it treats the case with different discount factors, which is essential for our results regarding the long-run behavior of tax distortions. Second, our model introduces capital as an

\(^{14}\)In fact, it can be verified that if the politician is risk neutral, the solution would involve \( x = 0 \) for a number of periods, then jumping to some level \( \bar{x} \) and remaining there forever.
additional state variable and allows for capital accumulation, which is essential for the analysis of long-run capital taxation. Theorem 1 shows that in the presence of this additional state variable, there may not be backloading even when the discount factor of politicians is greater than that of the citizens (see subsection 3.5 below).\footnote{Two additional differences between our technical results and those of Ray are as follows: (i) Ray makes the opposite of our Assumption 4 or 4’, so that in his analysis, the incentive compatibility constraint of the agent always binds. In contrast, under Assumption 4 or 4’, we show how the relevant constraint becomes slack and the corresponding equilibrium distortions disappear. (ii) As the proof of Theorem 1 illustrates, the equilibrium nature of our problem necessitates an analysis of situations in which allocations converge to the boundary of the sets $\mathbb{W}[K_t]$ and $\Lambda$. This requires a different strategy of proof.}

We end this subsection by considering the best stationary SPE in the environment without capital. With stationary strategies, $x_t$ has to be constant (conditional on the politician remaining in power).\footnote{A similar result can be stated for the economy with capital, but in this case stationary equilibria would allow the payment to the politician, $x_t$, to be a function of $K_t$, which complicates the analysis. The economy without capital allows us to emphasize the importance of focusing on general SPE in a clearer fashion.} As noted above, the previous literature has focused on this type of stationary equilibrium, in particular, assuming that individuals vote “retrospectively” according to some fixed threshold (see, for example, Persson and Tabellini, 2000, Chapter 4). We have the following simple result:

**Proposition 3** Consider the environment without capital in Proposition 2 and suppose that Assumptions 1, 3 and 4’ hold and that $U_C(0,0) > U_L(0,0)$. Then, in the best stationary SPE distortions never disappear.

**Proof.** In a stationary equilibrium without capital, along-the-equilibrium-path allocations must be constant in each period, i.e., $x_t = x$ and $L_t = L$. The sustainability constraint for the politician, (5), then becomes

$$
\frac{v(x)}{1 - \delta} \geq v(\eta L). 
$$

(22)

This constraint must bind in all periods with $\psi > 0$, since otherwise the solution to (3) would involve $x = 0$ and no distortions. The assumption that $U_C(0,0) > U_L(0,0)$ then implies that in this case $L > 0$, thus $x = 0$ would violate (22). The analogue of (20) with $F_L = 1$ then shows that there is a positive distortion on labor in all periods. ■

This proposition illustrates the role of general SPE in our analysis. Stationary equilibria do not allow the optimal provision of dynamic incentives to politicians and imply that political economy distortions never disappear, even when $\beta \leq \delta$.\footnote{An even more restrictive class of equilibria would be Markov Perfect Equilibria, where strategies can only be conditioned on state variables that are payoff relevant in the continuation game. Since in the economy without capital there are no such payoff-relevant variables, it is impossible to provide incentives to politicians and the unique Markov Perfect Equilibrium involves zero production.}
2.5 Renegotiation-Proof Equilibria

The comparison of stationary equilibria to general SPE shows the additional benefits from the latter. Nevertheless, one might be worried that the best SPE we focus on may have some undesirable properties, for example, it may feature highly Pareto inferior behavior in some subgames.\footnote{Another concern might be that the equilibrium supporting the allocation in Theorem 1 involves “complicated” strategies. This is not the case, however, as our equilibrium only requires the voters (or the randomly-chosen political decision maker) to know the “social plan” and observe $x_t$.} If this were the case, one might be concerned that such an equilibrium would not be realistic, because players would renegotiate away from Pareto inferior allocations. Anticipating such renegotiation might then destroy incentives in earlier subgames. The issue of how renegotiation should be handled in dynamic games is not settled, and there are many alternative notions of “renegotiation-proofness” in the literature (see, for example, Fudenberg and Tirole, 1994). Here, we adopt the simplest notion of renegotiation-proofness, which requires that the SPE play after any history $h^t$ should not allow all players to be made weakly better off (and some strictly better off). Given this definition, we have the following result:

**Theorem 2** The best sustainable mechanism in Theorem 1 can be supported as a renegotiation-proof SPE.

**Proof.** For any initial level of capital $K$, let the equilibrium discounted value of the (initial) politician be $w_0(K)$ and let the maximum value that can be promised to the politician be $\bar{w}(K)$ (see the proof of Theorem 1 in Appendix B). Recall the notation from the proof of Proposition 1, whereby $h^t \neq \hat{h}^t$ implies that the politician has deviated from the social plan. Consider the following continuation equilibria. If $\rho(h^t) = 1$ and $h^t \neq \hat{h}^t$, then the continuation equilibrium is a solution to (3), with initial value for the next politician $w' = w_0(K(h^t))$, where $K(h^t)$ is the capital stock after history $h^t$ (that is, after the deviation if there is any). If $\rho(h^t) = 1$ and $h^t = \hat{h}^t$, then the continuation equilibrium is a solution to (3), with initial value for the next politician $w' = \bar{w} = \bar{w}(K(h^t))$. Consequently, if $h^t = \hat{h}^t$, the continuation utility of the citizens is no higher when $\rho(h^t) = 1$, thus it is a best response for them to choose $\rho(h^t) = 0$ if $h^t = \hat{h}^t$. Similarly, if $h^t \neq \hat{h}^t$, it is a best response for the citizens to set $\rho(h^t) = 1$. Since both after $\rho(h^t) = 0$ and $\rho(h^t) = 1$, the continuation play involves the best SPE, it is not possible to make all players better off, establishing that the best sustainable mechanism is supported as a renegotiation-proof SPE.

Intuitively, renegotiation proofness is ensured by using the following type of continuation equilibrium: if citizens replace a politician that has followed the social plan so far, then the next politician receives the maximum feasible utility; if, instead, citizens replace a politician...
that has deviated from the social plan, then the continuation equilibrium gives the highest possible utility to the citizens.

3 Political Economy of Dynamic Optimal Taxation

In this section we show how we can combine an optimal dynamic mechanism design problem with the political economy setup described above. We first describe a general Mirrlees environment with heterogeneous agents and political economy frictions. We then show that we can separate the provision of incentives to politicians from the design of optimal mechanism for individuals, thus reducing the problem of designing sustainable dynamic mechanisms to the problem studied in the previous section.

3.1 Environment

We consider a general dynamic Mirrlees optimal taxation setup in an infinite horizon economy. There is again a continuum of individuals and we denote the set of individuals, which has measure 1, by $I$. The instantaneous utility function of individual $i \in I$ at time $t$ is given by

$$u(c^i_t, l^i_t \mid \theta^i_t)$$

where $c^i_t \geq 0$ is the consumption of this individual, $l^i_t \geq 0$ is labor supply, and $\theta^i_t$ is his “type”. This formulation is general enough to nest both preference shocks and productivity shocks.\(^{19}\)

Let $\Theta = \{\theta_0, \theta_1, ..., \theta_N\}$ be a finite ordered set of potential types, with the convention that $\theta_i$ corresponds to “higher skills” than $\theta_{i-1}$, and in particular, $\theta_0$ is the worst type. Let $\Theta^T$ be the $T$-fold product of $\Theta$, representing the set of sequences of length $T = 1, 2, ..., \infty$, with each element belonging to $\Theta$. We think of each agent’s lifetime type sequence $\theta^\infty$ as drawn from $\Theta^\infty$ according to some measure $\mu^\infty$. Let $\theta^{i, \infty}$ be the draw of individual $i$ from $\Theta^\infty$. The $t$-th element of $\theta^{i, \infty}$, $\theta^i_t$, is the skill level of this individual at time $t$. We use the standard notation $\theta^{i,t}$ to denote the history of this individual’s skill levels up to and including time $t$, and make the standard measurability assumption that the individual only knows $\theta^{i,t}$ at time $t$. No other agent in the economy will directly observe this history. We assume that each individual’s lifetime type sequence is drawn from $\Theta^\infty$ according to the same measure $\mu^\infty$ and independently from the draws of all other individuals, so that there is no aggregate uncertainty in the type distribution.\(^{20}\) In addition, to simplify the notation, we also assume (without loss

\(^{19}\)In particular, productivity shocks would correspond to the case where $u(c^i_t, l^i_t \mid \theta^i_t) = u(c^i_t, l^i_t / \theta^i_t)$.

\(^{20}\)This structure imposes no restriction on the time-series properties of individual skills. Both identical independent draws and arbitrary temporal dependence are allowed. For concreteness, one may wish to think that $\theta^i_t$ follows a Markov process.
of generality) that within each period, there is an aggregate invariant distribution of types denoted by $G$.

We impose the same assumptions on $u (c, l \mid \theta)$ for each $\theta \in \Theta$ that we made on $U (c, l)$ in Assumption 1. We also assume that the utility functions satisfy a natural single crossing property, whereby $u_c (c, l \mid \theta) / u_l (c, l \mid \theta)$ is increasing in $\theta$ for all $c$ and $l$ and for all $\theta \in \Theta$, with $u_c$ and $u_l$ denoting the partial derivatives of $u$. All individuals again have the same discount factor $\beta \in (0, 1)$, thus at time $t$, they maximize

$$E \left[ \sum_{s=0}^{\infty} \beta^s u \left( c_{t+s}^i, l_{t+s}^i \mid \theta_{t+s}^i \right) \right]$$

where $E \left[ \cdot \mid \theta_{t:t} \right]$ denotes the expectations conditional on having observed the history $\theta_{t:t}$.

