Priorities of Teaching Mathematics in Universities

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ABSTRACT

The research urgency is caused by necessity to build a clear hierarchy of objectives of teaching mathematics for different groups of professions (technical, economic, humanitarian) in Universities. Defining priorities allows focusing on the most important goals of teaching mathematics. In this regard, this paper is focused on the calculation of numerical values of priorities of the following strategic objectives in teaching mathematics: the formation of mathematical competences for the solution of professional tasks; the formation of logical thinking; the education of mathematical culture in teaching mathematics. These are the very goals which determine the mission of teaching mathematics in high school. A leading approach to the study of such problems is a method of hierarchies’ analysis by Thomas Saaty which allows bringing the qualitative (linguistic) expert assessment of the objectives’ importance into the quantitative values of their priorities. The author has modernized this method, organized the process of peer-review assessment of the objectives’ importance in teaching mathematics and first got the numerical values of the priorities, showing the importance of each of the objectives to achieve the goal of teaching mathematics in high school. The materials of the article are of practical value for teachers of mathematics, heads of departments of mathematics and academic managers in higher education institutions, since the hierarchy of objectives allows highlighting of the most important ones in the development of curricula, programs of mathematical disciplines and teaching methods.

KEYWORDS
Teaching; mathematics; university; goals; priorities

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Introduction

Requirements for specialist in a postindustrial society

Higher education should provide graduates with the opportunity successfully to tackle the current professional challenges and be prepared for additional knowledge to solve new problems. This is especially important in modern post-industrial era. There is a reduction of the product life cycle. In the sectors of high technology the life cycle of a product continues from a year and a half to
three years. 30 years ago it was 10-15 years. The company needs to have at the "output" new products before competitors. The business became limited in time. Products and technologies are becoming more complex. The volume of knowledge is growing constantly required for their production. Accordingly, an increasing amount of knowledge is necessary for the graduate of technical specialties that include information technology. However, in the competitive environment customers need not an abstract product, but the product that meet their requirements at a greater extent. The most important thing is not to produce goods, but sell it. This requires good skills from marketers and other graduates of economic specialties. Moreover, information about new products and services is distributed around the world almost instantly. Large companies simultaneously carry out hundreds of development projects for new products and services. There is a need in the new organizational culture, and the current graduates of management training have to create it. Information technology and biotechnology are developing so rapidly that humanity is faced serious moral problems. Graduates of the Humanities have to notice them timely and to recommend actions that will prevent the tragic development of events (Maron, 2010). All these issues must be taken into consideration by teachers of modern high school, (Salmi & Frumin, 2007; Radaev, 2008; Yumuşak et al., 2016; Pérez & Furman, 2016; Boran & Bağ 2016; Bilici, 2016). It is clear that in modern conditions the professional can not only rely on the knowledge acquired in his student years. Continuous training is needed (Filonovich, 2009). It should be built to meet the challenges of employers (Shaidullina, Merzon & Zakirova, 2015) and to provide knowledge needed to solve urgent problems. This concerns all forms and methods of organization of the postgraduate education (Masalimova & Sabirova, 2014).

The mission and purpose of teaching mathematics in university

The successful assimilation of the new is possible only on the basis of fundamental knowledge acquired at the University. Mathematics plays a special role in building such a Foundation. A well-known Russian mathematician Mikhail Vasilievich Ostrogradsky (1801 – 1862) pointed out that the mission of teaching mathematics in special educational institutions is to give knowledge necessary for studying of professional disciplines, and develop the mindset and mental skills of students (Gnedenko & Maron, 1958). It is obvious that professional disciplines are determined by the direction of the students' training and based on different sections of higher mathematics. Accordingly, the curriculum of mathematics in university should be profiled. The preparation of such curricula and the justification for the inclusion in them of different areas of mathematics, a lot of work is dedicated (Vygodsky, 1984), (Zimina, 2005), (Kornilova & Tikhomirov, 1990). Also the teaching methods are discusses in details. Almost all the authors mention the need to take into account the dual nature of the mission of mathematics - to provide knowledge and learn to think. It should be noted that another great Russian mathematician, teacher and organizer of science Nikolay Ivanovich Lobachevsky said about it (1796 – 1856). He believed that in teaching mathematics should be: a systematic and scientific rigor of the presented material; availability to students; the development of thinking in the study; accounting by the teacher of the age and individual characteristics of learners (Alexandrova, 1976), (Nagaeva, 1948) (Lobachevsky, 1956), (Modzalevsky, 1948). Of course, not every teacher can implement these
contradicting to each other to a certain extent principles. Not everyone can achieve the heights of teaching that have been reached by N.I. Lobachevsky and M.V. Ostrogradsky. Nikolay Ivanovich Lobachevsky was the rector of Kazan University. Thanks to his organizational talent the University of Kazan became one of the best universities of Russia and stays at this level until now. As rector, he read in his University almost all the basic courses of higher mathematics and many of his students glorified Russia (Sindalovsky, 1902). Mikhail Vasilievich Ostrogradsky in St. Petersburg, read public lectures, as we would say today – carried out informal supplementary education for persons interested in mathematics. As one of the listeners of these lectures wrote in the “Journal of communications” in May – June 1841, the lectures were attended by up to 61 students, “53 of which were in epaulets”, even the great Cauchy could not gather in Paris more than 20 listeners.

