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SIGNALING BANK QUALITY: AN EVOLUTIONARY APPROACH TO MONEY MARKET IN TRANSITION

Препринт WP9/2005/01
Серия WP9
Исследования по экономике и финансам
Research of economics and finance

The paper studies the evolution of markets for private deposits under asymmetric information from game-theoretic perspective. We study an evolutionary signaling game between large populations of individual investors and banks of two types who may invest in either risky or safe projects, and attract debt holders by sending either high or low signals. Both populations are assumed boundedly rational, and the adaptive evolution of their strategy is explicitly derived from several intuitive assumptions. Our analysis suggests that a systems of competitive banking under asymmetric information and without prudent regulation is inherently prone to banking crises caused by the competitive pressure on the banks to increase their profits by engaging in more risky projects. This situation, in particular, prevents efficient banks from attracting more debt holders and crediting better projects that require higher credit resources. Several conclusions for policy analysis are suggested, alongside with the properties of the ensuing equilibria.

JEL codes: C73, D82, D83, G2, P21.

Keywords: banking crises, transition, evolutionary games.

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1. The banking system in transition

Transformation of private savings into private investment is commonly understood as one of the main goals of and rationale for the existence of a banking system. However, fulfillment of this function is not straightforward: in practice, the banking system may fail to accumulate a substantial proportion of the nation’s saving, and/or supply its credit resources to the appropriate borrowers. Further, many banking systems, especially in emerging economies, appear to be intrinsically volatile and vulnerable to the various shocks. Periods of deposit and credit expansions are occasionally interrupted by banking crises and panics, which pour the banking systems to the states of increased insecurity and inefficiency (Koopman e.a., 2003).

Why some banking systems are stable and basically efficient, while some others appear to be volatile and unsettled over the long run? This fact might be attributed to a wide variety of reasons, including poor quality of bank management, unstable macroeconomic situation, inappropriate regulation of banks, or peculiar features of institutional setup in particular countries. Our paper investigates the reasons for such long-term inefficiency, and explains it in terms of multiplicity of money market equilibria and evolutionary fluctuations between them.

This approach to the problem is motivated by the recent experience of banking sector development in transition economies of Central and East European countries. In the largest of these countries, Russia, development of the banking sector emerged at first as one of a few success stories of transition. Prior to 1990, private banks did not exist in this country, yet by 1995 there were as many as 2,439 of them. At the time, the banking sector has been the leading Russian industry in terms of both operating profits and rates of mastering the new market environment. Nevertheless from the very beginning this system also suffered from several substantial drawbacks, including inefficient credit rationing, lack of prudence in risk management and poor quality of service offered to customers. Not surprisingly, the initial boom did not last for long: a first spectacular downfall of the system took place in 1994, with the

\footnote{E-mail: icef-research@hse.ru. The author is thankful to the participants of the Vth international conference of HSE and ICEF seminar audience for helpful comments. The usual caveat applies.}
quick expansion of financial pyramids: several small banks and financial companies played Ponzi games against a large population of private debtholders. These pyramids flourished in many transition economies, including (besides Russia) Ukraine, Bulgaria, Romania, Armenia, Albania; and their eventual collapse left diluted several million people, and caused massive impoverishment, public mistrust in the banking system, social unrest, and even one revolution (in Albania).

This first crisis was seemingly over by 1995, when the Russian government launched the state securities market (the so-called GKO and OFZ) with extremely attractive rates of return (50% and even more in real terms). This second wave of growth was accompanied by the processes of concentration of banking capital backed up by the state. Yet more severe was the effect of the systemic collapse in August 1998, when the government defaulted on its GKO/OFZ debt and jointly announced a drastic (more than 4 times) devaluation of the Ruble. Russian banks whose major assets were GKO and OFZ, lost about 40 bln. Rubles of capital, which constituted about 50% of the total capital of the banking system, and some 7.5% of GDP (Survey of economic policy in Russia in 1999, 2000). This crisis resulted in bankruptcy of many banks, including the largest ones, and the total number of banks dropped by nearly one thousand in a couple of years. Yet more importantly, this second crisis resulted in rapid loss of domestic and international confidence in the Russian banking system, and a substantial outflow of public deposits, accompanied by a reduction of the scale and scope of profitable credits operations. All this purged the industry in a deep systemic crisis whose consequences have not been completely exhausted till the mid-2000s.

Nevertheless, since 2000–2001, the banking industry in Russia went to the stage of yet another recovery. Over the year of 2003, all major banking indicators increased by one quarter or more. In that only year, total assets of the banking system went up from 3260 bln. RUR at the end of 2001 to 4015 bln. RUR, and constituted 36% of the country’s GDP; in 2003, it increased further to 5600 bln. RUR (42% of GDP), with an absolute increase of almost 40% over that year. Private deposits amounted to 1540 bln. RUR at the end of 2003 (up from 998 bln. RUR one year ago), while credits outstanding, at 2960 bln. RUR in 2003 went up from 1990 bln. RUR at the end of 2002 and 1230 bln. at the end

\[2 \text{US$} = \text{approx. 30 RUR.}\]
of 2001. The aggregate own capital of the banks amounted in 2003 to 794 bln. RUR (up from 570 bln. in 2002 and 455 bln. in 2001), in parallel with some concentration of the banking sector, as the total number of registered banks went down to 1666 (on January 1, 2002, there were 2003 of them), of which only 1329 were actually operating.

These changes might appear encouraging; however, quite a few characteristics of the system still imply it hardly has reached the stage of maturity. First of all it should be acknowledged that the Russian banking system remains inefficient by the world standards relatively to its own potential. This is illustrated in Table 1, which compares the Russian and the US banking systems in relative terms. The table shows that, despite some progress, the Russian banks still issue two times less loans (compared to GDP) and attract two times less deposits (compared to personal incomes) than their American counterparts do. These indicators are even larger for other developed banking systems: thus, loans-to-GDP ratio in Germany is about 0.80, and in the UK yet larger. Relative backwardness of the Russian banking system is hardly surprising: survey estimates suggest that bank deposits in Russia still hardly amount to 40% of private savings accumulated by the population, the rest being held in cash, mostly US dollars and Euro (Avraamova and Ovcharova, 1998; Ibragimova, 1999), or, as in recent years, in other forms, such as foreign bank accounts or real estate (Kuzina, 2005). Thus, the banking system fails to accumulate quite a substantial share of the private savings of the nation.

On the credit side, the situation might appear to be somewhat better, yet its growth can itself be a source of potential concern. First, the main assets of the Russian banks concentrate in particular sectors, such as export-oriented companies, real estate and consumer credits. Concentration on these effectively prevents diversification of assets, making the banking sector potentially vulnerable to any adverse shock in these areas. Second, by 2003 the Russian banks have nearly exhausted the pool of low-risk borrowers, and increasingly engage in crediting more and more risky projects (Solntsev and Khromov, 2003). Third, and most importantly, limited number of profitable and secure investment opportunities makes it unnecessary for the banks to accumulate substantial credit resources.

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3 General data on Russian banking system is taken from the Bulletin of the Central Bank of Russia, various issues.
and hence depresses their incentives to attract private deposits. As a result, total credits issued still exceed total deposits by almost 50%, suggesting that the Russian banks issue relatively more credits out of their own capital than their counterparts from more efficient banking systems\textsuperscript{4}. For this very reason the Russian banks lack credit resources to work with the best Russian borrowers, such as Gazprom, LUKOIL or RAO UES, who prefer to borrow from the consortia of foreign banks or in the world financial markets. To consider just one example, only in 2000 Gazprom (the state-controlled natural gas monopoly) borrowed 185 bln. RUR, of which more than 90% were supplied by the consortia of foreign banks, who are able to offer cheaper credits, and with only minor financial participation of the Russian banks (Matovnikov, 2001). This figure amounted to more than 1/10 of all credits issued by the Russian banking system, and more than 1/4 of credits issued by 100 largest Russian banks in that year.

Next, the Russian system remains heavily concentrated. As of early 2005, more than 70% of the total value of assets accrues to five largest banks — Sberbank, Vneshtorgbank (bank for foreign trade), Gazprombank (affiliated with Gazprom), MDM-Bank and Alfa-bank (see Table 2 for a summary of these and other indicators at the end of 2002). The first of these large banks, Sberbank, accumulates about 2/3 of total private deposits — this bank alone possesses a nationwide network of branches, which makes it unique and endows it with substantial monopoly power. High concentration of the national banking capital is accompanied by the concentration of its efficiency: as shown in Table 2, the share of credits outstanding to own capital steadily decreases with the bank’s size. This observation goes at odd with the world practice, and implies that a bulk of small banks in Russia cannot find a proper niche on the credit market\textsuperscript{5}. Certainly, this expression should be interpreted with some caution, as small banks may be of use for small towns and/or small business enterprises. Yet the problem is that even the best of these banks cannot raise the scale of their operations by diversifying their portfolios and

\textsuperscript{4} Direct comparison of the deposits-to-capital ratios of Russian and Western banks is likely to be misleading because the banks’ equity in Russia is often inflated for strategic reasons and/or due to regulatory requirements of the Central Bank.

\textsuperscript{5} As eloquently summarized by one of the Russian bankers, “had the bottom thousand of Russian banks disappeared, this would have been unnoticed by everyone except their employees”.