Policy decisions are made by a self-interested politician, with preferences specified as in Assumption 3 above. There is again a large number of identical politicians, and, at the end of every period, the citizens can replace the current politician with a new one. The main difference from the environment in the previous section is that now the politician has to choose dynamic tax structures (mechanisms) that also provide incentives to individuals. In general, the politician can design complicated mechanisms, where individuals send messages, and allocations depend on all messages. In Acemoglu, Golosov and Tsyvinski (2006), we considered such an environment. While all the results of interest in this paper apply in that environment, here we simplify the exposition and limit ourselves to a more specific game form and thus to more restrictive mechanisms. In particular, let $h_t^i = \{l_{i,0}, l_{i,1}, ..., l_{i,t}\}$ be the history of individual $i$’s labor supplies, with the set of all possible histories for individual $i$ denoted by $H_t^i$. We then adopt the following game form between politicians and citizens. At each time $t$, the economy starts with a stock of capital inherited from the previous period, $K_t$. Then:

1. Individuals make labor supply decisions, denoted by $[l_{i,t}]_{i \in I}$, where $l_{i,t} \geq 0$. Output $F (K_t, L_t)$ is produced, where $L_t = \int_{i \in I} l_{i,t} \, di$.

2. The politician chooses the consumption function $c_t : H_t^i \to \mathbb{R}_+$, which assigns a level of consumption for each complete history of labor supplies of each individual. He also decides the amount of rents $x_t$. We again assume that because of institutional constraints, $x_t$ cannot exceed $\eta F (K_t, L_t)$ for some $\eta \in (0, 1]$. The capital stock left for next period is $K_{t+1} = F (K_t, L_t) - C_t - x_t$, where $C_t = \int_{i \in I} c_t \left( h_t^i \right) \, di$ is aggregate consumption.

3. Elections are held and the citizens decide whether to replace the politician $\rho_t \in \{0, 1\}$.

To define an equilibrium, let public history at time $t$ be $\tilde{h}_t^i = (K_0, c_0, x_0, \rho_0, K_1, ..., c_t, x_t, \rho_t, K_{t+1})$, and denote the entire history of the game by $h_t^i = \left( \tilde{h}_t^i, [h_t^i]_{i \in I} \right)$. The set of entire histories
at date \( t \) is \( H^t \). An equilibrium in this economy is given by labor supply decisions \( \left[ l^i_{t,t} \right]_{i \in I} \) at time \( t \) given entire history \( h^{t-1} \), allocation and policy decisions \( \{ c^*_t, x^*_t, K^*_{t+1} \} \) by the politician in power given public history \( \hat{h}^{t-1} \), and individual histories \( \left[ h^i_t \right]_{i \in I} \), and electoral decisions by the citizens, \( \rho^i_t \) at time \( t \), given public history \( \hat{h}^{t-1} \) and individual histories \( \left[ h^i_t \right]_{i \in I} \), and individual histories

We will again focus on the best equilibrium (best sustainable mechanism) that will maximize the ex ante utility of citizens at time \( t = 0 \).

### 3.2 Restriction to Private Histories

As is well known the behavior of individual allocations in dynamic incentive problems can be very complicated even in the absence of sustainability constraints on politicians (e.g. Green, 1987, Phelan and Townsend, 1991, Atkeson and Lucas, 1992, Phelan, 1994). In order to highlight the effect of political economy interactions, we first simplify the analysis by focusing on environments with *private histories*—that is, we focus on environments where individual histories are not observed by politicians. In terms of the above timing of events, this implies that \( c_t \) can only condition on current labor supply (rather than the entire history of labor supplies). Naturally, individuals can still condition their action on their own entire history. In subsection 3.4 below, we generalize our results to the case where allocations can be conditioned on the entire history.

The restriction to private histories simplifies the analysis considerably. In particular, we can think of the politician (indirectly) choosing a sequence \( \{ c_t (\theta), l_t (\theta) \}_{\theta \in \Theta} \) at each \( t \), which implies that the politician chooses a function \( c_t \) such that if an individual supplies labor \( l_t (\theta) \) for some \( \theta \in \Theta \), then he receives consumption \( c_t (\theta) \). If \( l^i_t \neq l_t (\theta) \) for some \( \theta \in \Theta \), then \( c^i_t = 0 \). With this formulation, the incentive compatibility constraints, which ensure that appropriate incentives are provided to individuals to reveal their types and choose labor supply consistent with the social plan, can be written as

\[
\begin{equation}
\forall \hat{\theta} \in \Theta, \forall \theta \in \Theta, \forall t, \quad u(c_t (\theta), l_t (\theta) | \theta) \geq u(c_t (\hat{\theta}), l_t (\hat{\theta}) | \theta)
\end{equation}
\]

for all \( \hat{\theta} \in \Theta \), for all \( \theta \in \Theta \), and for all \( t \). Since at every date there is an invariant distribution of \( \theta \) denoted by \( G (\theta) \), when the constraints in (24) are satisfied, we can express aggregate labor supply and aggregate consumption as

\[
L_t = \int_{\Theta} l_t (\theta) dG (\theta) \quad \text{and} \quad C_t = \int_{\Theta} c_t (\theta) dG (\theta).
\]

---

21 Here “ex ante” utility refers to expected utility before any individual knows \( \theta_{i,0} \).

22 Moreover, given the single crossing property, (24) can be reduced to a set of incentive compatibility constraints only for neighboring types. Since there are \( N + 1 \) types in \( \Theta \), this implies that (24) is equivalent to \( N \) incentive compatibility constraints between neighboring types.

23 Here \( \int_{\Theta} \) denotes the Lebesgue integral, and in what follows, we will suppress the range of integration, \( \Theta \).
As in the previous section, let us first consider the constrained efficient allocation:

\[ U^{SM} = \max_{\{c_t(\theta^t), l_t(\theta^t), x_t, K_t\}_{t=0}^{\infty}} \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t u\left(c_t\left((\theta^t)\right), l_t\left((\theta^t)\right) \mid \theta_t\right) \right] \tag{25} \]

subject to some initial condition \( K_0 \), the resource constraint

\[ \int c_t(\theta) dG(\theta) + x_t + K_{t+1} = F\left(K_t, \int l_t(\theta) dG(\theta)\right), \tag{26} \]

a set of incentive compatibility constraints for individuals, (24), for all \( t \) and for all \( \theta \in \Theta \), and the sustainability constraint of the politician

\[ \sum_{s=0}^{\infty} \delta^s v(x_{t+s}) \geq v\left(\eta\left(F\left(K_t, \int l_t(\theta) dG(\theta)\right)\right)\right), \tag{27} \]

for all \( t \). The incentive compatibility constraints for individuals in (24) play a similar role to (2) in our formulation above. In particular, we can define

\[ \Lambda = \{ (C, L) \text{ such that } \exists \{c_t(\theta), l_t(\theta)\}_{\theta \in \Theta}_{t=0}^{\infty} \text{ satisfying (24), and} \}
\]

\[ C_t = \int c_t(\theta) dG(\theta), \text{ and } L_t = \int l_t(\theta) dG(\theta)\}. \tag{28} \]

We first have a direct generalization of Proposition 1. Since the proof is identical to that of Proposition 1, it is omitted.

**Proposition 4** The allocation of resources in the best equilibrium (best sustainable mechanism) is identical to the solution of the maximization problem in (25) and can be supported with no replacement of the initial politician along the equilibrium path.

### 3.3 Separation of Private and Public Incentives

Our analysis of the dynamic Mirrlees economy with self-interested politicians is simplified by separating the provision of incentives to individuals from the provision of incentives to politicians. To do this, for each \((C, L) \in \Lambda\) consider the following quasi-Mirrlees program:

\[ U(C, L) = \max_{\{c(\theta), l(\theta)\}_{\theta \in \Theta}} \mathbb{E} \left[ u\left(c_t(\theta), l_t(\theta) \mid \theta\right) \right] \tag{29} \]

subject to the incentive compatibility constraints, (24), and two aggregate constraints

\[ \int c_t(\theta) dG(\theta) \leq C_t, \tag{30} \]

\[ \int l_t(\theta) dG(\theta) \geq L_t, \tag{31} \]

where we refer to the derived function \( U(C, L) \) as the indirect utility.
The maximization problem (29) is the familiar static Mirrlees problem with two additional (aggregate) constraints. The first, (30), specifies that the aggregate amount of consumption is no larger than $C_t$; the second, (31), specifies that the aggregate amount of labor is no smaller than $L_t$. The solution to this problem exists. Moreover, in Appendix A we prove that, once we allow for randomizations, $U(C, L)$ is uniquely defined, continuously differentiable and jointly concave in $C$ and $L$, non-decreasing in $C$ and non-increasing in $L$, which are properties that will be useful below.

It is straightforward to see that because of our focus on “private histories”, the allocations of consumption and labor supply at any date $t$ in any mechanism, $\{c_t(\theta), l_t(\theta)\}_{\theta \in \Theta}$ can only depend on $C_t$ and $L_t$ and are independent of any $C_s$, $L_s$ with $s \neq t$. This implies that the maximization problem (25) is mathematically equivalent to problem (3), with two crucial differences. First, the objective function $\sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$ now refers not to individual utility, but to the indirect utility function derived from the quasi-Mirrlees program above. Second, the abstract set constraint (2) now takes the form given by (28).

This formulation establishes the following theorem, which will allow us to use the results from the previous section to characterize the best sustainable mechanism in the more general economy of this section (proof in the text).

**Theorem 3** The best sustainable mechanism for the dynamic Mirrlees model with private histories and with self-interested politicians solves the maximization problem (3) with $U(C, L)$ corresponding to the indirect utility function derived from maximization problem (29).

Consequently, the allocation induced by the best sustainable mechanism is a solution to a problem that maximizes the ex ante utility of the citizens as given in (29), but must also choose levels of aggregate consumption and labor supply consistent with the sustainability constraint of the politician in power.\textsuperscript{24} An important implication of this result is that political economy considerations do not fundamentally alter the optimal taxation problem; instead, they modify the aggregate constraints in this dynamic maximization problem. From a technical point of view, this theorem implies that we can separate the analysis of the political economy of dynamic taxation into two parts:

1. solve the problem of providing incentives to individuals given aggregate levels of consumption and labor supply;

2. provide incentives to politicians by choosing aggregate variables and the level of rents.