Curriculum and methods of its implementation is a means of achieving the objectives of teaching mathematics. In accordance with the provisions of modern strategic management, they must be selected after allocation of clear objectives and determination of their priorities as to achieve all at once with limited resources is impossible. Based on the mission of mathematics it is necessary to formulate the goals of teaching in the following way.

1. The formation of mathematical competences for the solution of professional tasks
2. The formation of logical thinking
3. The education of mathematical culture.

It is better to do the necessary clarifications. The first objective involves not primarily the direct application of mathematics to solve practical problems, such knowledge is given by secondary school, but knowledge of mathematical methods used in specialized disciplines of relevant specialty. Under mathematical culture is understood, first and foremost the following.

1.1. Requirements to the clear statement of the problem.
1.2. The requirement to justify the solution.
1.3. Awareness of the limits of applicability for mathematical methods.

The relative importance of objectives is different for different specialties: technical, economic, and humanitarian. So the technicians should first be given knowledge for solving professional tasks, as many tasks before them can be successfully solved by mathematical methods. The tasks of the historians cannot be solved by mathematical methods, and it is more important for them the formation of logical thinking on the basis of the studied documents. It should be noted that the limitations of mathematical methods is evident today not only in the humanitarian sphere. Modern technology has overtaken the development of mathematics. The functioning of a number of high-tech products cannot be adequately described by modern mathematical methods. This fact even led scientists in the second third of the twentieth century to talk about the crisis of mathematics.

The need to prioritize the goals of teaching mathematics in university

The majority of Russian Universities have already passed to two-level education system: bachelor, master. Mathematics is studied in Baccalaureate. With the number of hours allotted is reduced, relative to the curriculum of specialty's
curriculum. In these circumstances, in order to achieve success in the teaching of mathematics, it is necessary to prioritize the goals of teaching. It is not enough to simply provide the order like: "it's more important and it is less important." The mentality of the mathematics teachers is that they do not feel satisfaction from solving the problem until they see the numbers. This was stated even the great Euler, who, it should be noted, a significant part of his career has worked in Russia. This is true for modern mathematics teachers. Based on the ideas of T.L. Saaty (2011) it is possible to find reasonable numerical values of the relative priorities of the goals in learning mathematics. This is the very problem which is solved in this work. Besides only the technical, economic and Humanities in institutions are considered and the hierarchy of objectives in teaching mathematics in University education of mathematicians, physicists and other future professionals in the fields of fundamental Sciences is not considered (Gnedenko & Gnedenko, 1988).

Materials and Methods

Method of hierarchies' analysis and the possibility of its application to prioritize the goals of teaching mathematics in university

To determine the priorities of the various purposes typically the following method is used. The examiner is suggested to evaluate the importance of each goal with the help of some number of score points. Then the goals are ordered by total points. Such a simple traditional approach does not correspond to the real complexity of the task of prioritization. It has long been saying by all the experts. Moreover, it is known from Cybernetics that there is no simple methods for solving complex problems or, as some experts say; "every complex problem has a simple wrong way to solutions!" Ignorance of this provision leads to such things as replacement of traditional exam with tests. Negative examples can be continued. The complexity of the ordering can be explained as follows. Let's say we want to sort N purposes. The number of choices will be N! = 1*2*...N. For N=14 it is more than 87 billion. It is necessary to choose the best. Without inadequate simplification, the problem can be solved on the basis of the following. You need to enable the expert to carry out pairwise comparison of the objectives and on this basis to calculate the degree of importance and the priorities of the goals. Research of psychologists have shown that the accuracy of expert estimates, expressed at the verbal – linguistic level is much higher than with the direct use by experts of numeric scores. The use of linguistic variables and pairwise comparison greatly improves the accuracy of estimates. Therefore, it is necessary to allow the experts to use qualitative comparisons, such as: "Goal A is more important than goal B". However, the following questions arise.