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attracting more credit resources.

To summarize, the Russian banking system so far appears to be inefficient in accumulating private savings, and lacks resources for large-scale credit operations, thus failing to exercise their major economic role of transformation of savings into private investment. The stylized facts listed above suggests a potential explanation for that: the Russian banking system appears to be locked in a kind of underdevelopment trap. On the one hand, relatively high interest rate margins, and lack of public trust creates no incentives for most banks to attract private deposits at large, limiting their credit resources. Given this constraint, the banks have to work with relatively small-scale borrowers, whose needs can essentially be met with moderate resources based on own capital and a little bit of deposits. Inasmuch as this strategy of the banks is supported by the public’s unwillingness to bring their savings to the banks, this state of affairs corresponds to bad equilibrium with low interest rate on private deposits. On the other hand, inasmuch as the money market stabilizes, some banks get double new incentives: to intensify credit operations (even if at a cost of higher risk), and to attract more credit resources (given a gradual restoration of the public’s confidence). Still, the banks which do this need not necessarily be more skillful or safe, while the public in general lacks information about the quality of a particular bank. This informational asymmetry gives rise to the adverse selection problem, and is ultimately responsible for pushing the system back to the bad equilibrium.

The story could have been completely different had the banks been able to attract large credit resources from the public, and offer longer-term credits to the first-class lenders like Gazprom. Attracting large resources would require development of a retail banking network with large sunk costs, which would be worthwhile only if the inflow of deposits is large enough. If this condition is met, the good equilibrium is self-supporting as well; the problem is that the distance between the good and the bad equilibrium is rather substantial. Switching from the bad to the good equilibrium would require a simultaneous strategy change by many uncoordinated debtholders, and the ability of the good banks to offer best-quality and cost-efficient credits, which in itself calls for substantial restructuring of the banking sector. The intuition developed at the end of the previous paragraph suggests yet another reason why this transition might fail: failure of the public to discriminate among the
banks gives rise to the adverse selection problem.

We capture the above intuition by means of a signaling game between two players who represent the populations of individuals and banks as described in Section 3. In our model, banks act as senders, and issue different kind of costly signals we interpret as the interest rate at which they attract individual deposits\(^6\). Individuals, acting as receivers in this game, observe this signal, and make their decision about depositing money in a particular bank, or withholding from deposit at all. Those banks which are more fit to manage deposits and credit risks are said to be of "good" type; those who are not are termed "bad". One might expect that good banks (in the sense just defined) will have incentives to have better deposit management, leading to the eventual prevalence of such bank in the industry\(^7\). The question why this does not necessarily happens is interesting from policy viewpoint, and we suggest an explanation to this failure.

In our model, both individuals and banks maximize expected utility. However, beliefs by both players evolve as a result of past observations and the relative success of previously played strategies, i.e. they are boundedly rational in the sense of Simon (1955, 1978). In particular, individuals in our model have limited memory, and form their beliefs about the quality of a particular bank sending particular signal on a basis of the last \(m\) observations. Banks in our model learn more quickly: in particular, within a single period they send the signal which is most beneficial \(\text{ex ante}\), and invest in the projects which are most profitable \(\text{ex post}\), after they have learned the amount of attracted deposits. Learning dynamics built along these lines is specified and discussed in Section 4. Section 5 summarizes our findings.

\(^6\) Instead of (or alongside with) the interest rate, other signals might also be considered, such as the quality of service delivered to customers, the quality of bank’s office etc. In fact, any variable that is costly to the bank and monotonically increases with the number of debtholders would fit into the spirit of our signaling game, and leads to qualitatively similar conclusions.

\(^7\) In view of this possibility, it might sound attractive to extend the definition of type to a conjunction of performance in both active and passive operations. This can be done without any substantial change in our results; since these can be obtained in a simpler framework, we stick to the definition of type developed in the text.
2. Review of related literature

The above evidence suggest several lines of approach to the problem of banking sector development, including the theory of bank runs (Diamond and Dybvig, 1983; Chari and Jagannathan, 1988; Jacklin and Bhattacharya, 1988; Dewatripont and Tirole, 1994; Alonso, 1996; Allen and Gale, 1998; Chen, 1999), and bank monitoring (Diamond, 1984, 1991; Calomiris and Kahn, 1991; Holmstrom and Tirole, 1997). Both lines offer the general theoretical framework resulting in multiple equilibria in the game between the banks and the public, but most often without explicit discussion of the coordination on any of these and/or of the switching mechanisms. Such mechanisms have been developed in the literature on evolutionary games (Maynard Smith, 1982; Weibull, 1995; Samuelson, 1997; Hofbauer and Sigmund, 1998; Fudenberg and Levine, 1999) — in particular, the stochastic mutations approach by Kandori e.a. (1993) has been applied to banking problems in Temzelides (1995) and Temzelides and Adao (1995). Being explicit in terms of evolution of strategies, this literature is usually quite stylistic in terms of institutional settings.

In our model setup we focus on several specific issues pertinent to the savings market in transition, which have also been informally investigated in several papers. Spicer and Pyle (2002) and Avdasheva and Yakovlev (2000) describe the evolution of savings strategies and argue that one of the crucial determinants of public’s trust to the banking system is the evolution of institutions. Denizer and Wolf (2000) compared the actual savings rate before transition to the normal savings rate estimated using the data of similar economies, and claim that a great deal of the actual savings were involuntary. They also analyze the determinants of saving behaviour on the sample of transition and developed countries, and find that wars and the rate of decline affect savings negatively, while the ratio of M2 to GDP and the rate of CPI inflation contributes to it. Last, they also observe convergence in time between the factors of saving behaviour across transition and developed countries. All these factors do indeed contribute to the configuration of the banking system in Russia, but are exogenous to our model.

The model we build combines the institutional intuition discussed in the previous section with the analytical structure of Bayesian games with their evolutionary analysis. We adopt the multiple equilibrium frame-
work of signalling games and discuss the ways to select among these in the light of equilibrium refinements literature (Banks and Sobel, 1987; Cho and Kreps, 1987; Cho and Sobel, 1990; Rabin and Sobel, 1996; Mailath e.a., 1993). Most of these refinements differ in the ways they limit beliefs that the uninformed party holds after a signal off the supposed equilibrium path; in our framework this principle is also supported by the evolutionary consideration. In this respect, our paper is most close to the evolutionary studies of the signaling games along the lines pioneered by Rabin and Sobel (1996), Nödeke and Samuelson (1997) and Nödeke and van Damme (1990) who applied this setup to the signaling games of Spence (1973, 1974) type.

Although different in setup, our model is similar in spirit to Nödeke and Samuelson (1997). In their model, finitely many sellers send one of a finite set of signals each, and offer goods of one of two types to a single buyer. At the beginning of each period, the buyer forms her beliefs about the share of high-quality sellers corresponding to each signal, and announces a competitive price schedule which reflects her expected quality of the good that stands behind each signal. In order to work with finite Markov process, they voluntarily limit the cardinality of the set of possible beliefs, and impose the condition of competitive pricing on the buyer. We also work with finite set of beliefs of the myopic uninformed players (debtholders), as well as finite sets of signals and credit strategies. However, in our model the set of states is defined by the actual and believed number of debtholders per bank of each type, which is naturally finite, so that we do not need to impose this restriction. Our model uses simpler signal structure than Nödeke and Samuelson do, but our case is further complicated by the multiplicity of credit strategies.

3. The model

In this section we develop an analytical model of interaction between individuals and banks within the framework of a repeated signalling game in discrete time. There are $N$ individuals in the population of potential debtholders and $M$ independent banks in the population of banks. This specification assumes that banks are similar and small, and can be associated with the specific kind of projects they credit, as shall be developed shortly. Both populations are finite but large enough to appeal to the
law of large numbers. These assumptions are rather innocuous but useful from two viewpoints. First, they give the convenience to deal with finite state spaces; second, they allows us to represent the typical individual deposit by its average $D$, and own capital of a typical bank by its average $K$.

Credit resources of each bank in every period consist of its own capital $K$ plus the sum of its deposits $Dn_i$, so that $N = \sum_{i=1}^{M} n_i + n(w)$, where $n_i \geq 0, \forall i$, the first sum being the total number of debtholders in all $M$ banks and the last term, $n(w)$ — the number of withholders. In every period each bank uses all its resources to credit exactly one of two available technologies or projects. Gross returns of the banks on any project are stochastic, and can take on two values: low or high. Low returns on the project correspond to borrower’s default; these are normalized to zero. High returns occur if the project has been successfully completed. Values of these high returns depend on the scale of the credit issued, and are given by the return functions $g(n_i)$ that are increasing concave in $n$, with $g(n_i) > g(K)$ and $g(n_i) > n_i$, $\forall n_i > 0$ — that is, successful projects bring positive (net) returns and exhibit decreasing returns to scale. Furthermore, for each possible value of credit resources ($K + Dn_i$ if $n_i > 0$, $K$ if $n_i = 0$), projects can be of two kinds. Projects of the first kind bring higher revenue in case of success, but have lower chance of being successful, i.e. bring high rather than low (zero) returns. We term the former projects risky, the latter — safe. We also assume that, once the banks have learned how many resources they have in the current period, they choose the project of either risky or safe kind which brings them the highest possible expected return. We associate the projects with the highest possible revenue and denote them $\bar{g}$ and $\underline{g}$ for the risky and safe projects, respectively. Note that the properties of the $g(\cdot)$ functions imply that the expected returns $\mathbb{E}g(\cdot)$ on each project of each kind are increasing concave as well.