\textsuperscript{24}The key feature necessary for Theorem 3 is that politicians’ deviation payoffs depend only on aggregates. If, instead of $x_t = \eta F(K_t, L_t)$, the maximum consumption for the politician were a nonlinear function of the entire distribution of labor supplies, $[l_{i,t}]_{i \in I}$, Theorem 3 would not necessarily hold.
It is also useful to relate the full dynamic Mirrlees solution to this problem. The full dynamic Mirrlees program with private histories and with a benevolent government that can commit to future policies is a solution to the constraint maximization program (25) when we drop the sustainability constraint, (27). In this full dynamic Mirrlees case, one would interpret the sequence of payments to the politician, \( \{x_t\}_{t=0}^{\infty} \), as an exogenously given level of government spending that the economy needs to finance out of tax revenues. Accordingly, the best sustainable mechanism will be undistorted when it can achieve the same allocation as that of a full dynamic Mirrlees economy with the same sequence of \( \{x_t\}_{t=0}^{\infty} \) (which naturally involves no marginal distortions in addition to those implied by Mirrleesian optimal taxation). It is straightforward to observe that the full dynamic Mirrlees solution can be alternatively obtained by maximizing \( U(C, L) \) derived from (29) subject to the resource constraint, (26), and to the constraint that \( (C_t, L_t) \in \Lambda \) for all \( t \). This implies that the full dynamic Mirrlees program (without political economy) and the best sustainable mechanism considered here solve very similar maximization problems, and any difference between the two programs will arise only when the sustainability constraint on politicians, (27), binds.

### 3.4 The Best Sustainable Mechanism with Private Histories

Now combining Theorems 1 and 2 together with Theorem 3, we obtain our main characterization result for the behavior of distortions in the dynamic Mirrlees taxation problem with self-interested politicians. Notice also that Assumption 4 applies as before, except for the more specific definition of the set \( \Lambda \) in (28).

**Theorem 4** Consider the problem of dynamic optimal taxation with private histories and suppose that Assumptions 1-4 hold. Then the best sustainable mechanism involves no replacement of politicians and has the following features:

1. there are downward labor distortions at some \( t < \infty \) and downward intertemporal distortions at \( t - 1 \) (provided that \( t \geq 1 \));

2. when \( \beta \leq \delta \), the solution to the constrained efficient allocation problem, \( \{C_t, K_{t+1}, L_t, x_t\}_{t=0}^{\infty} \) converges (almost surely) to some \( (C^*, K^*, L^*, x^*) \). At this allocation, the labor and intertemporal distortions disappear asymptotically, i.e., (6) and (7) hold as \( t \to \infty \), and the allocation converges to that of a full dynamic Mirrlees program with an exogenous level of government spending given by \( x^* \);

3. when \( \beta > \delta \), then there are downward labor and intertemporal distortions, even asymptotically.
Moreover, this best sustainable mechanism can be supported as a renegotiation-proof sub-game perfect equilibrium (in the sense of Theorem 2).

Proof. In view of Theorem 3, the proof of this theorem is identical to those of Theorems 1 and 2 and is omitted. ■

This theorem therefore shows that all of the results from Theorem 1 generalize to this environment with dynamic taxes chosen to provide incentives to citizens. The theorem also states that as in the simpler environment of the previous section, incentives can be provided to politicians (and also to citizens) in a renegotiation-proof manner.

The first implication of Theorem 4 is that when politicians are as patient as (or more patient than) citizens, the best sustainable mechanism leads to an allocation that is asymptotically undistorted. The discussion in the previous subsection then implies that this limiting allocation is identical to the solution of a full dynamic Mirrlees program (without political economy and with full commitment), but with an exogenously specified level of government spending, \( x^* \), which is the limiting level of payments to the politician. As we approach this limiting allocation, distortions disappear exactly as in Theorem 1. The meaning of distortions disappearing can be clarified further by considering the marginal tax rate on the highest type of agent (proof omitted).

**Proposition 5** Consider a sequence of \( \{C_t, L_t\}_{t=0}^{\infty} \). Then, the marginal labor tax rate on the highest type \( \theta_N \), at time \( t \) is given by

\[
\tau_{N,t} = 1 + \frac{U_L(C_t, L_t)}{U_C(C_t, L_t) F_L(K_t, L_t) F_L(K_t, L_t)}
\]

and is equal to zero whenever (6) is satisfied.

This proposition implies that when \( \beta \leq \delta \) the asymptotic equilibrium involves no aggregate distortions and zero marginal labor tax on the highest type as in the standard Mirrlees program.

### 3.5 An Example

We now illustrate the results from the previous subsection by computing the best sustainable mechanism for a number of simple economies. For simplicity, we consider an economy with two types, i.e., \( \Theta = \{\theta_0, \theta_1\} \), and individual utility functions given by

\[
u(c, l | \theta) = \sqrt{c - l^2}.
\]

(32)

Suppose that type \( \theta_0 \) is disabled and cannot supply any labor, so \( \theta_0 = 0 \), and we normalize \( \theta_1 = 1 \). Let us also assume that a fraction \( \pi = 1/2 \) of the population is of type \( \theta_1 \) and that the utility function of the government is given by \( v(x) = \sqrt{x} \). We take a baseline case where the government is as patient as the citizens, \( \beta = \delta = 0.9 \). We also set \( \eta = 1 \) throughout.
Figure 1: Time path of normalized promised values to the government, \((1 - \delta) w_t\), with \(\beta = 0.9\) and \(\delta = 0.9, 0.8, 0.7,\) and \(0.6\). Higher curves correspond to higher values of \(\delta\).

We first consider the case without capital, so that the production function is \(F(K, L) = L\). Let us define normalized promised values to the politician by \((1 - \delta) w\). Figure 1 plots the time path of the promised value to the politician for the baseline case, with \(\delta = 0.9\), and also for a range of lower discount factors for the politician, \(\delta = 0.8, 0.7,\) and \(0.6\). The lowest curves is for \(\delta = 0.6\), and then, respectively, for \(\delta = 0.7, 0.8,\) and \(0.9\). Consistent with the results in Proposition 2, when \(\delta = \beta\), \(\{w_t\}_{t=0}^{\infty}\) is an increasing sequence and converges to some level \(w^*\). Interestingly, in these examples the sequence \(\{w_t\}\) is everywhere increasing even when \(\delta > \beta\).

Figure 2 depicts the evolution of the aggregate distortion, \(1 + U_L/U_C\) (which, from Proposition 5, is also equivalent to the marginal tax on type \(\theta_1\)) for the different levels of politician discount factors. The lowest curve shows the case where \(\beta = \delta\), and consistent with part 2 of Theorem 4, the aggregate distortion converges to zero. An interesting feature of the example is that the convergence of \(\{w_t\}_{t=0}^{\infty}\) and of distortions to their steady-state values is rather fast. Consequently, aggregate distortions disappear very rapidly. The figure also shows that, as predicted by part 3 of Theorem 4 and in contrast to the case with \(\beta = \delta\), when \(\delta < \beta\) aggregate distortions do not disappear asymptotically; in fact, the distortions could be quite sizable. For example, when \(\delta = 0.6\), the aggregate distortion converges to an asymptotic value of 0.15 (the highest curve in the graph).

Another question concerns how much of the economy’s output has to be allocated to the politician (as rents or government consumption). Figure 3 answers this question, again for \(\beta = 0.9\) and \(\delta = 0.9, 0.8, 0.7,\) and \(0.6\). When the politician’s discount factor is equal to
Figure 2: Time path of distortions with $\beta = 0.9$ and $\delta = 0.9, 0.8, 0.7, \text{ and } 0.6$. Higher curves correspond to lower values of $\delta$.

that of the citizens, he receives a very small fraction of the output even in the asymptotic equilibrium. As we consider lower discount factors for the politician, his temptation to deviate increases and consequently, he receives a higher fraction of the output. But even with $\delta = 0.6$, this is only 16% of total output. Recall also that all of these computations refer to the case where $\eta = 1$. As we consider lower levels of $\eta$ (i.e., better institutional controls), the share of aggregate output captured by the politician diminishes.

It is also useful to compare the best SPE allocations described above with those that arise under stationary equilibria (as discussed in Proposition 3). Our computations suggest that difference in distortions and welfare losses between economies with and without the restriction to stationary strategies depends on the concavity of the politician’s utility function. Let us consider the family of utility functions $v(x) = x^\alpha$. Our computations above are for the case where $\alpha = 0.5$. In addition, we also computed the corresponding equilibria for intermediate values between $\alpha = 0.5$ and $\alpha = 0.9$. When $\beta = \delta = 0.9$, the aggregate labor distortion in the stationary equilibrium varies from 1 percent to almost 8 percent as we change $\alpha$ from 0.5 to 0.9. The welfare loss from the restriction to stationary strategies, defined as the percentage increase in consumption that can be achieved with the best SPE instead of the stationary equilibrium, also changes with the degree of concavity of the politician’s utility function. For example, when $\alpha = 0.9$ the welfare losses are about 1.5 percent for $\beta = \delta = 0.9$ and about 4 percent for $\beta = \delta = 0.8$. These results show that the welfare losses and distortions resulting from the restriction to stationary equilibria can be quite significant.
We also computed the equilibria of a simple economy with capital, again using the same parameter values as in our benchmark computations, except that the production function now takes the Cobb-Douglas form \( F(K,L) = AK^{\nu}L^{1-\nu} \), with \( \nu = 0.5 \) and \( A = 5 \). We first describe the pattern of distortions starting with an initial capital stock of \( K_0 = 4 \), which is below the steady state level of capital. Since the evolution of labor distortions is very similar to the benchmark case, we only show the evolution of the intertemporal distortions. In Figure 4, we see that intertemporal distortions disappear in the long run when \( \beta = \delta = 0.9 \), but they can be as high as 13 percent when \( \beta = 0.9 \) and \( \delta = 0.6 \).

Finally, let us consider the case in which the initial capital stock is above the steady-state level, for example at \( K_0 = 8 \). In this case, the sustainability constraint of the politician, (5), binds only at \( t = 0 \). Consequently, there are no intertemporal distortions and there are labor distortion for at \( t = 0 \). Figure 5 shows that in this case both the per-period rents received by the politician, \( \{x_t\}_{t=0}^\infty \), and his discounted utility, \( \{w_t\}_{t=0}^\infty \), are decreasing over time. This contrasts with the backloading results in Proposition 2 and in Ray (2002).\(^{25}\) Clearly, the source of this difference is the presence of the additional state variable, the capital stock.

### 3.6 Extension: History-Dependent Mechanisms

We now briefly discuss how our results generalize to the environment without the restriction to private histories (where individual allocations can be conditioned on their entire history

\(^{25}\)In this case, not only \( \{x_t\}_{t=0}^\infty \) but also \( \{w_t\}_{t=0}^\infty \) is decreasing over time.
Figure 4: Time path of capital distortions with $\beta = 0.9$ and $\delta = 0.6, 0.7, 0.8, 0.9$. Higher curves correspond to lower values of $\delta$.