1. How to pass from linguistic variables to numbers?
2. How to pass from pairwise comparisons to streamline of the entire list of goals?

The answers to this give a method of hierarchies' analysis - the only method of multi-criteria optimization, which have received international recognition (Saaty, 2011; Saaty & Keris, 1991; Emelyanov & Larichev, 1985; Saaty, 1987; Jensen, 1984; Vargas, 1982). Based on this method in this paper calculation of the priorities of the goals in teaching mathematics to engineering, economic and Humanities specialties is carried out. Experts were faculty and graduates of the Moscow Aviation Institute and National Research University - Higher School of
Economics, as well as International Jewish Institute of Economics, Finance and Law.

Results

The table of pair comparison of importance of goals

On a specially organized seminar, with the participation of the consultant - moderator, the experts were asked to fill in a table of relative importance of objectives in teaching mathematics for engineering, economic and Humanities specialties of University. One table is for one group of professions. The table is square. In the title of its rows and columns the pointed in section 1.2 the goals of teaching mathematics are introduced. In the intersection of line i and column j is the importance of the goal A, relatively to the target B. Linguistic meaning that can be used when comparing is the following:

1. A is incomparably more important than B;
2. A is much more important than B;
3. A is more important than B;
4. A and B are equivalent;
5. A is less important than B;
6. A is much less important than B;
7. A is incomparably less important than B.

It is easy to notice, that in contrast to the scale T.L. Saaty (2011) doesn’t use nine, but seven values of the linguistic variable “Importance”. These values, as it is shown by practice, are most adequately perceived by the decision makers (DM).

It is necessary to fill only those cells of the table which are above its main diagonal. Because if, for example, A is more important than B, then it is obviously, that B is less important than A.

In the result the tables 1, 2, 3 were obtained for pairwise comparison of the importance of purposes which are given below.

<table>
<thead>
<tr>
<th>Table 1. Technical specialty: comparison of importance of goals.</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Table for comparisons of importance of objectives</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>GOAL</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The formation of mathematical competences for the solution of professional tasks</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The formation of logical thinking</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The education of mathematical culture</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>1</strong> The formation of mathematical competences for the solution of professional tasks</td>
<td>Much more important</td>
<td>More important</td>
<td></td>
</tr>
<tr>
<td><strong>2</strong> The formation of logical thinking</td>
<td></td>
<td></td>
<td>Equivalent</td>
</tr>
<tr>
<td><strong>3</strong> The education of mathematical culture</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2. Economic specialty: comparing of the importance of objectives

<table>
<thead>
<tr>
<th>GOAL</th>
<th>The formation of mathematical competences for the solution of professional tasks</th>
<th>The formation of logical thinking</th>
<th>The education of mathematical culture</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The formation of mathematical competences for the solution of professional tasks</td>
<td>Equivalents</td>
<td>Less important</td>
</tr>
<tr>
<td>2</td>
<td>The formation of logical thinking</td>
<td></td>
<td>Less important</td>
</tr>
<tr>
<td>3</td>
<td>The education of mathematical culture</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Humanities: comparing of goals

<table>
<thead>
<tr>
<th>GOAL</th>
<th>The formation of mathematical competences for the solution of professional tasks</th>
<th>The formation of logical thinking</th>
<th>The education of mathematical culture</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The formation of mathematical competences for the solution of professional tasks</td>
<td>Much less important</td>
<td>Less important</td>
</tr>
<tr>
<td>2</td>
<td>The formation of logical thinking</td>
<td></td>
<td>More important</td>
</tr>
<tr>
<td>3</td>
<td>The education of mathematical culture</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It is necessary to give necessary explanations to the experts' estimations.

Technical Sciences are characterized by a high degree of applicability of mathematical methods to them. For many, but not all, modern technical systems mathematical models are created that give fairly accurate results. That is why experts put the value "Much more important" at the intersection of the row corresponding to the goal "to Give the mathematical knowledge to solve professional problems" with the column "to Teach logical thinking." This is not belittling of the value of logical thinking. It is a statement of the fact that in modern conditions, the teacher of mathematics should focus exactly how to give the future engineer or technologist mathematical knowledge to solve
professional problems. Moreover, the logical thinking of the programmer, for example, will be taught at a special course in logic.