Besides the kind of projects and the amount of credit resources, expected returns depend on the intrinsic quality of the banks, understood as their abilities to efficiently transform private savings into investment. These abilities correspond to bank’s types, which can be either good, denoted $\theta$ or bad, denoted $\theta'$. The names are not intentional, just

\footnote{We assume there are no reserve requirements nor collateral; introducing them will not affect our qualitative conclusions, but shall make our reasoning more complicated.}
shorthand for more and less proper (skillful, prudent) banking institutions: good banks are better at both managing deposits and credit rationing than their bad counterparts are. Throughout the paper we assume both sets of banks to be nonempty. For each value of \( n_i \), safe projects \( g \) bring equal expected returns to both types of banks. By contrast, risky projects \( \bar{g} \) are better managed by the good banks of type \( \theta \) than by the bad bank of type \( \bar{\theta} \). These and related properties are formalized by several general assumptions which hold through the rest of the paper.

**Definition 1.** The **relevant range** is the range of \( n \) at which expected returns on risky projects of the bank are marginally larger than those on safe projects\(^9\).

The relevant range is denoted \( \tilde{N} \); all characteristics of the banks are considered over this range.

**Assumption (Dominance).** \( \forall n_i \in \tilde{N} : E\bar{g}(n_i|\bar{\theta}) > E\bar{g}(\tilde{n}_i|\bar{\theta}) \).

**Assumption (Genericity).** \( \forall n_i \in \tilde{N} \), there is a unique \( g^* \) such that \( E\bar{g}^*(\cdot) = \max_g E\bar{g}(n_i|\theta), g = \{g, \bar{g}\}, \theta = \{\theta, \bar{\theta}\} \).

**Assumption (Monotonicity).** \( \forall n_i, \tilde{n}_i \in \tilde{N} : n_i > \tilde{n}_i \Rightarrow E\bar{g}^*(n_i|\theta) > E\bar{g}^*(\tilde{n}_i|\tilde{\theta}), \text{ where } g = \{g, \bar{g}\}, \theta = \{\theta, \bar{\theta}\} \).

*Dominance* says that expected return on the risky projects is strictly higher for the good banks than for the bad ones over the relevant range of \( n_i \). This property directly follows from our interpretation of types: good banks are more able to ration out bad borrowers, which increases the expected return from their credits. *Genericity* implies that for every value of \( n_i \) there is a unique project which maximizes expected return (either risky, \( E\bar{g}(n_i|\bar{\theta}) \) or safe, \( E\bar{g}(n_i) \)) for both types of the banks. This assumption is not restrictive, and also common in the literature (e.g. Nödeke and Samuelson, 1997). *Monotonicity* says that the expected return on the best available project is strictly increasing in \( n_i \) over the relevant range for both safe and risky projects and both types of the banks. Hence from the revenue-maximization viewpoint, all banks prefer more deposits to less (and to no deposits at all).

\(^9\) Since we work with finite sets only, all marginal values here and in what follows are to be interpreted in terms of increments, not differentials.
The above three assumptions over the relevant range ensure that expected returns as functions of the project and type of the bank satisfying the above specifications are of the shape illustrated on Figure 1a. Expected returns on the safe project \( E_g(\cdot|\cdot) \) are drawn in bold, expected returns on the risky project for the bad banks \( E_{\bar{g}}(\cdot|\bar{\theta}) \) are in dashed, and those for the good banks, \( E_{\bar{g}}(\cdot|\theta) \) are in normal lines. Note that expected returns on the risky projects for the good banks and the bad banks intersect with \( E_g(\cdot|\cdot) \) only once, at the points \( n_1 \) and \( n_2 \) (ignore other notations in the figure for the moment).

Both types of banks are identical in terms of paying interests on individual deposits. The interest rates are treated in real terms (net of inflation), and can be either high or low, \( s = \{\underline{s}, \bar{s}\} \), \( 0 < \underline{s} < \bar{s} < 1 \). Every bank is liable to gross pay \( D(1 + s) \), \( s = \{\underline{s}, \bar{s}\} \) on every debtholder’s deposit at the end of the period; we assume the banks always keep their promises provided they have enough funds. Information is asymmetric, hence individuals do not know the types of each bank. All they can do is to form their beliefs over that type using the promised interest as signal to the present and potential debtholders.

These latter are \textit{ex ante} identical in terms of preferences and judgmental abilities, but can be heterogeneous in their beliefs and strategies. That is, we treat them as \textit{boundedly rational}, rather than \textit{substantively rational} agents (Simon, 1955; 1978), and shall explicitly qualify this in the next section. For now we may think of debtholders as of a single player whose set of pure strategies is \( I = \{I(\underline{s}), I(\bar{s}), I(w)\} \), standing for investment in the bank sending \( \underline{s} \), in the bank sending \( \bar{s} \), and withholding from investment, respectively.\(^{10}\) The number of debtholders who invest in the banks which sent signal \( \underline{s} \) (respectively, \( \bar{s} \)) is \( N(\underline{s}) \) (respectively, \( N(\bar{s}) \)), and these deposits are shared equally by all banks which send the respective signal, regardless of their type. Thus, if the share of those banks which sent signal \( \underline{s} \) is \( \phi \), the number of debtholders of every single bank which sent signal \( \underline{s} \) is \( n_i(\underline{s}) = \frac{N(\underline{s})}{M\phi} \); similarly, \( n_i(\bar{s}) = \frac{N(\bar{s})}{M(1-\phi)} \) is the number of debtholders of each bank sending an \( \bar{s} \) signal.\(^{11}\)

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\(^{10}\) The vector of proportions of the individuals playing each of these strategies can then be thought as the vector of mixed strategies over \( I \). We do not consider mixed strategies explicitly, because under our dynamic specifications these do not correspond to stationary states.

\(^{11}\) Below we shall omit subscript \( i \) inasmuch as it does not cause confusion.
The game consists in repeated replications of the stage games indexed by \( t \). In each stage game, the banks have to bear the cost of debt service \( s_t = \{ g, \bar{s} \} \) per unit deposit. Expected cash outlays of the banks consist of debt service plus regular outflows given by \( \zeta n_t(s_t) \), where \( \zeta \) is the fraction of deposits \( n_t(s_t) \) which represents normal net withdrawals for transaction purposes. The sum of these terms, \( D(sn_t(s_t) + \zeta n_t(s_t)) \equiv c(n_t, s_t) \) constitutes the expected outflow of resources, which is used by the bank to plan its expenses. This value, however, need not coincide with the factual one due to non-transactional net outflows \( \Delta n_t(s_t) \) which are due to bankruptcies and changed beliefs of the debtholders at the end of period \( t \). Factual outflows of credit resources thus equal \( c(n_t, s_t) + \Delta n_t(s_t) \), which is the cost of debt to the bank. Note that no bank is clearly willing to raise the signal unless it will result in higher profit.

Timing of each stage game is given in Figure 2. At the beginning of each stage game, banks of both types choose one of the two signals so as to maximize their ex ante expected profits. Individuals observe the signal and choose whether they will invest in high-signaling bank, in low-signaling bank, or withhold from investment at all. All deposits are shared between the respective banks, which invest the resulting credit resources in either risky or safe project. The banks of both types thus have four pure strategies, given by the possible combinations of the signal they send and the projects they choose. Next, production takes place, after which the banks receive random returns on the projects. As a result, some banks might fail to pay their bills, and become bankrupts. Such banks have to quit the game. We assume that the bankrupted bank will be replaced in the next period by another bank of the same type as the old one, which chooses its strategies alongside with the incumbent banks\(^\text{12}\). At the end of the stage game, individuals collect payoffs and update their beliefs about the quality of the banks sending each signal.

In every stage game, banks of either type maximize the ex ante expected profit \( \pi(\cdot) \) of their returns short of the expected cost and given the expected number of debtholders \( n \), i.e.

\[
\pi(s^*, g^*) = \max_{(s, g)} \pi(s, g|n, \theta) \equiv E_g(K + Dn(s)|\theta) - c(n, s)
\]

\(^{12}\) This assumption is most natural given the experience of Russian transition, where bank owners used to strip off the assets of a problematic bank, and use these to reopen their business under a new brand name at the expense of past debtholders.
where \( g = \{g, \bar{g}\} \), \( s = \{s, \bar{s}\} \) and \( \theta = \{\theta, \bar{\theta}\} \). This maximization takes place in two stages: \textit{ex ante} they choose the signal, and \textit{ex post} (upon observing the number of deposits) — the project. Note that \( \pi(s, g|n, \theta) \) is quasilinear in the cost function \( c(n, s) \), which is itself independent of the random return on the credited projects.