Figure 5: Time path of $\{x_t\}_{t=0}^{\infty}$ (Panel A) and $\{w_t\}_{t=0}^{\infty}$ (Panel B) for high level of initial capital stock.
of past actions or messages). A full analysis of this more general model requires a number of intermediate results. In particular, we first need to extend Theorem 3 to show that the provision of private and public incentives can still be separated. This necessitates additional technical results, which are presented in Acemoglu, Golosov and Tsyvinski (2006).\footnote{In particular, in Acemoglu, Golosov and Tsyvinski (2006), we consider a slightly more general game form in which citizens make reports about their types (or send general messages) rather than directly choosing labor supplies. We show that there is no loss of generality restricting attention to direct mechanisms and we provide a generalized Revelation Principle for this environment. This result is of independent interest, since it shows that a special class of dynamic incentive problems without commitment can be analyzed without abandoning the tools from the full commitment case (and thus contrasts with previous analyses in similar but finite-horizon environments where this could not be done, e.g., Roberts, 1984, Freixas, Guesnerie and Tirole, 1985, or Bisin and Rampini, 2005). We omit a further discussion of this generalized revelation principle here to save space.} Given these results, the broad outlines of the analysis are very similar to that presented above. One major difference is that the resulting indirect utility is no longer a simple function of the current level of aggregate consumption and labor, \((C, L)\), but depends on the entire sequence of aggregate consumption and labor levels. Thus the indirect utility is now a functional of the form 

\[ U(\{C_t, L_t\}_{t=0}^\infty) \]

defined over the infinite sequences \(\{C_t, L_t\}_{t=0}^\infty\). The set of feasible sequences \(\{C_t, L_t\}_{t=0}^\infty\) is now denoted by \(\Lambda^\infty\). We refer to an allocation as interior if \(\{C_t, L_t\}_{t=0}^\infty \in \text{Int} \Lambda^\infty\). Once this indirect utility functional is defined, the proof strategy is similar to that of Theorem 1.

Theorem 5 Consider the optimal dynamic Mirrlees economy with self-interested politicians and suppose that individual allocations can be a function of the entire individual history. Then:

1. there are downward labor distortions at some \(t < \infty\) and downward intertemporal distortions at \(t-1\) (provided that \(t \geq 1\)).

Let the best sustainable mechanism induce a sequence of consumption, labor supply and capital levels \(\{C_t, L_t, K_{t+1}\}_{t=0}^\infty\). Suppose a steady state exists such that \(t \to \infty, \{C_t, L_t, K_{t+1}\}_{t=0}^\infty \to (C^*, L^*, K^*)\), where \((C^*, L^*)\) is interior. Moreover, let \(\varphi = \inf \{ \varphi \in [0,1] : \lim_{t \to \infty} \varphi^{-t}U^*_C = 0 \}\), where \(\varphi < 1\). Then:

2. if \(\varphi = \delta\), then there are no asymptotic aggregate distortions on capital accumulation and labor supply;

3. if \(\varphi > \delta\), then aggregate distortions on capital accumulation and labor supply do not disappear even asymptotically.
Proof. See Appendix C.

This theorem shows that the major results from Theorem 4 continue to hold here. The most important difference is that instead of comparing the discount factor of the politician \( \delta \) to \( \beta \), we now compare it to \( \varphi \), which is the rate at which the ex ante marginal utility of consumption \( U_{C_t}^* \) is declining in the steady state. Clearly, in the case where \( U(\{C_t, L_t\}_{t=0}^\infty) \) is time separable as in Theorem 4, the rate at which \( U_{C_t}^* \) declines is exactly equal to \( \beta \), so that the results in this theorem are essentially identical to those of Theorem 4. In this more general case, \( \varphi \) is the “fundamental discount factor” of the citizens, since it measures how one unit of resources at time \( t \) compares with one unit of resources at time \( t+1 \) (from the viewpoint of \( t = 0 \)). Only in special cases (e.g., without any dynamic incentive linkages) does this fundamental discount factor coincide with \( \beta \). Therefore, the case of \( \varphi = \delta \) indeed corresponds to a situation in which the politician is as patient as the citizens.

The most important results are again contained in parts 2 and 3 of the theorem. Part 2 states that as long as \( U_{C_t}^* \) declines sufficiently rapidly, the multiplier on the sustainability constraint goes to zero. This establishes that the sequence \( \{C_t, L_t, K_t\}_{t=0}^\infty \) is asymptotically undistorted, with no aggregate labor supply and capital accumulation distortions. This generalizes the results from the economy with private histories to the more general environment here. Part 3, on the other hand, states that if the discount factor of the politician \( \delta \) is sufficiently low, then aggregate distortions will not disappear, even asymptotically.

4 Anonymous Markets Versus Governments

We have so far characterized the behavior of the best sustainable mechanism under political economy constraints. Although this was largely motivated by our objective of understanding the form of optimal policy in an environment with both informational problems on the side of agents and selfish behavior on the side of politicians, an additional motivation is to investigate when certain activities should be regulated by governments (operated using centralized mechanisms) and when they should be organized in anonymous markets. In this section, we begin this analysis. Space restrictions preclude a detailed discussion of how anonymous markets should be modeled, so we take the simplest conception of anonymous markets as one in which there is no intervention by the government, and consequently more limited insurance. For the purposes of the exercise in this section, we do not need to assume anything specific about how the anonymous markets work, except that there exists a well-defined anonymous market

\(^{27}\)The main difference between this theorem and Theorem 4 is that the current theorem is stated under the assumption that an interior steady state exists.

\(^{28}\)In part 2 of this theorem, we limit attention to the case in which \( \varphi = \delta \), since when \( \varphi > \delta \) we will not converge to an interior steady state.
equilibrium, which yields ex ante utility $U^{AM}$ to individuals before they know their type (i.e., again “behind the veil of ignorance”). The point to note is that $U^{AM}$ is independent of both the discount factor of politicians and any other institutional controls imposed on politician behavior (since there is no government involvement in the anonymous markets).

Given this, we can provide some simple comparisons between anonymous markets versus sustainable mechanisms. Our first comparative static result states that an increase in the discount factor of the politician, $\delta$, makes mechanisms more attractive relative to markets. Let $U^{SM}(\delta)$ be the ex ante expected value of the best sustainable mechanism as defined by (25) when the politician discount factor is $\delta$.

**Proposition 6** Suppose $U^{SM}(\delta) \geq U^{AM}$, then $U^{SM}(\delta') \geq U^{AM}$ for all $\delta' \geq \delta$. Moreover, suppose that $\eta = 1$. Then, as $\delta \to 0$, $U^{AM} > U^{SM}(\delta)$.

**Proof.** Let $S(\delta)$ be the feasible set of allocation rules when the politician discount factor is equal to $\delta$ (meaning that they are feasible and also satisfy the sustainability constraint (5)). Let $\{c_t(\delta), l_t(\delta), x_t(\delta)\}_{t=0}^{\infty} \in S(\delta)$ represent the best sustainable mechanism, where $c_t(\delta)$ and $l_t(\delta)$ are vectors of consumption and labor supply levels for different types. Since $\delta' \geq \delta$, $\{c_t(\delta), l_t(\delta), x_t(\delta)\}_{t=0}^{\infty} \in S(\delta')$—when the discount factor of the politician is $\delta'$, the left-hand side of (5) is higher, while the right-hand side is unchanged, so $\{c_t(\delta), l_t(\delta), x_t(\delta)\}_{t=0}^{\infty}$ satisfies (5). Therefore, $\{c_t(\delta), l_t(\delta), x_t(\delta)\}_{t=0}^{\infty}$ is feasible and yields expected utility $U^{SM}(\delta)$ when the politician’s discount factor is $\delta'$. This implies that $U^{SM}(\delta')$ is at least as large as $U^{AM}$, therefore $U^{SM}(\delta') \geq U^{SM}(\delta) \geq U^{AM}$.

The second part follows from the observation that with anonymous markets, individuals can always achieve the autarchy allocation, thus $U^{AM} \geq E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c^a(\theta_t), l^a(\theta_t) | \theta_t) \right]$, where $c^a$ and $l^a$ denote the optimal autarchy choices of an agent with type $\theta$. In contrast, with $\eta = 1$ and $\delta \to 0$, the centralized mechanism necessarily leads to a utility of $E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(0, 0 | \theta_t) \right] < E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c^a(\theta_t), l^a(\theta_t) | \theta_t) \right]$.

Because politicians operating centralized mechanisms are self-interested and unable to commit to policy sequences, not all equilibrium allocations without government intervention can be achieved by a mechanism operated by the government. Consequently, anonymous markets can be preferred to sustainable mechanisms. This contrasts with the typical results in the mechanism design literature (with benevolent governments and full commitment) where market allocations can always be achieved by centralized mechanisms.

Perhaps more interesting are the implications of institutional checks and balances on politicians. In our model, these institutional checks are represented by the parameter $\eta$. A lower $\eta$ implies more strict limits on the amount of resources that the politician in power can divert.
for his own consumption.

**Proposition 7** Suppose $U^{SM}(\eta) \geq U^{AM}$, then $U^{SM}(\eta') \geq U^{AM}$ for all $\eta' \leq \eta$. Moreover, as $\eta \to 0$, $U^{SM}(\eta) > U^{AM}$.

The proof of this proposition is similar to that of Proposition 6 and is omitted. It states the intuitive result that better institutional controls on politicians make mechanisms more desirable relative to markets. This proposition also shows that electoral accountability and other institutional controls on politicians are complementary. Institutional checks on politicians combined with electoral accountability would make centralized mechanisms more attractive relative to anonymous markets, which fail to provide the same degree of insurance and risk sharing across individuals.

5 Conclusions

The optimal taxation literature pioneered by Mirrlees (1971) has generated a number of important insights about the optimal tax policy in the presence of insurance-incentive trade-offs. The recent optimal dynamic taxation literature has extended these insights to a macroeconomic setting, focusing on the dynamic behavior of taxes and on issues of optimal capital taxation. A potential criticism against all of this literature is that these optimal tax schemes assume a benevolent government with full commitment power. A natural question is therefore whether the insights of this literature apply to real world situations where politicians care about reelection, self-enrichment or their own individual biases, and cannot commit to sequences of future policies or to taxation mechanisms.

