Rapid Forecast of Tsunami Runup using Shallow-water Modeling of Tsunami Propagation in the East (Japan) Sea

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ABSTRACT


A rapid forecast method is suggested to predict tsunami wave characteristics at the shore. Extensive computer resources and time have been required to directly compare numerical simulations of tsunami wave propagation from the source to the shore; this new method matches two different fast numerical procedures. During the first stage, 2D nonlinear shallow-water equations were applied to compute tsunami wave propagation in the open sea up to the coastal zone at a depth of 10 m; the fast computing can be done on rough grids of 1 km resolution. In fact, numerical simulations were performed in the domain bounded by a vertical wall at the 10 m depth. During the second stage, the wave water oscillations on the wall were re-computed on the shore using 1D shallow-water equations. This 1D numerical procedure is also fast and does not require extensive computer resources. This procedure was applied here to estimate the runup wave heights during the 1983 tsunami event and some prognosticated events in the East (Japan) Sea.

ADDITIONAL INDEX WORDS: Tsunami runup height, Rapid forecast, Analytical theory of long wave runup, Shallow-water system, Numerical modeling, Tsunami characteristics along the East Korean Coast.

INTRODUCTION

Computing wave runup on the coast is an important part of short and long-term predictions of tsunami hazards. Existing methods of tsunami risk assessment, such as the PTVA method, include the precise, high-resolution computation of tsunami flow on dry land that accounts for building and street locations, bush and forest on the coast and other factors influencing wave propagation. In the simplest runup simulation variant, the structures on the coast are modeled by the different roughness of the dry land (Gayer et al., 2010). The results of such computations are used to estimate tsunami risk in various areas of the world's oceans, including the Indian Ocean after a catastrophic 2004 tsunami (Dominney-Howes and Papathoma, 2007), the U.S. Pacific Northwest coastline and Acapulco, Mexico (Geist and Parson, 2006), Seaside, Oregon (Gonzalez et al., 2009), Sydney, Australia (Dall’Ozzo et al., 2009) and the Aeolian Islands, Italy (Dall’Ozzo et al., 2010). Runup computations are also important for estimating casualties due to tsunami inundation flow (Koshimira et al., 2006).

Due to the significant time required for runup computations, the total time for tsunami modeling from the source to the coast is prohibitive, and direct numerical models are difficult to use for short-term prediction of tsunami characteristics in operational practice. For instance, in the East (Japan) Sea, a tsunami usually arrives within 100 minutes at the Korean eastern coastline; thus, the runup height in coastal zones must be predicted within a very short time before the tsunami arrives. The database model (Korea Meteorological Administration) with a vertical wall assumption can provide warning messages, but there is low accuracy for predicting tsunami runup height. To increase the computational accuracy for tsunami propagation, various nested methods with different mesh resolution can be applied (Choi et al., 2003; Roger and Hebert, 2008; Roger et al., 2010).

In contrast to numerical methods, the analytical theory of long wave runup on the inclined beach is well developed (Carrier and Greenspan, 1958; Pedersen and Gjevik, 1983; Synolakis, 1987; Kaistrenko et al., 1991; Synolakis, 1991; Pelinovsky and Mazova, 1992; Tadeppali and Synolakis, 1994; Carrier et al., 2003; Kanoğlu, 2004; Tinti and Tonini, 2005; Kanoğlu and Synolakis, 2006; Didenkulova et al., 2006, 2008; Didenkulova and Pelinovsky, 2008; Madsen and Fuhrman, 2008; Didenkulova, 2009). The combination of numerical modeling for tsunami in the open sea and coastal zones with an analytical approach to describe the runup stage can assist in improving rapid forecasting of the tsunami runup height with no significant increase in computation time. In fact, such a combination of numerical and analytical methods has been used to roughly estimate tsunami runup heights (Choi et al., 2002a; Ward and Aspbaugh, 2003). In these previous reports, the shape of the incident wave was assumed to be a fixed sine wave or solitary wave (soliton).

The main purpose of this paper is to describe a developed, combined model based on numerical simulation far from the coast and analytical “runup” solutions for the rapid prediction of tsunami coastal characteristics that could prevent tsunami disasters. The runup heights are estimated with the use of the analytical theory of long wave runup on the plane beach. Numerical simulation of the 1983 tsunami event in the East (Japan) Sea is
performed, and the runup heights are calculated using the combined model that was developed.