In every stage game, the representative individual chooses the investment strategy \( I \) which maximizes his \textit{ex ante} payoff:

\[
v(I^*) = \max_I v(I|s, g, \theta) \equiv v(I)(1 - \Pr(B|s))
\]

where \( I = \{I(\bar{s}), I(s), I(w)\} \) is one of the three possible investment strategies, and \( \Pr(B|s) \) is the probabilistic belief that any bank sending signal \( s = \{s, \bar{s}\} \) will be bankrupt (the \( B \) event), given the bank’s type and the project it chooses. This probability is taken as exogenous when characterizing Bayesian equilibria; in the next section it shall also be derived from intuitively simpler rules of boundedly rational learning (Fudenberg and Levine, 1999).

The basic solution concept we use is that signaling equilibrium.

**Definition 2.** A signaling equilibrium is any profile of players’ strategies \( \{(s^*, g^*); (I^*, \Pr(B|s^*))\} \) such that

1. \( (s^*, g^*) \in \arg\max_{\{s, g\}} \pi(s, g|I^*, \theta) \) for \( \theta = \{\theta, \bar{\theta}\} \) and equilibrium individual strategy \( I^* \in \{I(\bar{s}), I(s), I(w)\} \);

2. \( I^* \in \arg\max_{\{I\}} v(I|s, g^*, \theta) \) for \( s = \{s, \bar{s}\} \) given \( \Pr(B|s^*) \);

3. \( \Pr(B|s^*) \) is consistent with Bayes rule whenever possible.

This definition is pretty standard, but because bank strategy consists of two actions, the equilibrium conditions are more complicated, even though \( g = g^* \) is satisfied automatically. To qualify this, consider again Figure 1a with two points \( n_1 \equiv \max\{n : E\bar{g}(n) \leq E\bar{g}(n|\theta)\} \) and \( n_2 \equiv \max\{n : E\bar{g}(n) \leq E\bar{g}(n|\bar{\theta})\} \) introduced earlier. In addition, let \( n' \) be the (unique) value of \( n \) at which the marginal increase of expected return due to the \( n^{th} \) debtholder equals the cost of this debtholder attracted at the low signal \( \bar{s} \), i.e. \( E\bar{g}'(n'|\cdot) = c(n', \bar{s}) \). Similarly, let \( n'' : E\bar{g}'(n|\theta) = c(n'', s) \) and \( \bar{n}' : E\bar{g}'(\bar{n}'|\bar{\theta}) = c(\bar{n}', \bar{s}) \) be the (unique) values of \( n \) at which the
marginal expected returns to the good (respectively, bad) bank due to the \( n^{th} \) debtholder equals the cost of this debtholder attracted at the high signal \( \bar{s} \). It follows from the definition of good banks that none of them will ever want to operate with more deposits than \( n' \) (\( g' \)) at low (high) cost of deposits\(^{13}\). For the bad banks the story is a bit different as, being "less appropriate", they may be more inclined to get higher share of deposits than they are able to "digest", operating at a point to the right of \( \bar{n}' \). In case of normal operations such an increase will not be profit-maximizing in the long run; yet this behaviour may be justified only by willingness to pay some debtholders at the expense of the others, and under poor control from the government authorities. The experience of transition warrants allowing this option; we call bad banks operating in the range \( n \in [\bar{n}', \infty) \) the Ponzi banks. Expected profit maximization implies that any bank which is not Ponzi will want to switch from safe to risky project only if 1) the expected volume of deposits does not exceed \( \bar{n}' \) (and \( \bar{n}' \)), and 2) the difference between additional return on risky project more than offsets the difference in costs between \( \bar{s} \) and \( s \). Threshold volumes of deposits above which this last property holds are denoted \( n^* \equiv \max\{n : E\bar{g}(n|\bar{\theta}) - c(n, \bar{s}) \leq E\bar{g}(n|\bar{\theta}) - c(n, \bar{s})\} \) for the good and \( \bar{n}^* \equiv \max\{n : E\bar{g}(n|\bar{\theta}) - c(n, \bar{s}) \leq E\bar{g}(n|\bar{\theta}) - c(n, \bar{s})\} \) for the bad banks. To make things interesting we assume such \( n^* \) and \( \bar{n}^* \) always exist; they are depicted on Figure 1a, where upward sloping straight lines denote the costs associated with high (dashed) and low (solid) signals\(^{14}\).

Now it is time to introduce a few more specific assumptions, which are supposed to hold for all \( n \in \tilde{N} \):

**Assumption 1.** \( \bar{n}^* < n^* < n' \).

**Assumption 2.** \( \bar{n}^* < n' < \bar{n}^* \).

**Assumption 3.** \( v(I|g, \bar{s}, \bar{\theta}) > v(I|g, s, \bar{\theta}) > v(w) > 0 \), \( v(w) > v(I|\bar{g}, \bar{s}, \bar{\theta}) > 0 \), and \( v(w) > v(I|\bar{g}, \bar{s}, \bar{\theta}) > 0 \).

**Assumption 4.** \( v(I|g, \bar{s}, \bar{\theta}) > v(I|g, s, \bar{\theta}) > v(w) > 0 \) and \( v(I|\bar{g}, \bar{s}, \bar{\theta}) > v(I|\bar{g}, \bar{s}, \bar{\theta}) > v(w) > 0 \).

\(^{13}\) By the nature of good banks, this also means that \( E\bar{g}(n|\bar{\theta}) > c(n, g) \), \( \forall n \).

\(^{14}\) For reference purposes it may worth to summarize the relationships among the above parameters, which remain true irrespectively of the assumptions 1 through 4:

- \( E\bar{g} < E\bar{g}'(\bar{\theta}) < E\bar{g}'(\bar{\theta}) \);
- \( n^* < n', \bar{n}^* > n' \);
- \( n_1 < n_2 \);
- \( \bar{n}^* > n_1, \bar{n}^* > n_2 \).
Assumption 1 says that for type $\theta$ the threshold value $n^*$ above which it wants to increase the signal from $s$ to $\bar{s}$ and switch from the $\bar{g}$ to $\bar{g}$ project is lower than the value $n'$ at which the marginal gain on the $\bar{g}$ project equals the cost of credit resources at low interest rate. Location of $n_2$ in this assumption follows from the regularity requirement; condition $n^* < \bar{n}^*$ also follows from earlier definitions. Assumption 2 claims that $n^* < \bar{n}^*$ — thus, while banks of type $\theta$ would be willing to switch at $n^*$ to $\bar{g}$ and $\bar{s}$, banks of type $\bar{\theta}$ would prefer to stick at the safe projects and low interests up to the point $n'$ and up to $\bar{n}^*$, respectively. In other words, should the banks of type $\bar{\theta}$ want to raise the signal and project type in the deposit game, they would have to opt for a jump in deposits from $n'$ to $\bar{n}^*$ as a result of this higher signal. Such nonempty intervals are called gaps. If $n^* > n'$, we speak of type $\theta$ gap; if $\bar{n}^* > n'$, we speak of type $\bar{\theta}$ gap. Clearly type $\bar{\theta}$ gap implies $\theta$ gap, but not the opposite. Unless otherwise specified, we explicitly assume that no bank ever wants to raise the signal if this results in a gap.

The remaining assumptions deal with individual preferences. Assumption 3 asserts that investment in the bad bank sending high signal is desirable only if this latter invests in safe projects; investment of these banks in risky project bring negative expected return, so individuals prefer withholding and receiving $v(w)$, which is the utility of current consumption. By assumption 4, investment in the good bank under high signal is preferred to $v(w)$ for any project credited. Note that the last two assumptions together imply that $v(I(\bar{g}, \bar{s}, \bar{\theta})) > v(I(g, s, \theta)) > v(w)$ for $\theta = \{\bar{\theta}, \bar{\theta}\}$.

Proposition 1. Provided assumptions 1, 3 and 4 hold, $v(\bar{s}) < n^*$, and $\mathbb{E}_\theta(v(\bar{s})) - c(n(\bar{s}), \bar{s}) > g(K) > 0$ for $\theta = \{\bar{\theta}, \bar{\theta}\}$, the following is the set of pooling equilibria:

1. Banks of both types send signal $\bar{s}$ and choose projects $\bar{g}$;
2. Individuals invest in either bank sending $\bar{s}$ with equal probability;
3. Individual beliefs $Pr(\hat{\theta}|\bar{s})$ and $Pr(\hat{\theta}|\bar{s})$ are both high enough to ensure $v(\bar{s})(1 - Pr(B|\bar{s})) > v(w) > v(\bar{s})(1 - Pr(B|\bar{s}))$.

Proof. In the pooling equilibrium, signals are uninformative, and thus investment depends on individual beliefs. By definition of $Pr(B|\bar{s})$ and assumptions 3 and 4, individuals would deposit in the banks sending $\bar{s}$, and withhold money from the banks sending $\bar{s}$. From assumption 1,
$n^* < \bar{n}^*$, and all deposits are divided equally among all banks sending the same signal. Hence all banks receive no more than $n^*$ deposits, and invest in $g$ by assumption 1 and because $E_g(n(s|\theta)) - c(n(s), s) > g(K)$. By definition of $\bar{n}^*$, no bank has incentives to increase the signal from $s$ to $\bar{s}$, as it will deprive it of any deposits given individual beliefs.

