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TREND INFLATION AND TAYLOR PRINCIPLE: DETERMINACY ANALYSES IN NEW KEYNESIAN MODEL WITH CAPITAL ACCUMULATION

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1. Introduction

Most of the existing New Keynesian general equilibrium literature within Calvo-pricing framework ignores a positive trend in the inflation process. Some papers, e.g. Clarida et al., (2000) assume log-linearization around zero inflation steady state. The main reason for this assumption is the analytical convenience. But it is obvious that equilibrium inflation even in the developed countries is positive. For example, Schmitt-Grohe and Uribe (2007) used the post-war data for the US GDP deflator and estimated the steady state inflation level at about 4.2%. Moreover, nowadays monetary authorities around the world do not consider price stability as zero inflation: most Central Bank’s policy is aimed to inflation level about 2%. And in the long run horizon the steady state inflation converges to Central Bank’s target.

A popular technique to eliminate steady state inflation in the equilibrium solution was proposed by Yun (1996). Yun assumed that firms which do not have an opportunity to re-adjust prices simply index them by the steady state inflation. Christiano et al. (2001) proposed indexation by the previous period’s inflation. But these assumptions seem to be empirically non-realistic: it was shown that not all firms adjust prices each period. Bils and Klenow (2004) produced an empirical study and showed that many prices remain constant during a long period of time. More importantly, from the theoretical point of view the concept of price indexation is not in line with the idea of “sticky” prices: the menu costs of price adjustment are still significant. Also, there is no such a phenomenon in the state-dependent sticky pricing literature.

The most recent literature deals with the problem in two ways. Some papers propose partial indexation based on the idea that firms face not only menu costs but also information costs. As a result, some fraction of them decides to avoid high information costs and to re-adjust prices by the known inflation of the previous period. But Cogley and Sborne (2008) have shown that assumption of stationary trend inflation eliminates the significance of partial indexation in estimated Phillips curve.

There is also emerging literature which deals with positive trend inflation without any indexation. Ascari (2004) derived the New-Keynesian Phillips curve (NKPC) in the presence of low positive trend in the inflation

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We analyze the effect of the presence of the trend inflation on the determinacy of the general equilibrium solution. For this purposes we derive and calibrate a standard New-Keynesian model in continuous time with three main blocks: investment-savings curve, modified New-Keynesian Phillips curve and Taylor-type monetary policy rule. As in the most of the existing literature, monetary authorities stabilize inflation and output gap around long-run targets using nominal interest rate. The main findings of the paper are the following. First, there is a significant set of plausible parameters in which the equilibrium is indeterminate under active policy. Second, the case of indeterminacy may have an empirical relevance: the indeterminate equilibrium can resemble the property of the data quite well. Third, the monetary policy should be not “very” active in order to guarantee the determinacy of the solution.

The rest of the paper is organized as follows. Section 2 describes the theoretical model with capital accumulation process. Section 3 provides analysis of the model. Then, Section 4 concludes.

2. The model

To analyze the consequences of positive trend inflation, we adopt the framework of Blanchard and Kiyotaki (1987), developed in Woodford (2003) and Gali (2008) and add capital accumulation process. The model is the standard version of sticky prices general equilibrium model. The model economy is composed of continuum of infinitely-living consumers-producers of one final and continuum of intermediate goods. We use standard functional forms for preferences and technology and assume perfectly competitive labor market.

2.1 Households

The representative infinitely-lived household seeks to maximize the objective function:

\[ \int_0^\infty \left( \ln C_t + b \ln \frac{M_t}{P_t} - \frac{L^{1+s}_t}{1+s} \right) \exp(-\theta t) dt \]

(1)
where \( C_t \) is the consumption of the single final good, and \( L_t \) denotes hours of work or employment, \( \theta \) is a discount factor and \( s \) is the inverse of the labor supply elasticity. \( M_t \) denotes holding of money and instantaneous utility function is increasing and concave in real balances \( \frac{M_t}{P_t} \). The instantaneous utility function is additively separable in consumption, labor and money.