In addition to historical tsunamiic earthquakes, possible tsunami scenarios were also considered. These scenarios include three hypothetical tsunamiic earthquakes located in the gap zones of the seismic map. According to The Headquarters for Earthquake Research Promotion (2004), possible earthquakes predicted for the next 30 years include one with a magnitude of more than 7.5 in northwest Hokkaido (magnitude 7.8, occurrence probability 0.006-0.1%), one north of Sado Island (magnitude 7.8, occurrence probability 3-6%) and one in the frontal area of Akita (magnitude 7.5, occurrence probability 3%). These three cases were considered in this study and are referred to as Hypoth1, Hypoth2 and Hypoth3, respectively (Table 1, Figure 3). Figure 1 shows the location of historical and three hypothetical tsunami sources, and Figure 2 presents the initial water displacements at the sources of historic and prognosticated events.

Figure 1. Topography and bathymetry of the East (Japan) Sea, which is the location of the initial tsunami source.

Figure 2. Initial water elevations of the 1983 tsunami event and three hypothetical tsunami events.

ANALYTICAL SOLUTION FOR THE RUNUP STAGE

Rigorous solutions for the 1D nonlinear shallow-water equations that describe the long wave runup were obtained for a plane beach using only the Carrier–Greenspan transformation (Carrier and Greenspan, 1958), and existing literature is cited in the Introduction. Significantly, here the linear and nonlinear theories predict the same maximum values for runup height if the incident wave is determined far from the shore. This result means that the linear theory can be applied to compute the maximal runup heights. The basic equation of the linear theory is the wave equation for water level displacement, \( \eta(x, t) \):

\[
\frac{\partial^2 \eta}{\partial t^2} = \frac{\partial}{\partial x} \left[ g h(x) \frac{\partial \eta}{\partial x} \right]
\]

(1)

where \( g \) is the acceleration due to gravity and \( h(x) \) describes the bottom profile. This equation can be solved analytically or numerically not only for a plane beach as in nonlinear theory but also for an arbitrary bottom profile. In the laboratory, runup modeling is usually studied for the plane beach combined with a flat bottom (Figure 3). In this case, the incident and reflected waves are separated above the flat bottom, and equation (1) can be solved analytically. In the case of runup for a sine wave (Shuto, 1972; Madsen and Fuhrman, 2008; Didenkulova, 2009):

\[
\frac{R}{A} = \frac{2}{\left(J_0^2(2kL) + J_1^2(2kL) \right)^{1/2}}
\]

where \( R \) is the maximal runup height, \( A \) is the incident wave amplitude, \( L \) is the beach width, \( k \) is the wave number of the incident wave and \( J_0 \) and \( J_1 \) are Bessel’s functions. The amplification factor \( R/A \) is shown in Figure 4. The oscillation of runup height with wave number is weak (weak shelf resonance), and curve (2) can be approximated by:

\[
\lambda_{\text{max}} = \frac{2\pi}{\lambda_0} \sqrt{\frac{2L}{\lambda_0}}
\]

(3)

where \( \lambda_0 \) is the wavelength of incident wave (shown in Figure 4 by dashed red line). The same formula can be obtained from nonlinear theory of long wave runup in the limited case of a large \( kL \).

The general solution for equation (1) can be found using the Fourier transformation:

\[
\tilde{\eta}(x, \omega) = \frac{1}{2\pi} \int \tilde{\eta}(x, \omega) \cdot e^{-i\omega t} \, d\omega
\]

(4)

and the water displacement of the shoreline is calculated using:

\[
R(t) = \frac{1}{\sqrt{\pi}} \int \left( \frac{\omega}{g} \frac{\lambda_0}{\lambda_0} \right)^{1/4} A(\omega) \cdot e^{-i\omega t} \, d\omega
\]

(5)

where we used the amplification ratio (3), which was re-written through the wave frequency.

Using the described equations, we can suggest one rapid estimation of runup height. During the first stage, the wave field is computed in the open sea with “no-flux boundary condition” walls. The wave field is then separated into the sum of incident and reflected waves, and the spectral amplitude of the incident wave \( A(\omega) \) is calculated using the inverse Fourier transformation. Finally, \( R(t) \) is computed using equation (5). This approach can be applied if the bottom near the “wall” is almost constant, which can be observed during the procedure for the selection of incident and reflected waves.

In real sea basins, the “computed wall” is located on the inclined bottom (Figure 5). In this case, another procedure to analytically compute the runup height can be suggested. If the bottom profile can be approximated by the linear function such that \( h(x) = \alpha x \) (\( \alpha \) is the bottom slope) everywhere (on both sides of the computed wall), the spatial structure of the monochromatic wave is described by the Hankel functions:

\[
\tilde{\eta}(x, \omega) = B(\omega) \cdot H_0^{(1)}(\zeta) + A(\omega) \cdot H_0^{(2)}(\zeta), \quad \zeta = 2\alpha x \sqrt{g/\omega}.
\]

(6)
Table 1: Fault parameters.