Since these equilibria involve low signal from both types, we call them **PL-equilibria**; the area of $n$ in the relevant range for which these equilibria hold are denoted by a thick leftward arrow on Figure 1a. PL-equilibria are "bad" in the sense of Section 1: low signals sent by all banks attracted few deposits, and the banks lack resources to invest in large-scale projects, even if they are well-suited to manage them.

A "better" equilibrium arises if the number of debtholders per bank increases, and the good banks have incentives to raise the signal. A necessary condition for this to be an equilibrium turns out to be the existence of a gap of type $\bar{\theta}$:

**Proposition 2.** Subject to assumptions 2, 3, and 4, $n(\bar{s}) \in (n', \bar{n}')$ and $E_{\bar{g}}(n(\bar{s}|\theta)) - c(n(\bar{s}), \bar{s}) > g(K) > 0$, the following is the set of separating equilibria:

1. Banks of type $\bar{\theta}$ send signal $\bar{s}$, banks of type $\theta$ send signal $s$;
2. Individuals invest in banks sending $\bar{s}$, and withhold from investment in the banks sending $s$;
3. Beliefs $Pr(\bar{\theta}|s)$ are high enough to ensure that $v(s)(1 - Pr(B|s)) < v(w)$.

**Proof.** In a separating equilibrium signals are fully informative, and if individuals believe that signal $s$ (s) came from banks of type $\bar{\theta}$ (\theta), they invest only in the banks which send the former signal because of the condition on $Pr(B|s)$ (which implies $Pr(\bar{g}|s) = Pr(\bar{g}|\theta)$) and by assumptions 3 and 4. Provided high signal brings deposits $n(\bar{s}) \in (n', \bar{n}')$ per bank of type $\bar{\theta}$, such banks find it more desirable than getting no credit resources at low signal by the condition $E_{\bar{g}}(n(\bar{s}|\theta)) - c(n(\bar{s}), \bar{s}) > g(K) > 0$. In the $(n', \bar{n}^*)$ range, banks of type $\bar{\theta}$ face a type $\bar{\theta}$ gap and find it not worthwhile to raise the signal by assumption 2, so these banks have to credit with own capital only. □
These equilibria are illustrated on Figure 1b, where the range of equilibrium values of $n$ is indicated by a horizontal arrow. Such separating equilibria, or $S$-equilibria are Pareto-optimal for the banks\footnote{In the literature, these are also known as the Riley equilibria, after Riley (1979).}; regrettably, they are both conditional upon existence of the type $\theta$ gaps, and vulnerable if banks (or banking regulation) are not prudential enough. Specifically, bad banks who are not happy with no deposits may want to raise the signal even under type $\bar{\theta}$ gap, which is dangerous because of possible mis-estimations of deposit inflows over the gap, and as it might result in more frequent bankruptcies of the bad banks from risky projects.

The above circumstances suggest that prevalence of particular equilibria and, more importantly, their succession in an evolutionary framework crucially depend upon the dynamics of individual beliefs. Such dynamics is explicitly constructed in the next section; for now we introduce a few more relevant equilibria.

**Proposition 3.** If assumptions 1, 3, 4 hold, $n(s) \in (n_1, \min(n', n''))$, and $Eg(n(s)|\theta) - c(n(s), s) > g(K) > 0$ for $\theta = \{\theta, \bar{\theta}\}$, the following is the set of pooling equilibria:

1. Banks of both types send signal $s$ and choose projects
   - for type $\theta$: $g$ if $n(s) \in (n_1, n')$; and
   - for type $\bar{\theta}$: $\begin{cases} g & \text{if } n(s) \in (n_1, n_2), \\ \bar{g} & \text{if } n(s) \in (n_2, n''). \end{cases}$
2. Individuals invest in either bank sending $s$ with equal probability;
3. Individual beliefs $Pr(\theta|s)$ and $Pr(\bar{\theta}|\bar{s})$ ensure $v(s)(1 - Pr(B|s)) > v(\bar{s})(1 - Pr(B|\bar{s})) > v(w)$.

**Proof.** Given individual beliefs that banks sending signals $s$ on average bring the highest expected return, individuals will be willing to deposit money at low interest rate, while being reluctant to invest in any bank sending $s$. If the public is happy to lend money at low interest, the banks of type $\theta$ ($\bar{\theta}$) will invest in the risky projects inasmuch as the number of deposits per bank exceeds $n_1$ (respectively, $n_2$); banks of type $\bar{\theta}$ will still invest in $g$ projects for $n(s) \leq n_2$, and would not be willing to raise the signal given individual beliefs. \qed
Proposition 4. Subject to assumptions 1, 3, 4, \( n(\bar{s}) \in (\bar{n}^*, \min(\bar{n}', n')) \) and \( \bar{E}_\theta(n(\bar{s})|\theta) - c(n(\bar{s}), \bar{s}) > g(K) > 0 \) for \( \theta = \{\bar{\theta}, \bar{\theta}\} \), the following is a set of pooling equilibria:

1. Banks of both types send signal \( \bar{s} \), and select the project \( \bar{g} \);
2. Individuals invest in banks sending \( \bar{s} \), and withhold from investment in the banks sending \( s \);
3. Individual beliefs \( Pr(\bar{\theta}|\bar{s}) \) and \( Pr(\bar{\theta}|s) \) ensure \( v(\bar{s})(1 - Pr(B|\bar{s})) > v(w)(1 - Pr(B|s)) \).

Proof. Given individuals’ beliefs and assumptions 3 and 4, it is optimal for them to invest in the banks sending high signals, which both banks can afford in an interval \( (\bar{n}^*, \min(\bar{n}', n')) \) by assumption 1. (We do not know whether \( n' \gtrless \bar{n}' \), hence the minimum condition). Because \( \bar{g} \) is increasing in \( n \), no bank will want to invest in \( g \), as they will have no deposit and lower profit because \( \bar{E}_\theta(n(\bar{s})|\theta) - c(n(\bar{s}), \bar{s}) > g(K) \). Pooling equilibria of proposition 3 increase the amount of deposits in comparison to PL equilibria over the nonempty range \( (\bar{n}^*, \min(n'^*, \bar{n}') \), these are called interim, or \( \text{PI-equilibria} \). PI-equilibria are illustrated on Figure 1a by dashed double arrow. Pooling equilibria of proposition 1 with high interest (\( \text{PH-equilibria} \)) are denoted on Figure 1a by small double arrow, and represent an instance of adverse selection in that bad banks can and want replicate the behaviour of good ones, which the individuals fail to discover. Indeed, if the public learns by experience that the banking system work well with low interests, its trust in the banking system would gradually increase, causing a shift from PL to PI-equilibria. Given an increasing number of deposits, the banks will start competing for the customers, and some of them (especially good ones, once the amount of their deposits reached \( \bar{n}^* \)) will want to raise the interest rate to \( \bar{s} \). As long as the public will trust this signal, as in PH-equilibria, and inasmuch as assumption 1 holds, this would cause further increase of deposits, followed by the bad banks as long as \( n(\bar{s}) > \bar{n}^* \). Note that there will be no separating equilibrium with \( n(s) < \bar{n}^* \) for the bad and \( n(\bar{s}) \in (\bar{n}^*, \bar{n}^*) \) for the good bank because if the signal identifies the bank type with certainty, all individuals would be willing to invest in the good bank.
PH-equilibrium could have been destroyed had the number of deposits surpassed min(\(n', n\)). However, given riskiness of the projects of bad banks, this is rather unlikely to happen because frequent bankruptcies of such banks will quickly restore public’s beliefs that high signals correspond to bad banks. Such beliefs will cause a sharp outflow of deposits, and the money market will pour back to the PL equilibrium.

A natural question is: can this vicious circle be destroyed somehow? One answer is given by proposition 2, which establishes sufficient conditions for a separating equilibrium. This, however, requires a type \(\bar{\theta}\) gap in the opportunities available to bad banks. This gap, once not available at the outset, is unlikely to arise on its own. Another lucky instance is the case of \(n' < n^* < n\) which is illustrated on Figure 1c, where all deposits attracted by the high signal will be in hands of the good banks only. The figure reveals this case is rather exotic in that it involves very high differences between high and low levels of both projects and signals. By contrast, robustness of the PL-PI-PH cycle are further supported by the possibility of Ponzi strategies, which precludes prevalence of the S-equilibria by undermining public’s trust in the reliability of the banking system. These considerations are further confirmed by our simulations presented in the next section; yet first we supply some more relevant characteristics to the equilibria just defined.

The intuition that justifies the substitution of PH equilibria for the PI one is conveniently captured by the notion of intuitive criterion (Cho and Kreps, 1987). Fix an equilibrium \(((s^*, g^*); (I^*(s^*) \Pr(\theta|s^*))\)), and take the signal \(\hat{s} \neq s^*\). Let \(\tilde{\theta}\) be the type who would never want to send \(\hat{s}\) because this signal would always bring him less than the equilibrium message. Let \(\tilde{\theta}\) be the other type, for which the signal \(\hat{s}\) is preferred to the equilibrium signal \(s^*\) against any best response whenever the second player (receiver) upon observing this signal, assigns probability zero to type of the first player (sender) being \(\tilde{\theta}\). Formally,

\[
\text{Definition 3. An equilibrium } ((s^*, g^*); (I^*, \Pr(\theta|s^*)) \text{ is said to fail the intuitive criterion if} \\
\exists \tilde{\theta} \text{ and } \exists \hat{s} : & \pi(s^*, g^*|n(s^*), \tilde{\theta}) > \max_{n, g} \pi(s, g|n(s), \tilde{\theta}) \\
\exists \tilde{\theta} \text{ and } \exists \hat{s}(\tilde{\theta}) : & \pi(s^*, g^*|n(s^*), \tilde{\theta}) < \min_{n|\theta \neq \tilde{\theta}, g} \pi(s, g|n(s), \tilde{\theta}).
\]

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Proposition 5. Provided \( n(s) \in (n_1, n_2) \) and \( \frac{N(s)}{M(1-\varphi)} > \frac{N(s)}{M} \), PI-equilibria fail the intuitive criterion.