Mathematical methods are used in Economics. For a number of tasks it is quite successfully, but often mechanically, without taking into consideration the limitations of the methods, influence of human factor which is difficult to take into account, and lack of precision of the data used. Therefore, for future economists it is "Less important" according to experts "to Give the mathematical knowledge to solve professional problems" than "Education of mathematical culture". As it is stated in section 1.3 a key factor of mathematical culture is "the awareness of the limits of applicability of mathematical methods".

In the Humanities the possibility of mathematical methods' application is rather limited. Methods for the solution of individual tasks from this area are established, in particular, a fuzzy mathematics based on the concepts of fuzzy sets, linguistic variable and membership functions (Van Laathoven, 1983), (Buckley, 1985). The method used in this paper also belongs to this group, but they are quite complex and cannot be included in the General mathematics course for the Humanities. That is why, when teaching mathematics to the Humanities, "to Give the mathematical knowledge to solve professional problems" according to the experts "is Much less important" than the "Formation of logical thinking".

**Priorities of the goals**

On the basis of tables 1, 2, 3 the calculation of priorities is carried out according to the method developed by the author on the base of approach of T.L. Saaty. Linguistic values described above are assigned the values: 7, 5, 3, 1 respectively 1/3, 1/5, 1/7. Priorities are computed as the eigenvector of the symmetric matrix derived from the table of comparisons of objectives' importance corresponding to its own main value. For its location an effective procedure is implemented (Demidovich & Maron, 2006), which calculates priorities with a high degree of accuracy. This procedure gives much more realistic priorities than organizing of objectives using numeric values directly offered by experts. Representative studies conducted with the assistance of psychologists showed that the accuracy of experts' estimates, expressed in qualitative linguistic level is much higher than with the direct use of numeric values by them. In response to opponents of the technique of T.L. Saaty (2011), we can quote the words of the greatest Soviet mathematician academician Andrey Nikolaevich Kolmogorov (1903-1987) that valuable is not something that is mathematically strict, but what is right. It should be noted that these words were said by the man who created the axiomatic of probability theory, without which this science until him had existed for several centuries.

**Table 4. Priorities of objectives in teaching mathematics**

<table>
<thead>
<tr>
<th>Goal</th>
<th>Specialty</th>
<th>Technical</th>
<th>Economic</th>
<th>Humanitarian</th>
</tr>
</thead>
<tbody>
<tr>
<td>The formation of mathematical competences for the solution of professional tasks</td>
<td>65,9%</td>
<td>20%</td>
<td>10,5%</td>
<td></td>
</tr>
<tr>
<td>The formation of logical thinking</td>
<td>15,6%</td>
<td>20%</td>
<td>63,7%</td>
<td></td>
</tr>
<tr>
<td>The education of mathematical culture</td>
<td>18,5%</td>
<td>60%</td>
<td>25,8%</td>
<td></td>
</tr>
</tbody>
</table>
As the data show given in table 4 the main goals in all cases have priorities about two – thirds or, in other words, about 60% of the 100%. In the context of these groups the following main goals for teaching mathematics for the considered specialty groups are obtained.

1. The main goal in teaching mathematics for technical students is "to give the mathematical knowledge to solve professional tasks.

2. The main goal in teaching mathematics for Economic specialty is "Education of mathematical culture".

3. The main goal in teaching mathematics for the Humanities is "Formation of logical thinking".

Discussions

Obtained in this work, the priorities of the goals (table 4) should not be interpreted as distribution of hours for different aspects of teaching mathematics. This simplistic approach would indicate an insufficient mathematical culture. Found priorities are guidelines for drawing up of the curriculum and development by the teacher of methodic for teaching course. Some recommendations should be given. Let’s start with the General recommendations for these specialty groups. It is essential to reconcile a curriculum for study mathematics with the curricula of the special disciplines of the syllabus. For this, ideally, the mathematics teacher should be familiar with the content of special disciplines and mathematical methods used in them. To achieve this in modern terms is difficult, but possible, properly organizing continuing education of mathematics teachers. Another way is to work with teachers of special subjects that can also be attributed to continuing education, but informal. Detrimental for this process is the development of syllabus by administrative staff of Universities.