<table>
<thead>
<tr>
<th>Case</th>
<th>Location (E) (N)</th>
<th>H (km)</th>
<th>θ (°)</th>
<th>δ (°)</th>
<th>λ (km)</th>
<th>L (km)</th>
<th>W (km)</th>
<th>D (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1983</td>
<td>138.8 40.2</td>
<td>2</td>
<td>22</td>
<td>40</td>
<td>90</td>
<td>40</td>
<td>30</td>
<td>7.6</td>
</tr>
<tr>
<td>Hypoth1</td>
<td>139.0 40.5</td>
<td>3</td>
<td>22</td>
<td>40</td>
<td>90</td>
<td>40</td>
<td>30</td>
<td>30.05</td>
</tr>
<tr>
<td>Hypoth2</td>
<td>139.1 45.8</td>
<td>5</td>
<td>180</td>
<td>60</td>
<td>105</td>
<td>140</td>
<td>22</td>
<td>12</td>
</tr>
<tr>
<td>Hypoth3</td>
<td>139.0 39.9</td>
<td>5</td>
<td>185</td>
<td>60</td>
<td>105</td>
<td>139</td>
<td>29.5</td>
<td>12</td>
</tr>
</tbody>
</table>

Figure 3. Usual geometry of the laboratory tank.

Using the asymptotic of the zero-order Hankel functions for large values of the argument, the second term in (6) clearly corresponds to the incident wave with amplitude A (propagated onshore), and the first term clearly corresponds to the reflected wave with amplitude B (propagated offshore). We assume that the characteristics of the incident wave are known; therefore, A(ω) can be found through inverse Fourier transformation of the incident wave.

To first analyze the wave runup on a vertical wall (Figure 1), the boundary condition on the wall is defined as:

\[
\frac{d\tilde{\eta}}{dx} \bigg|_{x=L} = 0
\]

which leads to:

\[B(\omega) \cdot H_1^{(1)}(\omega l) + A(\omega) \cdot H_1^{(2)}(\omega l) = 0,\]

where T is the travel time of the wave from the wall to the shore:

\[T = \frac{2l}{\sqrt{H}}\]

In equation (6) for \(x = L\) and equation (8) can be considered to be a system of equations to find amplitudes of the incident wave:

\[A = \frac{\tilde{\eta}(L)H_1^{(1)}(\omega l)}{H_0^{(1)}(\omega l)H_1^{(2)}(\omega l) - iH_1^{(1)}(\omega l)H_0^{(2)}(\omega l)}\]

in particular, if the wall is located far from the shore (\(\omega l >> 1\)), equation (10) is simplified to

\[A(\omega) = \frac{\omega l}{4} \tilde{\eta}(L, \omega)\]

thus, expressions (10) or (11) can be used to compute the spectral amplitudes of the incident wave through the Fourier spectrum of the water oscillations on the "computed wall".

To determine the wave runup on the same plane beach with no vertical wall, the incident monochromatic wave is assumed to have the same amplitude A(ω) as in the "wall" problem. In this case, the bounded (on the shore) solution of the wave equation is

\[\tilde{\eta}(x, \omega) = 2 \cdot A(\omega) \cdot J_0(\omega)\]

Elimination of A(ω) from (1) in equation (12) allows the calculation of the wave field on a plane beach versus the wave oscillation on the wall. By applying the Fourier transformation, the integral relation between runup oscillations and water oscillations on the wall can be obtained from:

\[R(x = 0, \tau) = \frac{\tilde{\eta}(L, \tau)}{\sqrt{(i-t)^3 - T^3}} \cdot \frac{d^3\tilde{\eta}(x = L, \tau)}{dt^3} d\tau\]

In equation (13), \(t = 0\) corresponds to the wave approaching the vertical wall, and in the initial moment, it is assumed that \(\tilde{\eta}(x = L, t = 0) = d\tilde{\eta}(x = L, t = 0)/dt = 0\). According to nonlinear theory, the extreme of \(R(x = 0, \tau)\) yields the maximal runup height of the tsunami wave on the coast.

Considering the two geometries of the nearshore: a plane beach (for the runup study) and a plane beach with a vertical wall at fixed depth (equivalent boundary condition), it has been shown that the runup height can be expressed through the characteristics of the water oscillations on the vertical wall. This procedure is rapid and simple using computers. As a result, the numerical simulation of the tsunami in the basin with a nearshore vertical wall can be used for tsunami runup estimations, although 1D theory could not reproduce the local bottom effects (refraction, focusing, etc.). The developed analytical approach was subsequently used in estimations of the tsunami runup heights for the 1983 event and three hypothetical events in the East (Japan) Sea.

Figure 4. Runup ratio of a tsunami in the case of the bottom configuration presented in Figure 3.