Proof. In definition 3 take \( \tilde{\theta} = \bar{\theta}, \bar{\theta} = \theta \) and \( \bar{s} = \bar{s} \). Type \( \bar{\theta} \) would never want to increase the signal from \( \bar{s} \) to \( s \) for \( n \in (n_1, n_2) \), hence any rise of the signal over that range could come only from the \( \bar{\theta} \) type. Holding this view, the public must believe that all banks signaling \( \bar{s} \) are good, and invest in them by assumption 4; the good banks want this to happen because of an increase of deposits over the PI-equilibrium by the condition \( \frac{N(s)}{M(1-\varphi)} > \frac{N(s)}{M} \).

Recall that PI-equilibria dominate PL ones for the banks; now proposition 5 implies that PI equilibria will be dominated by PH ones. However, the PH-equilibria are not sustainable either, if the bankruptcy rates for the \( \bar{g} \) projects are significantly larger for the bad banks than for the good ones. Even boundedly rational individuals, upon observing more frequent failures following the \( \bar{s} \) signal, will quickly revise their beliefs, destroying these equilibria. The above intuition can be captured by another equilibrium refinement, the perfect sequential equilibrium (Grossman and Perry, 1986). This refinement is based not on the specific conjectures about types, but on the entire belief sets for all possible histories of the game (even those that have not been reached) by interpreting each move of the informed party as a signal of its type, and updating own beliefs upon this signal. This definition can be stated as follows:

Definition 4. The equilibrium is called perfect sequential in a multi-period game if i) upon observing a given message \( s \), \( \exists \theta : \pi(s, g|n, \theta) > \pi(s', g|n, \theta), s' \neq s \), the receiver has to update his belief \( \Pr(B|s) \) that \( s \) has been sent by type \( \theta \) using Bayes’ rule; and ii) sender’s strategy should be the best response to any such belief.

Proposition 6. The PH-equilibria are not perfect sequential in the multiperiod game.

Proof. The public knows that both types of banks are present in PH equilibria, while holding erroneous beliefs that bad banks choose safe technologies. Repeated plays of this profile would result in more frequent bankruptcies of the banks sending \( \bar{s} \), forcing upward revision of beliefs \( \Pr(B|\bar{s}) \) up to the point that sending \( \bar{s} \) is not anymore optimal for either type of the banks.
An explanation to the circle PL–PI–PH–PL in terms of equilibrium refinements can be obtained using a single criterion, known as undefeated equilibrium and introduced by Mailath e.a. (1993). In our case it can be stated as follows:

**Definition 5.** The equilibrium \((s^*, g^*); (I^*, Pr(B|s^*, \theta))\) is said to be defeated by the equilibrium \((\hat{s}, \hat{g}); (\hat{I}, Pr(B|\hat{s}, \hat{\theta}))\) if there exists a disequilibrium message \(\hat{s} \neq s^*\) s.t. i) there exists a type \(\hat{\theta}\) which prefers to send this message; ii) type \(\hat{\theta}\) would prefer the latter (hat) equilibrium to the former (asterisk) one; and iii) beliefs of the receiver which justify strategy \(I^*\) are not consistent with the message \(\hat{s}\) sent by type \(\hat{\theta}\).

**Proposition 7.** Subject to assumption 3 and 4, if \(n(s) \in (n_1, n^*)\) PI-equilibria defeat PL-equilibria; if \(n(s) \in (n^*, \min(\bar{n}' , n'))\), PH-equilibria defeat PL-equilibria; and PL-equilibria defeat PH-equilibria provided bankruptcies of type \(\bar{\theta}\) banks are frequent enough to ensure \(v(\hat{s})(1 − Pr(B|\hat{s})) > v(\hat{w}) > v(\hat{s})(1 − Pr(B|\hat{s}))\).

**Proof.** Follows directly from the definition. Over the range \((n_1, n^*)\), strategy \(\hat{g}\) is superior to strategy \(g\), hence the banks will want to deviate from PL to PI, and individual beliefs that all banks sending low signal invest in \(g\) strategy are wrong. For the second part, recall that monotonicity ensures all banks prefer more deposits to less, hence both types of banks are willing to raise the signal over the range \((n^*, \min(\bar{n}' , n'))\), which is inconsistent with individual beliefs that all banks send low signals. For the last part, it suffices to notice that frequent bankruptcies of the banks sending \(\hat{s}\) become inconsistent with individual beliefs at PH-equilibria.

This proposition shows that neither of the pooling equilibria remain robust against some other pooling equilibrium, suggesting that the above vicious circle is likely to persist as a sequence of inefficient equilibria. It is easy to show that the S-equilibria will survive all these refinements — however, these are unlikely to communicate with the set of states which generate the above cycle. To find out the exact conditions under which the dynamics can evolve in this direction we have to consider explicitly the dynamics of individual beliefs in the deposit game.
4. The evolution of strategies

The dynamic we consider is analogous to Spence (1974), but constructed in the spirit of evolutionary games (Samuelson, 1998; Weibull, 1995). In developing it, we relax the assumption of substantially rational individuals who possess perfect foresight and calculation abilities in favor of less demanding assumption of bounded rationality in the sense of Simon (1955; 1978). Boundedly rational players are still willing to get as much as they can out of their scarce resources, but have only limited capacities to do so immediately or even reasonably quickly (perhaps because of price stickiness or inertia) in a dynamic environment.

In the literature several dozens of dynamic specifications for beliefs have been proposed. From the prescriptive viewpoint it is most attractive to consider a simple form, known as partial best response dynamics for the individuals (Fudenberg and Levine, 1999). In this dynamics, a fraction $\alpha$ of the individuals, drawn at random in every period, receives the learn draw, and adopts the strategy that has been most profitable in the past time period. In our application, not less important is the role of bankruptcies of some banks which serves as an additional and obvious reason for strategy change as given by (2).

We construct these dynamics in per bank terms, where each bank behaves according to (1), choosing the best signal at the beginning of each time period and the project upon collecting the deposits. Bankruptcies occur at random according to the strategy chosen and type of the bank. Recalling our original definitions, and assuming that uncertainty over the projects is additive with the spread $\eta(\theta)$, the probability of any given bank to be bankrupt is\textsuperscript{16}

$$\Pr(B|s) = \Pr[g(K + Dn(s)) + \eta(\theta) < c(n(s))]$$

(3)

for the $s = \{s, \bar{s}\}$ and $g = \{g, \bar{g}\}$ chosen by the individual banks. For the expected profit of the bank $\pi(s, g|n, \theta) = E_g(K + Dn(s)|\theta) - c(n, s)$ given its type $\theta = \{\theta, \bar{\theta}\}$ and the number of allotted deposits per bank, assume that $\eta$ is uniformly distributed on $\left[\frac{\pi(s, g|n, \theta)}{2}, \frac{3\pi(s, g|n, \theta)}{2}\right]$. The probability of bankruptcy given the signal $s = \{s, \bar{s}\}$ then becomes

\textsuperscript{16} Assuming here that the return on $g(K)$ is not random.
\[
\Pr(B|s) = F[c(n(s)) - Eg(K + Dn(s)|\theta)]
\]
\[
= \max \left[ 0, \frac{Dn(s)(1 - \zeta - s) - Eg(K + Dn(s)|\theta)}{\pi(s, g|n, \theta)} \right], \tag{4}
\]

where \( s = \{g, \bar{g}\} \), \( \zeta < 1 \) is a time-invariant transactional part of deposits’ dynamics, and \( F(\cdot) \) denotes the cumulative distribution of \( \eta \). Preference of investment in the bank that has sent signal \( g (\bar{g}) \) depends on this probability, which is itself a function of the bank type, project and signal. Random realization of this probability gives the frequency \( q_t(s|g, \theta) \) of factual bankruptcies of the banks sending each signal. Individuals then form their beliefs that the bank which sends signal \( s = \{g, \bar{g}\} \) is going to be bankrupt according to the following specification, which is a variant of the familiar fictitious play (Fudenberg and Levine, 1999):

\[
\mu_t(s) = \mu_{t-1}(s)\frac{m+1}{m} + q_t(s|g, \theta) \frac{1}{m} \tag{5}
\]

where \( \mu_t(s) = \Pr(B|s) \) is the current believed probability of bankruptcy and \( m \) is the length of individual memory. Decision about investment is the bank sending each signal is then governed by a simple rule:

\[
[1 - \mu_t(\bar{g})] (1 + \bar{g}) \geq [1 - \mu_t(g)] (1 + g). \tag{6}
\]