Now more specifically on the methodology for technical specialties. It should not be hidden that for them to develop good technique is easier than for economists, and the Humanities. Although, this brings to mind the words of N. I. Lobachevsky: "I am ready to think that if the teaching of mathematics which is characteristic of the human mind, remains for many unsuccessfully, that in justice it ought to be ascribed to the deficiencies in the art and method of teaching." The engineering students chose a science which interests lies outside mathematics. Therefore its teaching must be built so that future engineers constantly would feel the value of mathematics for solving problems in their practice. The desire to do without mathematics educates students’ false idea that in today's engineering studies it is possible to do approximate reasoning or without knowing the numerical analysis methods, to apply ready-made programs. A renowned teacher and writer Elena Sergeevna Ventzel (1907 – 2002), with a bitter smile says that math in a technical degree plays the same role as the feathers in the combat attire of the savage. It should be noted, in fairness, that her excellent textbooks on probability theory and operations research have saved from this many engineers. It seems appropriate a combination of rigorous presentation of mathematical methods at lectures with the use of mathematical packages for solving applied tasks in practical classes. Besides, the student’s attention certainly ought to be focused on how the choice
of specific numerical methods and initial approximations affects the final result. This is particularly important now, when in the Universities studies youth, which is a "generation Y". The term used for the generation of people born from 1983 to 2003 (Kondrakova, 2014). For these people, critically important is the speed of obtaining of information. Because in the Internet it is possible to obtain the necessary data by pressing only a single button. It is not surprising that today's student sees no sense to keep something in mind, and to be engaged in systemic analysis, since access to information is always there. Understanding that not all of this information is accurate does not come immediately.

Ideally, the transition from theory to calculations should occur on one lesson. Practically this is possible when the training is conducted in small groups, as in the International Jewish Institute of Economics, Finance and law. It should be noted that this system of instruction goes back to study in Yeshivas - Jewish religious schools. It is close to the scholastic method of teaching that dominated in medieval universities.

In most Universities mass training is applied, the technology of which is created by the works of Komensky, Pestalozzi and other scientists. In the conditions of mass education, the transition from theory to the calculations in one lesson is not always appropriate to carry out. It should be noted that the rigor of presentation does not imply the mandatory to use formalisms in the style of Bourbaki. The experience of this understanding of rigor in our country was quite sad. Especially for school education. Although it should be noted that the development is a spiral, and in modern mathematical packages the recording of operations using vector notations, abbreviating recording of actions with matrices and other formalisms are often the most simple and effective means to obtain the required solution for engineering problems.

Teaching economists in mathematics needs to get from them a clear understanding of the boundaries of applicability of different methods. The economist – the Manager should not use average wages in organization as an indicator of the level of the employees' wellbeing if the salary of a Manager and employee differ in 100 times. He must understand the impossibility of application of probabilistic methods where there are no mass and homogeneous random phenomena. He needs to understand that simulation modeling of economic processes by methods of system dynamics, as easily and clearly implemented by modern software such as PowerSim, are based on the integration of systems of differential equations, and the integral may diverge.

Russian speaking humanities specialist can be taught logical thinking only by mathematics teacher who is able to present interesting material in good Russian. Not knowing how to do it is doomed to failure, whatever knowledge he possessed. So one of the most memorable annoying factors of the reform in teaching mathematics, which was conducted in the sixties of the last century until 1978 in the Soviet high school was the following fact. According to the terminology of a new textbook in geometry it was impossible to tell that the two lines are equal, since equal can only be numbers, not geometric shapes. Those segments that coincide with the overlay should be called congruent. How here again not to remember the great N. Lobachevsky, who said, though turning not only to teachers of mathematics: "If we see that in the better class they neglect their language and there is vanity from knowledge of foreign language, we ought to regret it and call it a miserable event in our time". Accounting of the language
culture is a required element when working with experts, whose knowledge is often the only adequate method for solving problems in decision making support. It should be noted that in the above proposed method of determining the priorities of objectives in teaching mathematics, by pair comparison of importance of objectives the scale with fewer values is used than was suggested by T.L. Saaty (2011). This is due to the fact that in the Russian language to distinguish good from bad less gradations is used, for example, than in Persian or Arabic. Accounting of this has allowed obtaining from experts reliable answers.

**Conclusion**

In the result of research the following results are obtained.

1. The hierarchy of objectives in teaching mathematics for engineering, economic and Humanities specialties was constructed. Numerical values of the priorities of the goals were calculated.

2. It is established that the main goal of teaching mathematics for technical students is "to give the mathematical knowledge to solve professional problems. The main goal of teaching mathematics for students of Economics is "Education of mathematical culture". The main goal of teaching mathematics for students of Humanities is "to teach logical thinking - the ability to draw conclusions". The main goals in all cases have priorities about two – thirds or, in other words, about 60% of the 100%.

3. Obtained in this work, the priorities of the goals (table 4) should not be interpreted as distribution of hours for different aspects of teaching mathematics. Found priorities are guidelines for drawing up of the curriculum and development by the teacher of methodic of teaching course.

**Notes on contributors**

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