The budget constraint takes the form
\[
\left( \frac{B_t}{P_t} + \frac{M_t}{P_t} + \dot{K} \right) \leq W_t L_t - C_t + (r + \delta) K_t + \frac{i_t B_t}{P_t} + T_t
\]

where \( i \) is an interest rate, \( B_t \) is a quantity of one-period, nominally riskless discount bonds, \( K_t \) is aggregate capital in the economy, \( W_t \) is real wage, \( r + \delta \) is real user cost of capital and \( T_t \) represents lump-sum additions or subtractions to the income, expressed in real terms. The maximization of (1) subject to (2) leads to the following solution:
\[
W_t = L_t C_t
\]
\[
\frac{\dot{C}_t}{C_t} = i_t - \pi_t - \theta
\]
\[
\frac{\dot{M}_t}{P_t} = C_t i_t^{-1} b
\]
\[
r_t = i_t - \pi_t
\]

where \( \pi_t = \frac{\dot{P}_t}{P_t} \) is the inflation rate.

### 2.2 Firms

The economy produces a single final good and a continuum of intermediate goods indexed by \( j \) where \( j \) is distributed over the unit interval \( (j \in [0,1]) \). The production of final good is perfectly competitive and is subject to the following production function
\[
Y_t = \left[ \int_0^{1} y_{t,j}^{\frac{-1}{\eta}} \, dj \right]^{\frac{1}{\eta}}
\]

where \( Y_t \) is the quantity of the final good produced, \( Y_{t,j} \) is the quantity of intermediate good produced by the firm \( j \) and \( \eta \) stands for elasticity of substitution between different inputs. As it was mentioned above, the final good is consumed by the household.

Perfect competition in a final good’s sector ensures that the demand which faces the firm \( j \) is given by
\[
Y_{t,j} = \left( \frac{P_{t,j}}{P_t} \right)^\eta
\]

\( P_{t,j} \) is a price of intermediate good \( j \) and \( P_t \) is an aggregate price which takes the following form
\[
P_t = \left( \int_0^1 P_{t,j} \, \frac{1}{\eta} \, dj \right)^{\eta-1}
\]

There is monopolistic competition in the markets for intermediate goods: each intermediate good is produced by a single firm subject to the constant return to scale production function
\[
Y_{t,j} = K_{t,j}^{-\alpha} L_{t,j}^{1-\alpha}
\]

where \( L_{t,j} \) is the labor-input and \( K_{t,j} \) is the capital input for the production of the firm \( Y_{t,j} \) and in the equilibrium
\[
L_t = \int_0^1 L_{t,j} \, dj
\]
\[
K_t = \int_0^1 K_{t,j} \, dj
\]

The aggregate capital accumulation process is given by
\[
\dot{K} = Y_t - C_t - \delta K_t
\]

where \( \delta \) is depreciation rate.

As in Calvo (1983), firms are not allowed to change their prices unless they receive a random “price change signal”. It is expressed by exogenous Poisson process with arrival rate \( \psi \) and expected time between price changes \( \frac{1}{\psi} \). At the moment of realization \( t_0 \) the probability that the firm will
2.3 Policy

Fiscal policy is conducted with lump-sum taxes and subsidies so that \( B_t = 0 \). Monetary policy is conducted using simple Taylor-type interest rule. Namely, monetary authorities target inflation and output to their steady state levels.

\[
i_t - i^* = (1 + a_i)(\pi_t - \pi^*) + a_t(\ln Y_t - \ln Y^*)
\]

where \( i^* \), \( \pi^* \), \( Y^* \) are non-stochastic steady states levels of interest rate, inflation and output.

2.4 Non-stochastic steady state

Equilibrium is given by 3–13 and 15–20. Non-stochastic steady-state is derived from the assumption that \( \dot{C} = 0 \) and \( \dot{\pi} = \pi^* \). The steady states levels of variables \( i^* \), \( MC^* \), \( Y = \frac{Y}{K} \) and \( X = \frac{X}{P} \) are given by

\[
i^* = \theta + \pi^*
\]

\[
\left( \frac{X}{P} \right)^* = \left( \frac{\psi}{\psi - \pi(\eta - 1)} \right)^{\frac{i}{1-i}}
\]

\[
(MC)^* = \left( \frac{\eta - 1}{\eta} \right)^{\frac{\theta + \psi - \pi \eta}{\theta + \psi - \pi(\eta - 1)}} \left( \frac{X}{P} \right)^*
\]

\[
\left( \frac{Y}{K} \right)^* = \frac{\theta + \delta}{\alpha} \left( \frac{1}{MC} \right)^*
\]

\[
\left( C \right)^* = 1 - \frac{\delta}{\frac{1}{\alpha} \left( \frac{C}{Y} \right)^*}
\]

As shown in Ascari and Ropele (2007) the presence of steady-state inflation lowers \( Y^* \).
2.5 Log-linearization