These beliefs, alongside with transaction demand and forced outflows of debtholders due to bankruptcies determine the dynamics of individuals playing each strategy. According to this criterion, let \( \delta(\bar{g}) \) be the indicator function of decision to invest in the banks with low signal, \( \delta(\bar{g}) \) — the analogous indicator for the high signal; \( \delta(\bar{g}) = \delta(\bar{g}) = 0 \) correspond then to the decision to withhold. From the current number of debtholders \( n_t(\bar{g}) \) deduce first the number of transactional withdrawals \( \zeta n_t(\bar{g}) \), and then from the remainder — the average number of withholders due to bankruptcies of the banks of both types that sent this signal, yielding \( n_t(\bar{g})\left(1 - \zeta - (1 - \zeta)\left[q_t(g|g, \bar{\theta})M(\bar{\theta}) + q_t(g|g, \theta)M(\theta)\right]\right) \), where \( M(\bar{\theta}) \) and \( M(\theta) \) are shares of the good and bad banks, respectively. The remaining debtholders can learn, i.e. decide to switch to the strategy they find more attractive according to (6). A fraction \( \alpha \) of all current investors and withholders receives the ’learn draw’, and revises their current strategy, adopting the one that has been most profitable,
while the remaining \((1 - \alpha)\) individuals play the same strategy as they did in the last period. Summing up these values, we obtain the following discrete-time dynamics for the investment in low and high signals:

\[
\begin{align*}
\text{n}_{t+1}(s) &= \text{n}_t(s) \left[1 - \zeta - (1 - \zeta)Q_t(s)\right] \times [1 - \alpha\text{n}_t(\bar{s})\delta(\bar{s})] \\
&\quad + \alpha\delta(s) \left[n_t(s) + \text{n}_t(w)\right] \\
\text{n}_{t+1}(\bar{s}) &= \text{n}_t(\bar{s}) \left[1 - \zeta - (1 - \zeta)Q_t(s)\right] \times [1 - \alpha\text{n}_t(\bar{s})\delta(\bar{s})] \\
&\quad + \alpha\delta(s) \left[n_t(s) + \text{n}_t(w)\right]
\end{align*}
\]

where \(Q_t(s) \equiv (q_t(s|g, \theta)M(\theta) + q_t(s|g, \bar{\theta})M(\bar{\theta}))\), and in the same way, \(Q_t(\bar{s}) \equiv (q_t(\bar{s}|g, \theta)M(\theta) + q_t(\bar{s}|g, \bar{\theta})M(\bar{\theta}))\). The remainder goes to withholders, namely

\[
\begin{align*}
\text{n}_{t+1}(w) &= \text{n}_t(w) \left[1 - \alpha(\delta(s) + \delta(\bar{s}))\right] \\
&\quad + \text{n}_t(s) \left[(1 - \zeta)(1 - q_t(s|g, \theta)M(\theta) - q_t(s|g, \bar{\theta})M(\bar{\theta}))\right] \\
&\quad + \text{n}_t(\bar{s}) \left[(1 - \zeta)(1 - q_t(\bar{s}|g, \theta)M(\theta) - q_t(\bar{s}|g, \bar{\theta})M(\bar{\theta}))\right].
\end{align*}
\]

This dynamics has to be coupled with that of the banks. In general form, assume that the fraction \(\beta\) of banks revises its strategies in every time period, and switches to the one of the four strategies which is more rewarding. For most of the paper we shall be considering the case when banks are closer to substantial rationality than individuals are — in particular, for now we assume \(\beta = 1\). The choice of strategy by all banks is assumed to take part in stages: first, after bankruptcy took place, the banks choose the signal to be sent in the next time period. This rule may be expressed in terms of shares of the banks sending low signal:

\[
\phi_t = \begin{cases} 
\phi_{t-1} + \beta(1 - \phi_{t-1}) & \text{if } (\bar{s}, g) = \arg\max_{\{g, \bar{s}\}} \pi \left[g(K + n(s)|\theta) - c(n(s))\right] \\
\phi_{t-1} - \beta(2\phi_{t-1} - 1) & \text{if } (s, g) = \arg\max_{\{g, s\}} \pi \left[g(K + n(s)|\theta) - c(n(s))\right]
\end{cases}
\]

with the complementary dynamics for the other signal, and all investment strategies and types. The investment strategy to be chosen is determined in the next period, after the individuals have reacted to the bank’s signal just sent.
This dynamics has been simulated in MATLAB 6.1 with the following parameter values characteristic of the banking system of Russia. As basic parameters, we have taken $M = 1000, N = 10000000, K = 1000, D = 1, \alpha = 2, \zeta = 0.1, \bar{s} = 0.05, \bar{s} = 0.15$. We have taken technologies of the power form $g = k(K + \theta n(s))^\gamma$ with scale parameter $k = 1.5$, type parameters $\theta = 3$ and $\theta = 2$ and values of $\gamma$ of .77, .88 and .80 for the good banks with risky projects, bad banks with risky projects and safe projects, respectively. We also set memory length $m = 8$ and let the share of good banks to be .65, again in line with the Russian experience.

The sample dynamics is presented in Figures 3–5. Figure 3 depicts the share of banks which send low and high signals, respectively. The shares are clearly zero-one, yet the good banks send low signals substantively more often than the bad banks do. This suggests that raising the signal might indeed be used as a mean to attract more deposits, which is yet more wanted by the bad banks. To confirm this intuition, consider the corresponding dynamics of depositors as presented on Figure 4. The dynamics is clearly cyclical, with all the outflows of deposits from the low-signaling banks (asterisks with solid lines) are immediately followed by the temporary inflows of deposits to the bad ones (circles with dashed lines). This increase, however, is always temporary, to reflect higher rates of bankruptcies, which corresponds to a change from PH to PL equilibrium, with substantial outflow of deposits. Restoration of publics’ credence, by contrast, is associated with low signals from the good banks which invest in risky (i.e. more rewarding) projects, corresponding to PI-equilibria. This is most clearly seen on Figure 5, which shows the projects chosen by bank types in accordance with the signals they send. The figure reveals that upward trends in deposits correspond exactly to the periods of investments in risky projects, just as required by the PI-equilibria. These periods, however, are always succeeded by the high signals in PH-equilibria, which, in turn, result in gradual increase of deposits followed by the next decline.

Our numerical model leads to a broad variety of dynamics; yet the general tendencies are quite robust across specifications, and also highly consistent with the actual cyclical dynamics of the deposit market. The cyclical character of deposit inflows are never substituted by the good S-equilibrium, implying that the banking systems in transition can be prone to a kind of underdevelopment trap. Their immanent properties, including asymmetric information with poor verifiability of the bank
quality, insufficient regulatory interference and vulnerability to the external shocks are all responsible for the failure of banks to serve as the true translators of the national savings into private domestic investment.

5. Conclusion

This paper develops a signaling model which has been set to capture the evolutionary process of the banking system in transition, incorporating several features peculiar of modern Russia. The main conclusion which follows from our analysis is that persistent failures of many transitional countries (first of all, Russia) to build efficient and reliable banking systems are not occasional nor a coincidence of unfortunate circumstances. These can be explained as purely equilibrium phenomena in a game under asymmetric information with different types of banks and boundedly rational individuals. We have considered several inefficient equilibria in this game, and explored the reason why the system fails to reach the Pareto-efficient (Riley) separating equilibrium. Numerical simulation presented in Section 4 supports the theoretical analysis: it implies that failure of the banking system to become a stable and efficient channel of money transmissions across the economy can arise simply because this system itself represents an indistinguishable mixture of good banks that are able to manage large credit resources, and bad banks that cannot, but have incentives to mimic the behaviour of the former. Under these conditions, any attempt of the good bank to enlarge the scale of its operations by raising its signal are effectively blocked by the bad ones, whose invasion results in an increasing number of bankruptcies, and quickly restores public’s mistrust in the system as a whole. As a result, the main prediction of the model is that the system continuously circulates across the set of recursive states corresponding to pooling equilibria of the PL, PI and PH type. By contrast, a systemic shift is required to switch from this circle to the Riley S-equilibrium which prevails in developed economies, and in which good (large) banks would be able to attract most of the nations’ savings and credit large-scale reliable projects, while bad (small) banks will limit themselves to local operations with own capital.

This failure can clearly be attributed to the absence of prudential regulation from the part of the Central Bank. One might argue that had the regulator been able to fulfil its duties in a proper way, the vicious cir-
cle of underdevelopment and mistrust could have been broken. However appealing in principle, this task is not easy to implement in practice. In Russia the Central Bank has to deal with more than 1300 commercial banks all (or almost all) of whom have developed skills of misleading and nontransparent bookkeeping, so typical of many firms in transition. This creates additional informational asymmetry, which is further aggravated by many personal ties between the private bankers and state bureaucrats backing them. And even though the banking system of Russia was growing quite rapidly in 2000–2005, the mini-crisis of credibility which took place in the Summer 2004 persuasively shows that the persistence of this pattern should not be overestimated. The present-day growth of the banking system is largely due to the inflow of income from oil exports, and is rather caused by the state of capital account rather than by the fundamental changes in the structure of domestic savings.