NUMERICAL MODEL

All of the available topographic data were compiled along with digitizations of hydrographic charts, including Chinese and Russian navigation charts and Korean hydrographic charts, to produce a 1-min, gridded topography and bathymetry dataset for the region located between 117°–143° East longitudes and 24°–52° North latitudes (Figure 2) with refined 1-arc-second topography and bathymetry west of 135° East longitudes (Choi et al., 2002b). The earthquake parameters were determined previously (Aida, 1984). The initial surface profile of the source was determined by the method of Manshina and Smylie (1971).

The finite-difference model (Choi et al., 2003) was constructed to simulate tsunami generation and propagation using the linear shallow-water equation with a spherical coordinate system covering the East Sea area: mesh dimensions, 959 × 1118; mesh size, 1 angular min; and time step, 2 sec. The basic equations are:

\[\frac{\partial \eta}{\partial t} + \frac{1}{R \cos \phi} \left( \frac{\partial P}{\partial x} - \frac{\partial (Q \cos \phi)}{\partial \phi} \right) = 0,\]

\[\frac{\partial P}{\partial t} + \frac{gh}{R \cos \phi} \frac{\partial P}{\partial x} - fQ = 0,\]

\[\frac{\partial Q}{\partial t} + \frac{gh}{R \cos \phi} \frac{\partial \eta}{\partial x} - fP = 0.\]
In the equations above, \( \phi \) and \( \chi \) are the latitude and longitude, respectively, \( P \) and \( Q \) are discharge per unit width in the direction of \( \phi \) and \( \chi \), respectively, \( R \) is the radius of the earth and \( f \) is the Coriolis parameter.

Boundary conditions near the coast are "no-flux boundary conditions"; the normal component of the velocity or flow discharge to the boundary is zero, which corresponds to the vertical wall as the central location of the last sea points. In addition, the bottom profile can be approximated by the linear function of \( h(\phi, \chi) \) as the last sea point depth; \( h(0) \) can be assumed at the coastline (depth = 0). Here, \( dx \) indicates the computational grid size of the numerical model.

This numerical shallow-water model was used previously in our study of a tsunami in the East (Japan) Sea (Choi et al., 2002a, 2003). As a result, the water displacement in the last sea points is calculated, and the maximal runup heights are then estimated with use of the integral (13).

**NUMERICAL RESULTS**

The tsunami that occurred in 1983 is well documented for the Eastern Korean Coast (Choi et al., 1994; Choi et al., 2002a), and this event was used to check the applicability of the developed, combined model. Figure 7 displays the time series of the water displacement along the vertical wall that was computed by the finite-difference model (dashed line) and the displacement of the water on the shore (solid line) that was calculated using the analytical solutions for several coastal locations (Figure 6). The amplification factor for the tsunami nearshore is about 2-2.2. The observed maximal runup heights in these locations are given by numbers as well as dash-point lines. The runup heights at Gungchon and Onyang beach were underestimated, but the runup heights at Geunduk and Yongwa were underestimated. The most critical reason of this disagreement was that the observed records were based on the testimony of eyewitness ten years later on the 1983 tsunami occurred.

The maximum wave heights and runup heights for three hypothetical tsunamis were estimated to provide useful information for tsunami disaster prevention. In the case of hypothetical tsunamis, we focused on the beach along the Eastern Korean Coast with a theoretical approximation suitable for a gentle slope at the beach. Figure 6 shows the results of the three hypothetical cases; square, triangle and circle symbols represent the maximum water level of oscillation on the 'wall' from the numerical results, and lines (dash, solid and gray dash) represent the maximal runup height obtained from the analytical solution. Based on the simulation results, the regions where tsunami heights are relatively larger than at other beaches can be summarized. These regions are Jungdongjin (almost 7 m) and Imwon (over 8 m) beaches. In addition, hypothetical case 1 is significantly affected at the Eastern Korean Coast although its occurrence probability is 0.006-0.1%. Hypothetical case 2 is significant in terms of runup height and its occurrence probability (3-6%).

![Figure 5. Schematic presentation of the coastal zone.](image)

![Figure 6. Maximum runup height at each beach for the three hypothetical tsunami events.](image)

![Figure 7. Computed time series of water displacement on the vertical wall (dash line) and on the shore (solid line). Observed runup heights for the 1983 event are shown by the dash-point line.](image)
CONCLUSION

A rapid forecast method for tsunami runup on coastlines is proposed based on the combined use of a 2D numerical model and a 1D analytical runup theory. During the first step, the 2D numerical simulations of tsunami generation and propagation are performed using no-flux boundary conditions on the last sea points at a depth of 5-10 m (equivalent wall). The time-series of the water oscillations on the wall are then re-calculated for the runup heights using the analytical integral expression derived from 1D theory. The applicability of this approach was checked for the historic 1983 tsunami event in the East (Japan) Sea.

LITERATURE CITED


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