In this paper, we take a first step towards a political-economic analysis of optimal taxation. We focus on the best sustainable equilibrium, i.e., the best equilibrium that satisfies the incentive compatibility constraints of politicians. Political economy considerations introduce additional constraints on the optimal taxation problem, but these constraints are relatively simple. In particular, we show that the provision of incentives to politicians can be separated from the provision of incentives and insurance to agents. Political economy constraints, instead, take the form of additional constraints on aggregate consumption and labor supply in the economy. These constraints then lead to new (political economy) distortions and thus change the structure of taxation. Our analysis provides a characterization of these distortions and their evolution over time. We show that when politicians are as patient as, or more patient than, citizens, aggregate capital and labor distortions disappear in the long run. The politician in power still receives rents, but these rents are provided without additional distortions. This result therefore implies that the insights from Mirrlees’ classical analysis and from the more
recent dynamic taxation literature may generalize to certain environments featuring political economy constraints and commitment problems. However, we also show that when politicians are less patient than the citizens, aggregate distortions remain positive even asymptotically. In this case, in contrast to the classical results in optimal taxation, there will be positive distortions and positive aggregate capital taxes even in the long run.

Throughout the paper, we have focused on subgame perfect equilibria, but we have also shown that all our results hold with renegotiation-proof equilibria. We believe that the focus on subgame perfect equilibria is important, since these enable more effective (electoral) controls on politicians than either stationary or Markovian equilibria that much of the previous literature has focused on. For example, we show that with stationary equilibria, distortions never disappear, which contrasts with the results for subgame perfect equilibria.

Our analysis relies on the infinite horizon nature of the economy and especially on the infinite planning horizon of the politicians. Nevertheless, we believe that similar insights apply even when politicians have finite horizons, and a detailed investigation of this issue would be an interesting area for future research. For example, we conjecture that in a model with either finitely-lived politicians or with term limits, a society consisting of infinitely-lived citizens or overlapping generations of citizens will be able to commit to providing a continuation value (e.g., “pension”) to politicians that have not deviated from the social plan. In this case, even though distortions will not disappear in the long run, they will decline during the tenure of the politician. Such a model would also enable an analysis of the effects of term limits and other realistic institutional constraints on politicians.

The results in this paper can also be extended to an environment with competing parties or interest groups. For example, in Acemoglu, Golosov and Tsyvinski (2007), we consider a model in which political power fluctuates between different parties. We show that distortions decline as a particular party remains in power for longer and they increase when power switches to a new party (see also Dixit, Grossman and Gul, 2000). Another interesting area for future work may be to extend the analysis to different types of government intervention in the economy. For example, an important role that governments play in practice is contract enforcement. However, the power delegated to governments to enforce contracts can be misused in the same way as their taxation powers are potentially misused in this paper. A similar analysis might reveal what types of constraints political economy considerations will place on equilibrium contracting institutions.
6 Appendix A: Properties of the Indirect Utility Function $U(C, L)$

To establish the concavity of $U(C, L)$, we follow Prescott and Townsend (1984a, 1984b) and allow for stochastic mechanisms ("lotteries"). Here we focus on the case with private histories (see Acemoglu, Golosov and Tsyvinski, 2006, for the more general case). Let $z_t$ be a publicly-observed payoff-irrelevant random variable that can be used for randomization. Let $Z^t$ denote the history of the realizations of $z_t$ and $Z^t_t$ the set of all such histories. Then we can restrict attention to mechanisms that specify consumption-labor pairs $M_t \equiv (c_t, l_t) : Z^t \times \Theta \to \Delta (\mathbb{R}_+ \times [0, \bar{l}])$. Recall that $U(C, L)$ is a solution to a finite-dimensional maximization problem. Recall also that only $(C, L) \in \Lambda$ will enable this maximization program to be well defined by making the constraint set non-empty.

Let $C = \{(c, l) \in \mathbb{R}^2 : 0 \leq c \leq \bar{Y}, 0 \leq l \leq \bar{L}\}$ be the set of possible consumption-labor allocations for agents, where $\bar{Y}$ is defined in the text as the maximum level of per capita income. Let $\mathcal{P}$ be the space of $N + 1$-tuples of probability measures on Borel subsets of $C$. Thus each element $\zeta = [\zeta(\theta_0), ..., \zeta(\theta_N)]$ in $\mathcal{P}$ consists of $N + 1$ probability measures for each type $\theta_i \in \Theta$. Let us also denote the fraction of individuals with type $\theta$ at any point in time by $\pi(\theta)$, where clearly $\sum_{i=0}^{N} \pi(\theta_i) = 1$.

Then the quasi-Mirrlees problem can be defined in the following way

\[
U(C(z^t), L(z^t)) \equiv \max_{\zeta(\theta) \in \mathcal{P}} \sum_{\theta \in \Theta} \pi(\theta) \int u(c, l; \theta)\zeta(d(c, l), \theta | z^t)
\]

subject to

\[
\int u(c, l | \theta_i)\zeta(d(c, l), \theta_i | z^t) \geq \int u(c, l | \theta_i)\zeta(d(c, l), \theta_{i-1} | z^t) \quad \text{for all } i = 1, ..., N
\]

\[
\sum_{\theta \in \Theta} \pi(\theta) \int c\zeta(d(c, l), \theta | z^t) \leq C(z^t)
\]

\[
\sum_{\theta \in \Theta} \pi(\theta) \int l\zeta(d(c, l), \theta | z^t) \geq L(z^t)
\]

for $(C(z^t), L(z^t)) \in \Lambda$.

Before deriving properties of the function $U(C, L)$, we need to ensure regularity. Let (34), (35) and (36) define the constraint mapping.

**Lemma 1** The solution to (33) is a regular point of the constraint mapping.

**Proof.** The proof follows from the fact that from single-crossing property, all incentive compatibility constraints in (34) are linearly independent from each other, and also linearly
independent from (35) and (36), thus the constraint mapping has full rank, \( N + 2 \), and is thus onto.

Our main result on the function \( U(C, L) \) is:

**Lemma 2** \( U(C, L) \) is well-defined, continuous and concave on \( \Lambda \), nondecreasing in \( C \) and nonincreasing in \( L \) and differentiable in \( (C, L) \).

**Proof.** First, we show that \( U(C, L) \) is well-defined, i.e., a solution exists. For this, endow the set of probability measures \( \mathcal{P} \) with the weak topology. Since \( C \) is a compact subset of \( \mathbb{R}^2 \), \( \mathcal{P} \) is compact in the weak topology, and the constraint set is compact in the weak topology as well (see Acemoglu, Golosov and Tsyvinski, 2006, for details). Moreover, the objective function is continuous in any \( \zeta \in \mathcal{P} \), thus establishing existence.

Next, to show that \( U(C, L) \) is continuous, note that with the lotteries, the constraint set is convex. From Berge’s Maximum Theorem for arbitrary topological spaces (Aliprantis and Border, 1999, Theorem 16.31, p. 539), \( U(C, L) \) is continuous in \( (C, L) \). Concavity then follows from the convexity of the constraint set and the fact that the objective function is concave in \( \zeta \in \mathcal{P} \). \( U(C, L) \) is also clearly nondecreasing in \( C \), since a higher \( C \) relaxes constraint (35), and is nonincreasing in \( L \), since a higher \( L \) tightens constraint (36).

Finally, differentiability of \( U(C, L) \) follows from Lemma 1, which established that the solution to (33) is a regular point of the constraint mapping, combined with the fact that the objective function in (33) is continuously differentiable everywhere (see Acemoglu, Golosov and Tsyvinski, 2006, for details). This completes the proof of the lemma.

The necessary properties of the set \( \Lambda \) are derived in the next lemma.

**Lemma 3** \( \Lambda \) is compact and convex.

**Proof.** *(Convexity)* Consider \((C^0, L^0), (C^1, L^1) \in \Lambda \) and some \( \zeta^0, \zeta^1 \) feasible for \((C^0, L^0)\) and \((C^1, L^1)\) respectively. For any \( \alpha \in (0, 1) \), \( \zeta^\alpha \equiv \alpha \zeta^0 + (1 - \alpha)\zeta^1 \) is feasible for \((\alpha C^0 + (1 - \alpha)C^1, \alpha L^0 + (1 - \alpha)L^1)\). Thus this set is non-empty. Moreover, since \( \zeta^0, \zeta^1 \) satisfy (34), (35) and (36), \( \zeta^\alpha \) satisfies all three of these constraints, establishing convexity.

*(Compactness)* \( \Lambda \) is clearly bounded, so we only have to show that it is closed. Take a sequence \((C^n, L^n) \in \Lambda \). Since this sequence is in a bounded set, it has a convergent subsequence, \((C^n, L^n) \to (C^\infty, L^\infty)\). We just need to show that \((C^\infty, L^\infty) \in \Lambda \). Let \( \zeta^n \) be a feasible element for \((C^n, L^n)\), and since \( \mathcal{P} \) is compact under the weak topology, \( \zeta^n \to \zeta^\infty \in \mathcal{P} \), which implies that \( \zeta^\infty \) satisfies (34)-(36) and so \( \zeta^\infty \) is feasible for \((C^\infty, L^\infty)\), therefore \( \Lambda \) is closed.

Now define a promised utility for the politician for some sequence \( x = \{x_t\}_{t=0}^\infty \) as \( w = \sum_{t=0}^\infty \delta^t v(x_t) \). Let the set of feasible promised utilities starting with capital stock \( K \), be denoted
by $\mathbb{W}[K]$. Define the maximum utility that can be given to the politician when the capital stock is equal to $K_t$ as

$$\bar{w}(K_t) \equiv \max_{\{C_{t+j}, K_{t+j}, x_{t+j+1}\}_{j=0}^{\infty}} \sum_{j=0}^{\infty} \delta^j v(x_{t+j})$$  \hspace{1cm} (37)

subject to $(C, L) \in \Lambda$, the resource constraint (4) and the sustainability constraint (5).