Standard log-linearization procedure of equations 3, 4, 6 and 15–21 around non-stochastic steady-state, described in the previous section leads to the following system:

\[ \dot{x}_i = (\psi + \theta - \eta \pi^*) (xp_i - mc_i) + \pi^* (1 - \eta)z_i \] (26)

\[ \dot{\rho}_i = \psi + (1 - \eta) \pi^* (xp_i) \] (27)

\[ \dot{c}_i = i_i - \pi_i \] (28)

\[ i_i = (1 + a_i) \pi_i + a_2 y_i \] (29)

\[ \dot{k} = \left( \frac{Y}{K} \right) y_i - \left( \frac{C}{K} \right) c_i - \delta k_i \] (30)

\[ r_i = i_i - \pi_i \] (31)

\[ y_i - l_i = w_i - mc_i \] (32)

\[ y_i - k_i = \frac{r_i}{\theta + \delta} - mc_i \] (33)

\[ mc_i = \alpha r_i + (1 - \alpha) w_i \] (34)

\[ w_i = c_i + s_i \] (35)

\[ \dot{z}_i = -\pi_i + (\theta + \psi - \pi^* (\eta - 1)) z_i + \frac{\dot{c}_i}{\eta - 1} \] (36),

where \( z_i = (\theta + \psi - \pi^* (\eta - 1)) \int_0^\infty \exp\left(\theta + \psi - \pi^* (\eta - 1)\right) \left( p_i - p_i - \frac{c_i - c_i}{\eta - 1} \right) d\tau \)

and the lower case letters denote logs of initial values in deviations from their steady-state levels except interest rates and inflation which are deviations from steady-state in levels. For nominal variables (\( x \) and \( p \)) small case stands for log deviations from balanced-growth path levels deflated by steady state inflation level. For example, \( x_i = \log[X_i \exp(-\pi^* t)] \).

New-Keynesian Phillips curve is derived from 26–27 and takes the following form, analogously to Ascarry (2004) in discrete time:

\[ \pi_i = (\theta - \pi^*) \pi_i - \psi mc_i - \pi^* \pi (\eta - 1)z_i \] (37)

\[ \phi^* = \psi - \pi^* (\eta - 1) \quad \phi = \phi^* (\phi + \theta - \pi^*) \]

As in Ascarry (2004) today’s inflation depends not only on expectations of its change today which is the case of convenient Philips curve but on expectations of all path of future inflation.

The equilibrium solution of (27–37) results in the following system:

\[ AX = BY \]

where \( Y_i = \begin{bmatrix} k \\ c \\ \pi \\ r \\ i \\ mc \\ y \end{bmatrix} \)

\( A = \begin{bmatrix} -s & 1 & 0 & 0 & 0 & 0 \\ -(1 - \alpha) & 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -a_2 \end{bmatrix} \)

\( B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 + a_1 & 0 \end{bmatrix} \)

Given equations \( \dot{Y} = CY + DX = JY \) (38) where

\[ C = \begin{bmatrix} -\delta & -\left( \frac{C}{K} \right)^* & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & \theta - \pi^* & -\pi^* (\eta - 1) \phi^* \\ 0 & 0 & \frac{a_i}{\eta - 1} & -1 & \phi^* + \theta \end{bmatrix} \]
The solution of the system (38) even if can be derived analytically is too complicated to provide intuition. That is why the solution of the model was obtained using numerical methods with the standard calibration of parameters.

3.2 Calibration of basic parameters

Models parameters are calibrated according to values accepted as plausible in the business cycles literature: $\alpha$, the Cobb-Douglas parameter of production function, is fixed at 1/3; $\delta$, depreciation of capital rate, is set to 0.08 per annum and $\theta$, consumer’s discount factor, is assumed to be 0.02 per annum. For the analytical simplicity, consumer’s instantaneous utility function is linear with respect to labor: in this case $s$ equals 0 which means that labor supply is infinitely elastic as for example in Hansen’s indivisible labour model (1985).

According to the recent empirical findings by Bils and Klenow (2004) and Nakamura and Steinsson (2008) we set the Poisson parameter of Calvo scheme ($\psi$) to correspond to opportunity for firms to change their prices every 6 and 12 month. The same logic was used in Coibon and Gorodnichenko (2008). The shorter time period was proposed in the first paper and longer time horizon corresponds with recent paper by Nakamura and Steinsson. We also adopt two different values of firm’s steady-state markups: 10% and 20%. This means that corresponding values of $\eta$ equals to 11 and 6. To illustrate how the increase in steady state inflation influences the determinacy of equilibrium we analyze the steady state inflation level of 2%, 4% and finally 7%.