At the same time, experience of successful banking system of the developed countries suggests that there must be some way out, and finding one is an interesting policy question. Our analysis does is directly aimed at finding these measures, yet it can lead to some insights into what these can be. First, good (large) banks can be stimulated to participate in large-scale projects — by means of, e.g., tax policy or cheaper credits issued by the Central Bank. At the same time, bad (small) banks should face a limited window of opportunities to work with debtholders’ money. In our model, this move would correspond to raising the costs of attracting deposits of some banks by a flat tax on the number of branches, leaving it to the banks to decide whether they want to pay it for extra credit resources, or just forget it. This measure, however unpopular it might be among the bankers, could launch the self-selection mechanism that would make it more difficult for the bad banks to follow the good ones in the PH equilibria. Yet another, and perhaps the most important way could consist of a combination of planned enforcement of public’s trust in the banking system, followed by a sharp increase in the attractiveness of deposits in the large banks which could attract most of the country’s deposits and realize their scale economies. This solution, however, would clearly require discrimination of the various types of banks by the regulator, and probably require some form of subsidies. All these measures might be problematic from economic policy viewpoint; yet it seems that without such measures the inefficient banking systems are likely to stay in the vicious underdevelopment circle for a very long time.
### Tables and figures

#### Table 1. Comparison of the US and the Russian banking system (end of year data)

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>US, bln US$</td>
<td>Russia, bln RUR</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total assets</td>
<td>6436.5</td>
<td>6964.1</td>
<td>7324.7</td>
<td>3155.9</td>
<td>4015.1</td>
<td>5600.6</td>
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<td>Loans + investments</td>
<td>5438.8</td>
<td>5895.0</td>
<td>6253.3</td>
<td>2010.2</td>
<td>2659.7</td>
<td>3750.4</td>
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<tr>
<td>including loans to nonbanking sector</td>
<td>3942.4</td>
<td>4170.8</td>
<td>4396.8</td>
<td>1286.1</td>
<td>1711.0</td>
<td>2599.6</td>
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<tr>
<td>Deposits (time + transactional)</td>
<td>4226.0</td>
<td>4486.5</td>
<td>4742.2</td>
<td>1272.7</td>
<td>1401.3</td>
<td>2518.4</td>
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<tr>
<td>GDP</td>
<td>10100.8</td>
<td>10480.8</td>
<td>10987.9</td>
<td>8943.6</td>
<td>10834.2</td>
<td>13304.7</td>
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<tr>
<td>Personal income</td>
<td>8713.1</td>
<td>8910.3</td>
<td>8293.7</td>
<td>5293.5</td>
<td>6698.2</td>
<td>8749.2</td>
</tr>
<tr>
<td>Personal savings</td>
<td>127.2</td>
<td>183.2</td>
<td>170.0</td>
<td>475.2</td>
<td>707.7</td>
<td>1058.7</td>
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<tr>
<td>Total assets / GDP</td>
<td>0.64</td>
<td>0.66</td>
<td>0.67</td>
<td>0.35</td>
<td>0.37</td>
<td>0.42</td>
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<tr>
<td>Loans+Investments / Deposits</td>
<td>1.29</td>
<td>1.31</td>
<td>1.32</td>
<td>1.58</td>
<td>1.90</td>
<td>1.49</td>
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<tr>
<td>Loans to nonbanking sector/Deposits</td>
<td>0.93</td>
<td>0.93</td>
<td>0.93</td>
<td>1.01</td>
<td>1.22</td>
<td>1.03</td>
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<tr>
<td>Loans+Investments / GDP</td>
<td>0.54</td>
<td>0.56</td>
<td>0.57</td>
<td>0.22</td>
<td>0.25</td>
<td>0.28</td>
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<tr>
<td>Loans to nonbanking sector / GDP</td>
<td>0.39</td>
<td>0.40</td>
<td>0.40</td>
<td>0.14</td>
<td>0.16</td>
<td>0.20</td>
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<td>Deposits / Personal income</td>
<td>0.49</td>
<td>0.50</td>
<td>0.57</td>
<td>0.24</td>
<td>0.21</td>
<td>0.29</td>
</tr>
<tr>
<td>Deposits / Personal savings</td>
<td>33.22</td>
<td>24.49</td>
<td>27.90</td>
<td>2.68</td>
<td>1.98</td>
<td>2.38</td>
</tr>
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</table>


#### Table 2. Characteristics of the Russian banking system, end of 2002, bln. RUR

<table>
<thead>
<tr>
<th>Banks, ordered by assets</th>
<th>1 to 20</th>
<th>21 to 50</th>
<th>51 to 200</th>
<th>201 to 1000</th>
<th>1000 to 1332</th>
<th>Total</th>
<th>Share in assets</th>
</tr>
</thead>
<tbody>
<tr>
<td># of branches</td>
<td>1549.0</td>
<td>327.0</td>
<td>602.0</td>
<td>773.0</td>
<td>78.0</td>
<td>3329.0</td>
<td></td>
</tr>
<tr>
<td>Credits issued to nonbanking sector</td>
<td>1272.8</td>
<td>230.1</td>
<td>295.7</td>
<td>189.0</td>
<td>4.5</td>
<td>1992.0</td>
<td>0.50</td>
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<tr>
<td>including credits to firms</td>
<td>1014.1</td>
<td>188.3</td>
<td>219.0</td>
<td>141.0</td>
<td>2.9</td>
<td>1565.4</td>
<td>0.39</td>
</tr>
<tr>
<td>to individuals</td>
<td>86.3</td>
<td>5.8</td>
<td>27.5</td>
<td>24.9</td>
<td>1.3</td>
<td>145.8</td>
<td>0.04</td>
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<tr>
<td>to banks</td>
<td>105.6</td>
<td>31.0</td>
<td>38.8</td>
<td>18.1</td>
<td>0.1</td>
<td>193.8</td>
<td>0.05</td>
</tr>
<tr>
<td>Securities</td>
<td>352.2</td>
<td>26.0</td>
<td>19.7</td>
<td>11.6</td>
<td>0.3</td>
<td>409.8</td>
<td>0.10</td>
</tr>
<tr>
<td>Business accounts and deposits</td>
<td>318.2</td>
<td>87.7</td>
<td>127.4</td>
<td>106.8</td>
<td>3.2</td>
<td>643.2</td>
<td>0.16</td>
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<tr>
<td>State funds held</td>
<td>77.8</td>
<td>13.9</td>
<td>10.2</td>
<td>6.1</td>
<td>0.1</td>
<td>108.1</td>
<td>0.03</td>
</tr>
<tr>
<td>Individual deposits</td>
<td>819.6</td>
<td>45.2</td>
<td>71.4</td>
<td>60.6</td>
<td>1.4</td>
<td>998.4</td>
<td>0.25</td>
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<tr>
<td>including Sberbank (on 1.01.2002)</td>
<td>489.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>489.0</td>
<td>0.00</td>
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<tr>
<td>deposits excluding Sberbank</td>
<td>330.8</td>
<td>45.2</td>
<td>71.4</td>
<td>60.6</td>
<td>1.4</td>
<td>509.4</td>
<td>0.13</td>
</tr>
<tr>
<td>equity</td>
<td>236.3</td>
<td>82.5</td>
<td>131.4</td>
<td>115.5</td>
<td>4.9</td>
<td>570.6</td>
<td></td>
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<tr>
<td>all assets</td>
<td>2508.8</td>
<td>459.8</td>
<td>593.8</td>
<td>440.1</td>
<td>12.6</td>
<td>4015.1</td>
<td></td>
</tr>
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</table>

Figure 1a. Expected returns, costs of credit resources and pooling equilibria
Figure 1b. Separating equilibrium with gap
Figure 1c. Separating equilibrium with gap, case II
Figure 2. Timeline of each period

- Signals sent by the banks
- Individuals observe signals and make deposit decisions
- Realization of random returns
- Payoff collection given factual bankruptcies
- Revision of all beliefs given the experience
- Banks receive deposits and choose investment
Figure 3a. Shares for good banks
- Banks with low signal
- Banks with high signal

Figure 3b. Shares for bad banks
- Banks with low signal
- Banks with high signal
Figure 4. Number of investors

- Investors in low signal
- Investors in high signal
- Withholders
Figure 5a. Shares of projects, low-signaling banks

Figure 5b. Shares of projects, high-signaling banks
References


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Публикуемые в серии работы были представлены на научных семинарах, организованных в МИЭФ в рамках научной программы МИЭФ — ГУ ВШЭ, координируемой Международным академическим комитетом МИЭФ. Программа реализуется с 2003 г. при участии Директора проекта МИЭФ со стороны Лондонской школы экономики и политических наук профессора Ричарда Джекмана и старшего академического советника Амоса Витцума.

Papers published in this series were presented at the ICEF research seminars within the frame of its research programme coordinated by the International Academic Committee of ICEF. The programme has been implemented since 2003 and supervised by ICEF Project Director at LSE Professor Richard Jackman and Senior Academic Advisor Dr. Amos Witztum.

Препринт WP9/2005/01
Серия WP9
Исследования по экономике и финансам
Research of economics and finance

Белянин Алексей Владимирович

Эволюция рынка частных сбережений: теоретико-игровой анализ
(на английском языке)

Публикуется в авторской редакции

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Редактор О.А. Шестопалова
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