**Lemma 4** $\mathbb{W}[K] = [0, \bar{w}(K)]$.

**Proof.** Since $v(0) = 0$, it is clear that 0 is the minimal element. By definition $\bar{w}$ is the maximal element. Moreover, clearly any $w \leq \bar{w}(K)$ is also achievable with lotteries, so $\mathbb{W}$ must take the form $[0, \bar{w}(K)]$. ■

**Lemma 5** The solution to the maximization problem (3) starting with the capital stock of $K_0$ is equivalent to the solution to the program (9)-(13) combined with a choice of initial promised value to the politician, $w_0$, such that $w_0 = \arg \max_{w \in \mathbb{W}[K_0]} V(K_0, w)$.

**Proof.** The proof follows from Thomas and Worrall (1990). Clearly any solution to (9)-(13) gives a sustainable mechanism. Moreover, the ex ante utility for the citizens from any sustainable mechanism can be obtained as $V(K_0, w)$ from (9)-(13) by an argument analogous to the principle of optimality. It then follows that $V(K_0, w_0) = \max_{w \in \mathbb{W}[K_0]} V(K_0, w)$ gives the best sustainable mechanism. ■

Next note that the constraint set in the program (9)-(13) is not convex, and randomizations over the current consumption and the continuation value of the politician may further improve the value of the program (which is the reason why we introduced the payoff-irrelevant public histories $z^t$). So analogously to the quasi-Mirrlees problem, we now consider further randomizations. Now let $q = (C, K^+, L, x, w^+) \in \mathbb{R}^5$, $\mathcal{C}(w) = \{ q \in \mathbb{R}^5 : (9)-(13) \text{ are satisfied for given } w \}$, and let $\mathcal{Z}$ be the set of Borel subsets of $\mathcal{C}(w)$. Then let the triple $(\mathcal{C}(w), \mathcal{Z}, \bar{\mu})$ be a probability space. Let $\mathcal{P}(w)$ be the space of probability measures on $\mathcal{C}(w)$ endowed with the weak topology. Incorporating randomization, we can write the recursive formulation as:

**Problem A1**

$$V(K, w) = \max_{\xi \in \mathcal{P}(w)} \int [U(C, L) + \beta V(K^+, w^+)] \xi(dq)$$  \hspace{1cm} (38)

subject to

$$C + x + K^+ \leq F(K, L) \text{ } \xi\text{-almost-surely}$$  \hspace{1cm} (39)

$$v(x) + \delta w^+ \geq v(\eta F(K, L)) \text{ } \xi\text{-almost-surely}$$  \hspace{1cm} (40)
\[
w = \int [v(x) + \delta w^+]\xi(dq)
\]
and
\[(C, L) \in \Lambda \text{ and } w^+ \in \mathbb{W}[K^+] \quad \xi\text{-almost-surely.}
\]

Note that the resulting solution to this program will correspond to stochastic sequences \(\{x_t(z^t)\}_{t=0}^\infty\) and \(\{w_t(z^t)\}_{t=0}^\infty\).

**Lemma 6** \(V(K, w)\) is concave in \(w\).

**Proof.** Consider any \(w_0\) and \(w_1\) and \(\xi_0\) and \(\xi_1\) that are the solution to the maximization problem. Consider \(w = \alpha w_0 + (1 - \alpha)w_1\) for some \(\alpha \in (0, 1)\). Let \(\xi_\alpha = \alpha \xi_0 + (1 - \alpha)\xi_1\). Constraints (39) and (40) hold state by state, and are satisfied for both \(\xi_0\) and \(\xi_1\), and therefore must be satisfied for \(\xi_\alpha\). Constraint (41) is linear in \(\xi\), therefore \(\xi_\alpha\) also satisfies this constraint. Since the objective function is linear in \(\xi_\alpha\), we have \(V(K, \alpha w_0 + (1 - \alpha)w_1) \geq \alpha V(K, w_0) + (1 - \alpha)V(K, w_1)\), establishing the concavity of \(V\).

The above lemma establishes the concavity of \(V\) using arbitrary randomizations in the maximization problem (38). The next lemma shows that a particularly simple form of randomization, using only two points, is sufficient to achieve the maximum of (38).

**Lemma 7** There exists \(\xi \in \mathcal{P}(w)\) achieving the value \(V(K, w)\) with randomization between at most two points, \((C_0, K_0^+, L_0, x_0, w_0^+)\) and \((C_1, K_1^+, L_1, x_1, w_1^+)\) with probabilities \(\xi_0\) and \(1 - \xi_0\).

**Proof.** To achieve convexity, we only need the constraint set to be convex. The constraint set here is \(\mathcal{C}(w) \in \mathbb{R}^5\). From Caratheodory’s Theorem (e.g., Proposition 1.3.1 in Bertsekas, Nedic and Ozdaglar, 2003, pp. 37-38), the convex hull of \(\mathcal{C}(w)\) can be achieved with 6 points (see Acemoglu, Golosov and Tsyvinski, 2006).

Suppose, to obtain a contradiction, that there are more than two points with positive probability. We consider a case of three points (the same argument applies to any finite number of points). Suppose that randomization occurs between \((C_0, K_0^+, L_0, x_0, w_0^+)\), \((C_1, K_1^+, L_1, x_1, w_1^+)\) and \((C_2, K_2^+, L_2, x_2, w_2^+)\) with probabilities \(\xi_0, \xi_1, \xi_2 > 0\). Suppose without loss of generality that \(v(x_0) + \delta w_0^+ \leq v(x_2) + \delta w_2^+ \leq v(x_1) + \delta w_1^+\) and let \(\alpha \in [0, 1]\) be such that \(v(x_2) + \delta w_2^+ = \alpha [v(x_0) + \delta w_0^+] + (1 - \alpha) [v(x_1) + \delta w_1^+]\). Suppose first

\[U(C_2, L_2) + \beta V(K_2^+, w_2^+) > \alpha[U(C_0, L_0) + \beta V(K_0^+, w_0^+)] + (1 - \alpha)[U(C_1, L_1) + \beta V(K_1^+, w_1^+)].\]

Then element \(\hat{\xi} \in \mathcal{P}(w)\) assigning probability \(\xi_2 = 1\) to \((C_2, K_2^+ L_2, x_2, w_2^+)\) is feasible and yields higher utility than the original randomization, yielding a contradiction. Next suppose that

\[U(C_2, L_2) + \beta V(K_2^+, w_2^+) < \alpha[U(C_0, L_0) + \beta V(K_0^+, w_0^+)] + (1 - \alpha)[U(C_1, L_1) + \beta V(K_1^+, w_1^+)].\]
Now consider an alternative \( \hat{\xi} \in P(w) \) assigning probability \( \xi_0 + \alpha \xi_2 \) to \( (C_0, K_0^+, L_0, x_0, w_0^+) \) and probability \( \xi_1 + (1 - \alpha) \xi_2 \) to \( (C_1, K_1^+, L_1, x_1, w_1^+) \), which is again feasible and gives a higher utility than original randomization, once again yielding a contradiction. Therefore, \( \xi \) must satisfy

\[
U(C_2, L_2) + \beta V(K_2^+, w_2^+) = \alpha [U(C_0, L_0) + \beta V(K_0^+, w_0^+)] + (1 - \alpha)[U(C_1, L_1) + \beta V(K_1^+, w_1^+)].
\]

But then the optimum can be achieved by simply randomizing between \( (C_0, K_0^+, L_0, x_0, w_0^+) \) and \( (C_1, K_1^+, L_1, x_1, w_1^+) \) with respective probabilities \( \xi_0 + \alpha \xi_2 \) and \( \xi_1 + (1 - \alpha) \xi_2 \). \( \blacksquare \)

Lemma 7 implies that we can focus on randomizations between two points. We denote the solutions for any \( w \) by \( C_i(w), K_i^+ (w) \), \( L_i(w), x_i(w), w'_i(w), \xi_i(w) \) for \( i \in \{0, 1\} \), and rewrite Problem A1 in equivalent form:

**Problem A2:**

\[
V(K, w) = \max_{\{\xi_i, K_i^+, C_i, L_i, x_i, w_i^+}\}_{i=0,1} \sum_{i=0,1} \xi_i [U(C_i, L_i) + \beta V(K_i^+, w_i^+)] \tag{43}
\]
subject to

\[
\begin{align*}
C_i + x_i + K_i^+ &\leq F(K, L_i) \text{ for } i = 0, 1 \tag{44} \\
v(x_i) + \delta w_i^+ &\geq v(\eta F(K, L_i)) \text{ for } i = 0, 1 \tag{45} \\
w &= \sum_{i=0,1} \xi_i [v(x_i) + \delta w_i^+] \tag{46}
\end{align*}
\]

\( (C_i, L_i) \in \Lambda \) for \( i = 0, 1 \) and \( w' \in W[K_i^+] \). \tag{47}

Finally, since at each date there is randomization between at most two points, the aggregate public history can be taken as \( z' \in \{0, 1\}^t \).

Next we would like to establish that \( V(K, w) \) is differentiable in \( w \). This is made complicated by the presence of the term \( V(K_i^+, w_i^+) \), which may not be differentiable. Instead, we can apply an argument similar to that of Benveniste and Scheinkman (1979) to prove differentiability.

**Lemma 8** \( V(K, w) \) is differentiable in \( w \) and \( K \).

**Proof.** We provide the proof of differentiability in \( w \). The proof for \( K \) is similar. From Lemma 7, in Problem A1 when \( w = w_0 \), the optimal value can be achieved by randomizing between \( (\bar{w}_i^+(w_0), \bar{K}_i^+(w_0)) \) with probabilities \( p_i \) for \( i = 0, 1 \), where \( w_i^+(w_0) \in W[K_i^+(w_0)] \).