Finally, the solution of the system (38) was obtained according to the calibration of basic parameters presented in Table 1.

Table 1. Calibration parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>Steady-state inflation rate</td>
<td>2%; 4%; 7%</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Inverse of average time of price fixity</td>
<td>1; 2</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Elasticity of substitution among intermediate goods</td>
<td>11; 6</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share in output</td>
<td>1/3</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.08</td>
</tr>
<tr>
<td>$s$</td>
<td>Inverse of labor supply elasticity</td>
<td>0</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Discount factor</td>
<td>0.02</td>
</tr>
</tbody>
</table>
3.3 Analysis of indeterminacy regions and policy implications

The results of solution are presented on Figures 1–4. The axes are $a_1$ and $a_2$ values — the monetary policy rule coefficients in equation (20). The shaded area indicates the region with one eigenvalue with negative real part. This is the region of determinant equilibrium. The white area is the region with two eigenvalues with negative real part. It is the indeterminacy region. It should be stressed that the indeterminacy is what is called “real indeterminacy”, i.e. not only is the dynamics of nominal variable such as inflation indeterminate but also the dynamics of real variables such as output or consumption.

To compare and illustrate how our results correspond to the main finding by Duport (2001) and Carlstrom and Fuerts (2005) first of all we analyze the case of zero-trend inflation. The first graph on Figures 1—4 represents eigenvalues regions for this case and we are interested in colored zone — determinacy. One can see that monetary policy which responds only to inflation variation leads to indeterminate equilibrium. To guarantee the uniqueness of equilibrium monetary authorities should also respond to variations in output. This result perfectly corresponds with one presented in papers stated above.

From Figures 1–4 one can see that monetary policy parameter’s regions which guarantee the determinacy of equilibrium become smaller with an increase of steady state inflation. Higher firm’s mark-ups lead to broader range of possibilities for monetary authorities to respond to variation in output and inflation which leads to a unique equilibrium.

The result, that active monetary policy rule in response to inflation does not provide a determinant equilibrium, obtained for zero trend inflation holds. In absence of the policy reaction to output fluctuations the economy is guaranteed to be in sunspot equilibrium. According to results presented on Figures 1–4, if trend inflation is greater than discount rate monetary authorities should react not too active to changes in output. The idea that monetary authorities still should react to output gap contradicts findings of Coibon and Gorodnichenko (2008), but they did not include capital accumulation process in the theoretical model. Also, our result about the reaction to output extends the analysis of Carlstrom and Fuerts (2005) who argued that the equilibrium is determinant once the reaction to output parameter is greater than some value (as graphs for zero inflation case clearly show). We argue that if the reaction is parameter is too large then either equilibrium becomes indeterminate again, so the policy should not over-react to fluctuations of output.

For most parameter calibrations stated above the “classical” Taylor rule $(a_1 = 0.5, a_2 = 0.5)$ for trend inflation greater than 2% does not lead to a unique equilibrium. This means that the policy based on Taylor principle does not necessarily lead to determinant equilibrium. And as it was stated above good policy should not lead to indeterminacy of equilibrium. This finding extends to all the results from previous literature: Taylor principle breaks down in presence of non-zero trend inflation.
Fig. 1. Mark-up 10%, firms change prices every 6 month

Fig. 2. Mark-up 20%, firms change prices every 6 month
Fig. 3. Mark-up 10%, firms change prices every 12 month
and the process for shock is

\[ \dot{a}_t = -\rho a_t, \]

where \( a = \ln A. \)

The monetary policy shock is introduced into the policy rule (19) which is changed to

\[ i_t - \pi^* = (1 + a_t) (\pi_t - \pi^*) + a_t (\ln Y_t - \ln Y^*) + m_t, \]

and the process for shock is

\[ \dot{m}_t = -\rho m_t. \]

The third non-fundamental shock can be chosen arbitrarily and constructed for example as an inflation forecast error shock and may be any i.i.d. random variable. This shock may be independent of fundamental shocks or at the other extreme may simply be a linear compilation of them in which case there is no non-fundamental uncertainty. It is worth mentioning that the equilibrium is still indeterminate even in this case.

The graphs below present impulse response of output to two fundamental shocks for the equilibrium in which inflation forecast error is equal to 0.3 and the following parameterization: \( \pi = 4\%; \eta = 1\%; \psi = 1. \)

We set autoregression parameter for both shock to 0.35 which corresponds to half-life of a shock equal to two years.

Indeterminacy leads to different equilibrium paths for the same fundamentals. So in the indeterminate case there may exist equilibrium which is closer to empirical results that in determinacy case.

In order to demonstrate our point we expand the model by allowing for 2 types of fundamental uncertainty: technology and monetary policy shocks. Technology shock is introduced into production function (9) which is changed to

\[ Y_{t,j} = K_{t,j}^\alpha (A L_{t,j})^{1-\alpha} \]

(9')

and the process for shock is

\[ \dot{a}_t = -\rho a_t, \]

where \( a = \ln A. \)

The monetary policy shock is introduced into the policy rule (19) which is changed to

\[ i_t - \pi^* = (1 + a_t) (\pi_t - \pi^*) + a_t (\ln Y_t - \ln Y^*) + m_t, \]

and the process for shock is

\[ \dot{m}_t = -\rho m_t. \]

The third non-fundamental shock can be chosen arbitrarily and constructed for example as an inflation forecast error shock and may be any i.i.d. random variable. This shock may be independent of fundamental shocks or at the other extreme may simply be a linear compilation of them in which case there is no non-fundamental uncertainty. It is worth mentioning that the equilibrium is still indeterminate even in this case.

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We set autoregression parameter for both shock to 0.35 which corresponds to half-life of a shock equal to two years.

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**Fig. 4.** Mark-up 20%, firms change prices every 12 month
Monetary policy rule’s parameters which lead to determinacy depend on the level of trend inflation (inflation in the steady state). Higher levels of trend inflation reduce the space of parameters value suitable for the unique solution of the model. We have also shown that a “classical” Taylor principle leads to indeterminate equilibrium for a wide range of plausible values of model’s parameters.

Finally, we add some fundamental disturbances to the model and find out that the reaction of output to these shocks looks alike the empirical results of most of the literature. But to justify the empirical relevance the country’s case study will be helpful. So, the next step will be devoted to the comparison of models simulation results with the empirical result for the particular country with inflation target of Central Bank greater than 2%.

It would be also very interesting to find out how the model’s results change if monetary authorities conduct policy using interest rate smoothing in their rule. It is not obvious that the model will provide the same results as Coibon and Gorodnichenko (2008): interest rate smoothing increases sufficiently the probability of determinate equilibria. And it is also interesting to show to what measure of output should react monetary authorities. In other words, what variable (output gap or output growth or maybe even consumption) in monetary policy rule provide policymakers with wider range of possibilities of reaction which guarantee the uniqueness of equilibrium.

4. Conclusions

This paper investigates the influence of positive trend inflation on the equilibrium determinacy in the typical New-Keynesian model with capital accumulation process. In this model the monetary policy is conducted using simple interest rates rule. The recent findings in literature that “active” monetary policy rule does not guarantee the uniqueness of the equilibrium correspond to our results. We show that presence of capital accumulation affects the indeterminacy region. In short, the result of Carlstrom and Fuerts (2005) that determinacy is restored under policy which is active enough in response to output fluctuations does not hold. We show that if the reaction is larger than some value of this parameter the equilibrium is also indeterminate. In other words, the response to output should be active; but not too active.

Figure 5. Impulse response to fundamental shocks

Those impulse response functions look qualitatively alike the impulse response function often found in empirical studies. Of course, this can only be considered as a hint on the empirical relevance of the model. The thorough answer requires careful examination of the data from some country and calibration and simulation of the model based on this data. The final version of this paper will include this analysis as well.
References


В работе анализируется влияние ненулевой трендовой инфляции в неокейнсианской модели с ценообразованием по Кальво и накоплением капитала. Мы опираемся на работы Дюпорта (2001) и Аскари, Рупела (2007), которые рассматривали эти эффекты по отдельности. Мы показываем, что одновременное присутствие положительной инфляции и накопления капитала значительно изменяет свойство единственности равновесия: для его поддержания денежные власти должны не только активно реагировать на инфляционные шоки, но и значительно менять процентную ставку при колебаниях выпуска. С другой стороны, слишком большая реакция на выпуск также может привести к неединственности равновесия. Мы также показываем, что для большого набора разумных параметризаций стандартное правило Тейлора влечет множество равновесий. Мы также рассматриваем ряд альтернативных правил денежной политики.