Now consider the maximization problem:
Problem A3:

\[ Q(K, w) = \max_{\{\xi_i, C_i, L_i, x_i\}_{i=0,1}} \sum_{i=0,1} \xi_i \left[ U(C_i, L_i) + \beta V(\bar{K}_i^+ (w_0), \bar{w}_i^+ (w_0)) \right] \]

\[ C_i + x_i + \bar{K}_i^+ (w_0) \leq F(K, L_i) \text{ for } i = 0, 1 \quad (48) \]

\[ v(x_i) + \delta \bar{w}_i^+ (w_0) \geq v(\eta F(K, L_i)) \text{ for } i = 0, 1 \quad (49) \]

\[ w = \sum_{i=0,1} \xi_i \left[ v(x_i) + \delta \bar{w}_i^+ (w_0) \right] \quad (50) \]

\[ (C_i, L_i) \in \Lambda \text{ for } i = 0, 1. \quad (51) \]

Note that \( V(K^+, w^+) \), \( w^+ \) and \( K^+ \) are held constant at \( V(\bar{K}_i^+ (w_0), \bar{w}_i^+ (w_0)) \) and \( \bar{K}_i^+ (w_0) \) for \( i = 0, 1 \). By the same argument as in Lemma 6, \( Q(K, w) \) is clearly differentiable—since \( V(\bar{K}_i^+ (w_0), \bar{w}_i^+ (w_0)) \)'s for \( i = 0, 1 \) are just constants here, and all other terms are differentiable. In addition, we have

\[ Q(K, w) \leq V(K, w) \quad (52) \]

and

\[ Q(K, w_0) = V(K, w_0). \quad (53) \]

Now from Lemma 6, \( V(K, w_0) \) is concave in \( w_0 \), and therefore \(-V\) is convex. If \( f \) is convex, there exists a closed, convex and nonempty set \( \partial f \) such that for all \( \nu \in \partial f \) and any \( x \) and \( x' \), we have \( f(x') - f(x) \geq v(x' - x) \) (see Bertsekas, Nedic and Ozdaglar, 2003, Chapter 4). Let \(-\partial V(K, w)\) be the set of subdifferentials of \(-V\), i.e., all \( -\nu \) such that \( -V(K, \hat{w}) + V(K, w) \geq -\nu \cdot (\hat{w} - w) \). By definition, \(-\partial V(K, w)\) is a closed, convex and nonempty set. Consequently, for any subgradient \(-\nu \) of \(-\partial V(w_0)\), we have

\[ \nu \cdot (w - w_0) \geq V(K, w) - V(K, w_0) \geq Q(K, w) - Q(K, w_0) \]

where the first inequality is by the definition of a subgradient, and the second follows from (52) and (53). This implies that \(-\nu\) is also a subgradients of \(-Q(K, w_0)\). But since \( Q(K, w_0) \) is differentiable, \(-\nu\) must be unique, therefore \( V(K, w_0) \) is also differentiable.

7 Appendix B: Proof of Theorem 1

Since \( V \) is differentiable from Lemma 2 and concave from Lemma 6, the first-order conditions are necessary and sufficient for the maximization (43). Assign the multipliers \( \xi_i, \lambda_i \) to the
constraints in (44), \( \xi_i \psi_i \) to those in (45) and \( \gamma \) to constraint (46), and let \( V_w(K, w) \) be the derivative of \( V(K, w) \) with respect to \( w \), we have

\[
\begin{align*}
\beta \xi_0 V_w(K_0^+, w_0^+) + \delta \psi_0 \xi_0 + \delta \gamma \xi_0 & \leq 0 \\
\beta \xi_1 V_w(K_1^+, w_1^+) + \delta \psi_1 \xi_1 + \delta \gamma \xi_1 & \leq 0
\end{align*}
\]

with both equations holding as equality for \( w_i^+ \in \text{Int} \mathbb{W}[K_i^+] \). Therefore,

\[
\frac{\partial}{\partial w} V_w(K_i^+, w_i^+) \leq -\psi_i - \gamma,
\]

again with equality for \( w_i^+ \in \text{Int} \mathbb{W}[K_i^+] \). Moreover, since \( V \) is differentiable, the envelope condition implies that

\[
V_w(K, w) \geq -\gamma
\]

again with equality for \( w \in \text{Int} \mathbb{W}[K_i^+] \).

In addition, combining the first-order conditions for \( C_i, L_i \) and \( K_i^+ \), we have that for \( (C_i, L_i) \in \text{Int} \Lambda \),

\[
F_L(K, L_i) U_C(C_i, L_i) + U_L(C_i, L_i) = \psi_i \eta \eta' \left( \eta F(K, L_i) \right) F_L(K, L_i) \text{ for } i = 0, 1,
\]

and

\[
\beta \sum_{j \in \{0, 1\}} \xi_j^+ \left[ F_K \left( K_i^+, L_j^+ \right) U_C \left( C_j^+, L_j^+ \right) + \psi_j^+ \eta \eta' \left( \eta F \left( K_i^+, L_j^+ \right) \right) \right] = U_C(C_i, L_i) \text{ for } i = 0, 1.
\]

**Part 1:** Suppose, to obtain a contradiction, that (45) is slack for all \( t \) and \( i = 0, 1 \). Then the solution to (43) will involve \( x_{i,t} = 0 \) for all \( t \) and \( i = 0, 1 \), and thus \( w_0 = 0 \). Recall that from Assumptions 1 and 2, without any distortions \( F(K_0, L_0) > 0 \), thus the politician can deviate to \( x_0 = \eta F(K_0, L_0) > 0 \) and increase his utility, yielding a contradiction. Therefore, (45) must bind at some \( t \) and \( i \) with \( \psi_{i,t} > 0 \). Then (56) implies that there will be downward labor distortions at that \( t \), and (57) implies that there will be downward intertemporal distortions at \( t - 1 \).

**Part 2:** Fix some \( w \in \text{Int} \mathbb{W} \). Since \( \beta \leq \delta \) and \( V_w(K, w) \leq 0 \), (54) implies

\[
V_w(K_i^+, w_i^+) \leq -\psi_i - \gamma \text{ for } i = 0, 1.
\]

Combining this with (55) and \( \psi_i \geq 0 \) yields:

\[
V_w(K, w) \geq V_w(K_i^+, w_i^+) \text{ for } i = 0, 1.
\]
This implies that \( \{V_w(K_t, w_t)\}_{t=0}^{\infty} \) is a nonincreasing (stochastic) sequence,\(^{29}\) and necessarily converges on the extended real line. There are three cases to consider.

**Case 1:** \( \{V_w(w_t, K_t)\}_{t=0}^{\infty} \) converges to some \( V_w > -\infty \) and there exists a subsequence of \( \{w_t\}_{t=0}^{\infty} \) converging to some \( w_\infty \in \text{Int}[0, \bar{w}(K_t)] \). This is only possible if \( \{\psi_t\}_{t=0}^{\infty} \to 0 \). Equations (56) and (57) then imply the desired result.

**Case 2:** \( \{w_t\}_{t=0}^{\infty} \) converges to \( w_\infty \in \text{Bd}[0, \bar{w}(K_\infty)] \). We will now show through a series of lemmas that distortions also disappear in this case and then return to Case 3.

Recall that \( \bar{w}(K_t) \) denotes the maximum value that can be given to the politicians starting with capital stock \( K_t \). The next lemma states that if we ever reach the upper boundary of the set \( W[K_t] \equiv [0, \bar{w}(K_t)] \), we will always remain at the upper boundary of future \( W[K_t] \)'s.

**Lemma 9** Let \( \{C^*_{t+j}, K^*_{t+j}, L^*_{t+j}, x^*_{t+j}\}_{t=0}^{\infty} \) be the solution to the problem (37). Then, we have that for any \( t \)
\[
\bar{w}(K^*_{t+j}) = \sum_{s=0}^{\infty} \delta^s v(x^*_{t+j+s}).
\]

Therefore, if \( w_t = \bar{w}(K_t) \) for some \( t' \), then \( w_t = \bar{w}(K_t) \) for all \( t \geq t' \).

**Proof.** Suppose to obtain a contradiction that this is not the case. Then there exists some feasible sequence \( \{C_{t+j}, K_{t+j}, L_{t+j}, x_{t+j}\}_{j=0}^{\infty} \) and \( K_{t+j^*} = K^*_{t+j^*} \) for some \( j^* > 0 \) such that \( \sum_{s=0}^{\infty} \delta^s v(x_{t+j^*+s}) > \sum_{s=0}^{\infty} \delta^s v(x^*_{t+j^*+s}) \). Now form the following new sequence \( \{\tilde{C}_{t+j}, \tilde{K}_{t+j}, \tilde{L}_{t+j}, \tilde{x}_{t+j}\}_{j=0}^{\infty} \) such that
\[
\begin{align*}
(\tilde{C}_{t+j}, \tilde{K}_{t+j}, \tilde{L}_{t+j}, \tilde{x}_{t+j}) &= (C^*_{t+j}, K^*_{t+j}, L^*_{t+j}, x^*_{t+j}) \quad \text{for all } j < j^* \\
(\tilde{C}_{t+j}, \tilde{K}_{t+j}, \tilde{L}_{t+j}, \tilde{x}_{t+j}) &= (C_{t+j}, K_{t+j}, L_{t+j}, x_{t+j}) \quad \text{for all } j \geq j^*.
\end{align*}
\]
This new sequence is feasible in view of the fact that \( K_{t+j^*} = K^*_{t+j^*} \), and it gives value
\[
\bar{w}(K_t) = \sum_{s=0}^{j^*} \delta^s v(x^*_{t+j^*+s}) + \sum_{s=0}^{\infty} \delta^s v(x_{t+j^*+s})
\]
\[
> \sum_{s=0}^{j^*} \delta^s v(x_{t+j^*+s}) + \sum_{s=0}^{\infty} \delta^s v(x^*_{t+j^*+s})
\]
\[
= \bar{w}(K^*_t),
\]
yielding a contradiction and establishing the lemma. \( \blacksquare \)

**Lemma 10** Suppose that Assumption 4 holds and that \( w_{t'} = \bar{w}(K_{t'}) \) for some \( t' \geq 0 \). Then \( w_t > v(\eta F(K_t, L_t)) \) for all \( t \geq t' \).

\(^{29}\)Here “nonincreasing” implies that every realization of \( V_w \) at time \( t \) is no less than its value at \( t - 1 \). Throughout this proof, to reduce notation, we often suppress the stochastic nature of the sequences.
Proof. Suppose that \( w_{t'} = \bar{w} (K_{t'}) \) for some \( t' \). Then, Lemma 9 implies that \( w_t = \bar{w} (K_t) \) for all \( t \geq t' \). Now to obtain a contradiction, suppose that at some \( t \geq t' \) we have \( w_t = v (\eta F (K_t, L_t)) \). Consider two cases.

Case B: \( F (K_t, L_t) \leq F (\bar{K}, \bar{L}) \). By the second part of Assumption 4, a feasible variation is as follows: \( L_{t+s} = \bar{L} \) and \( C_{t+s} = \bar{C} \) for all \( s \geq 0 \), \( K_{t+s} = \bar{K} \) for all \( s \geq 1 \), and

\[
x_{t+s} = F (\bar{K}, \bar{L}) - \bar{C} - \bar{K} \quad \text{for all } s \geq 1.
\]

This variation gives the politician utility

\[
w' = v (F (K_t, \bar{L}) - \bar{C} - \bar{K}) + \frac{\delta}{1 - \delta} v (F (\bar{K}, \bar{L}) - \bar{C} - \bar{K})
\]

\[
\geq \frac{\delta}{1 - \delta} v (F (\bar{K}, \bar{L}) - \bar{C} - \bar{K})
\]

\[
> v (\eta F (\bar{K}, \bar{L}))
\]

\[
\geq v (\eta F (K_t, L_t)),
\]

where the penultimate inequality exploits the first part of Assumption 4 and the last inequality exploits the hypothesis of Case A. The string of inequalities leads to a contradiction.

Case B: \( F (K_t, L_t) > F (\bar{K}, \bar{L}) \) (which naturally implies that \( K_t > \bar{K} \)). Consider the following variation, which is feasible in view of Assumption 4 and the fact that \( K_t > \bar{K} \): \( L_{t+s} = \bar{L} \) and \( C_{t+s} = \bar{C} \) for all \( s \geq 0 \),

\[
x_{t+1} = F (K_t, \bar{L}) - \bar{C} - \bar{K}
\]

\[
x_{t+s} = F (\bar{K}, \bar{L}) - \bar{C} - \bar{K} \quad \text{for all } s > 1.
\]

This variation gives the politician utility

\[
w' = v (F (K_t, \bar{L}) - \bar{C} - \bar{K}) + \frac{\delta}{1 - \delta} v (F (\bar{K}, \bar{L}) - \bar{C} - \bar{K})
\]

\[
> v (F (K_t, \bar{L}) - F (\bar{K}, \bar{L})) + v (\eta F (\bar{K}, \bar{L}))
\]

\[
> v (\eta F (K_t, \bar{L}) - \eta F (\bar{K}, \bar{L})) + v (\eta F (\bar{K}, \bar{L}))
\]

\[
\geq v (\eta F (K_t, \bar{L}))
\]

\[
\geq v (\eta F (K_t, L_t)),
\]

where the first inequality uses the first part of Assumption 4 and the fact that \( F (\bar{K}, \bar{L}) > \bar{C} + \bar{K} \) (again from Assumption 4). The second inequality uses the fact \( \eta \leq 1 \). This third inequality follows from the fact that for a concave function \( f (x) \geq 0, f (x) \leq f (x - y) + f (y) \) for \( y \leq x \), and the final inequality uses \( L_t \leq \bar{L} \). The string of inequalities again leads to a contradiction, establishing that \( w_t > v (\eta F (K_t, L_t)) \) for all \( t \geq t' \). □
The conclusion from this lemma is that even if \( \{w_t\}_{t=0}^{\infty} \) converges to \( w_\infty \in \text{Bd}[0, \bar{w}(K_\infty)] \), the sustainability constraint, (5) will ultimately become slack, so that \( \psi_t \to 0 \) and the desired result follows.

Case 3: \( \{V_w(K_t, w_t)\}_{t=0}^{\infty} \to -\infty \) and there exists a subsequence of \( \{w_t\}_{t=0}^{\infty} \) converging to some \( w_\infty \in \text{Int}[0, \bar{w}(K_\infty)] \). This implies that either \( \{\gamma_t\}_{t=0}^{\infty} \to \infty \) or \( \{\psi_t\}_{t=0}^{\infty} \to \infty \). Then the first-order condition \( v'(x) = \lambda/\gamma + \psi \) implies that either \( x = \infty \) or \( \lambda = \infty \). The former is impossible in view of the resource constraint combined with the observation that \( Y_t \leq \bar{Y} < \infty \) for all \( t \). The latter would imply that \( U_C(C_t, L_t) \to \infty \). Since \( U \) is concave, \( U_C \to \infty \) is only possible when \( C \to 0 \). Since \( (C, L) \in \Lambda \), this implies \( L \to 0 \), and from Assumption 2, \( x \to 0 \) and thus \( w \to 0 \). However, Lemma 10 implies that the best sustainable mechanism cannot involve \( w \to 0 \).

These three cases together imply that either \( \{V_w(w_t, K_t)\}_{t=0}^{\infty} \) converges to some \( V_w > -\infty \) or \( \{w_t\}_{t=0}^{\infty} \) converges to some \( w_\infty \in \text{Bd}[0, \bar{w}(K_\infty)] \). In both cases, this implies \( \{C_t, K_t, L_t\}_{t=0}^{\infty} \to \{C^*, K^*, L^*\} \), which also implies that \( \{x_t\}_{t=0}^{\infty} \to x^* = F(K^*, L^*) - K^* - C^* \), completing the proof of Part 2.

Part 3: Suppose that \( \beta > \delta \). Then, if \( \{V_w(K_t, w_t)\}_{t=0}^{\infty} \) converges to some \( V_w \), then (54) and (55) imply that \( \psi_t \to \psi_\infty > 0 \). Then equations (56) and (57) immediately imply that the asymptotic allocation is distorted downwards. Next, suppose that \( \{V_w(K_t, w_t)\}_{t=0}^{\infty} \) does not converge. Nevertheless, it has a convergent subsequence (which may converge to \( -\infty \), but this is ruled out by the same argument as in the previous part). Suppose to obtain a contradiction that for all such convergent subsequences \( \psi_{i,t} \to 0 \). But this would imply convergence to a steady state since we would have \( \psi_{i,t} = 0 \) for \( i = 0, 1 \) and for all \( t \), yielding a contradiction. Therefore, there must exist a convergent subsequence with \( \psi_i > 0 \), so that \( \limsup [F_L(K, L)] U_C(C_t, L_t) + U_L(C_t, L_t)] > 0 \) and \( \limsup [\beta F_K(K^+, L^+_t)] U_C(C^+_t, L^+_t) - U_C(C, L)] > 0 \). Consequently, distortions do not disappear asymptotically, completing the proof. \( \blacksquare \)

8 Appendix C: Proof of Theorem 5

We provide a sketch of the proof here. Let us write the problem of characterizing the best sustainable mechanism non-recursively as

\[
\max_{\{C_t, L_t, K_t, x_t\}_{t=0}^{\infty}} \mathcal{L} = \mathcal{U}(\{C_t, L_t\}_{t=0}^{\infty}) + \sum_{t=0}^{\infty} \delta^t \left\{ \mu_t v(x_t) - (\mu_t - \mu_{t-1}) v(F(K_t, L_t)) \right\} \tag{58}
\]

subject to

\[
C_t + x_t + K_{t+1} \leq F(K_t, L_t), \quad \text{and} \quad \{C_t, L_t\}_{t=0}^{\infty} \in \Lambda^\infty,
\tag{59}
\]
for all $t$, where $\mu_t = \mu_{t-1} + \psi_t$ with $\mu_{-1} = 0$ and $\delta\psi_t \geq 0$ is the Lagrange multiplier on the constraint (27). The differentiability of $U(\{C_t, L_t\}_{t=0}^\infty)$ (see Acemoglu, Golosov and Tsyvinski, 2006) implies that for $\{C_t, L_t\}_{t=0}^\infty \in \text{Int} \Lambda^\infty$, we have:

$$U_{L_t} - \delta'(\mu_t - \mu_{t-1})v'(K_t, L_t)F_{L_t} = -U_{C_t} \cdot F_{L_t}$$ (60)

$$U_{C_t} = [U_{C_{t+1}} - \delta'(\mu_{t+1} - \mu_t)v'(F(K_{t+1}, L_{t+1}))] F_{K_{t+1}}$$ (61)

Since $\mu_t \geq \mu_{t-1}$, there will be downward labor and intertemporal distortions whenever $\mu_t > \mu_{t-1}$ and $\mu_{t+1} > \mu_t$, i.e., whenever $\psi_t > 0$ and $\psi_{t+1} > 0$.

**Part 1:** The result follows from the same argument as in the proof of Part 1 of Theorem 1.

**Part 2:** The first-order condition with respect to $x_t$ implies:

$$\frac{U_{C_t}}{\delta'v'(x_t)} = \mu_t \leq \mu_{t+1} = \frac{U_{C_{t+1}}}{\delta'v'(x_{t+1})}.$$ (62)

By construction, $\mu_t$ is an increasing sequence, so it must either converge to some value $\mu^*$ or go to infinity. Suppose that $(C_t, L_t, K_t)$ converges to some interior $(C^*, L^*, K^*)$ as hypothesized in the theorem, and that $x_t$ converges to $x^* = F(K^*, L^*) - C^* - K^*$.

Since as $t \to \infty$, an interior steady state $(C^*, L^*, K^*, x^*)$ exists by hypothesis and $U_{C_t}^*$ is proportional to $\varphi^t$, (62) can be written as

$$\frac{\varphi^tU_{C_t}^*}{\delta'v'(x^*)} = \mu_t \leq \mu_{t+1} = \frac{\varphi^{t+1}U_{C_t}^*}{\delta'v'(x^*)} \text{ as } t \to \infty.$$ (63)

Since $\varphi = \delta$, we have that (63) implies that as $t \to \infty$, $|\mu_{t+1} - \mu_t| \to 0$ and $\mu_t \to \mu^* \in (0, \infty)$ (where the fact that $\mu^* > 0$ follows from Part 1, since $\mu_{t+1} \geq \mu_t$ and $\mu_t > 0$ for some $t$). Therefore, $(\mu_t - \mu_{t-1})/\mu_t \to 0$, and since $\mu^* < \infty$, this is only possible if $\mu_t \to \mu^* = 0$, and the conclusion follows.

**Part 3:** Suppose that $\varphi > \delta$. In this case, (62) implies that $U_{C_t}^*$ is proportional to $\varphi > \delta$ as $t \to \infty$. This implies that $(\mu_t - \mu_{t-1})/\mu_t > 0$ as $t \to \infty$, so from (60) and (61), aggregate distortions cannot disappear, completing the proof.

### 9 References


