Acousto-Optic Interaction in Cells with Wedge-Shaped Transducers Excited at High Harmonics

V. I. Balakshy, A. V. Revenko
M. V. Lomonosov Moscow State University, Department of Physics, Leninskiye Gory, 119991 Moscow, Russia.
balakshy@phys.msu.ru

Summary
Diffraction of light in a non-homogeneous acoustic field produced by a wedge-shaped piezoelectric transducer is studied theoretically. Electrical, acoustic and acousto-optic characteristics of cells with the wedge-shaped transducers are calculated. Most attention is focused on special features of these cells operation at transducer’s third harmonic frequency. It is shown that the acoustic field has a complicated amplitude and phase structure which varies with ultrasound frequency. The dependence of the acousto-optic diffraction efficiency on the acoustic wave amplitude and the phase mismatch is studied. It is established that the diffraction efficiency can approach 100% despite a noticeable phase mismatch. Optimal values of acoustic power and light incidence angles are found. It is revealed that due to the wedge-like form of the transducer the frequency bandwidth at the third harmonic can be several times increased with retaining a high value of the electric-to-acoustic power conversion.

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1. Introduction

For acoustics, the problem of excitation of wedge-shaped piezoelectric transducers is not absolutely new [1, 2, 3, 4, 5]. This problem is of indubitable interest from two points of view at least. Firstly, when manufacturing a plate-like transducer it is impossible to make ideally parallel planes. Therefore, it is important to know the value of critical deviation from parallelism. Secondly, the wedge shape of the transducer with a noticeable wedge angle can significantly change characteristics of the acoustic field; this effect can be useful for developing various devices.

In the papers mentioned above, the wedge-shaped piezoelectric transducers were studied in the context of problems of ultrasonic frequency band broadening and electronic scanning of the ultrasonic beam radiation diagram. However, applications of similar transducers in acousto-optics have special features. The knowledge of integral characteristics, such as the frequency bandwidth and the electric-to-acoustic conversion coefficient, is not sufficient to make conclusions about the quality and the usefulness of the transducer for acousto-optic (AO) purposes. The acoustic field structure in the AO cell and its changing with ultrasound frequency are of great importance as well. In papers [6, 7, 8, 9] it has been shown, that the wedge-like form of the transducer results in amplitude and phase non-homogeneity of the excited acoustic beam. The amplitude non-homogeneity affects only the value of acoustic power that is necessary for reaching a specified value of the diffraction efficiency. The phase non-homogeneity, signifying the curvature of the beam wave front, influences stronger. In this case, one of the most important AO parameters – the Bragg angle – loses its sense because this angle is conventionally measured from the wave front [10, 11, 12]. By this is meant that no incidence angle of light can satisfy the phase matching condition; a phase mismatch does always exist and it is different in different points of the acoustic field. Therefore the question arises concerning the maximal diffraction efficiency. The phase non-homogeneity changes significantly amplitude, angular and frequency characteristics of AO interaction and certainly affects AO devices functioning. In particular, such an important characteristic, as the frequency dependence of the optimal incidence angle (known as the frequency dependence of the Bragg angle in the case of the homogeneous acoustic field), takes a more complicated form just in the area of the most efficient ultrasound excitation [1, 9].

This work is a continuation of investigations presented in papers [7, 8, 9]. The calculation of electric, acoustic and AO characteristics of cells with wedge-shaped transducers is performed in the approximation of a small value of the wedge angle. Most attention is focused on peculiar features of the cells operating at the third harmonic frequency in comparison with the fundamental harmonic which is used conventionally in acoustic and AO devices.
1.1. Basic relations

Figure 1 illustrates the statement of the problem. A piezoelectric plate with varying thickness $h(x)$ is attached to a flat surface of an AO medium. The plate is fed by the sinusoidal continuous voltage of a frequency $\Omega = 2\pi f$ from a HF generator having an electromotive force $E_0$ and an internal resistance $R_i$. The dependence of the transducer thickness $h$ on the coordinate $x$ can be expressed as

$$h(x) = h_0 + ax,$$  \hspace{1cm} (1)

where $a$ is the wedge angle and $h_0$ is the thickness of the piezoelectric plate in its centre at $x = 0$. The total length of the plate along the $x$-axis is supposed to be $l$.

Let us assume that no matching elements are between the generator and the transducer. In reality, such elements are always present in any practical device [13]. However, the problem of broadband matching of complicated frequency-depended impedances is a separate problem which is well studied in electronics [14]. Here, we will not touch it because the goal of this work consists in elucidation of those new peculiarities that appear due to the wedge-like form of the transducer.

Considering the wedge angle $\alpha$ sufficiently small, we can use a known solution of the problem of homogeneous piezoelectric plate excitation [15, 16] and write down the following expression for the complex admittance $dY$ of a small part $dx$ of the plate:

$$dY = \frac{\Omega^2 \epsilon h}{V_0 F(x)} \left\{ 1 - k^2 \frac{Z \sin F(x) + 2j [1 - \cos F(x)]}{F(x) - Z \cos F(x) + j \sin F(x)} \right\}^{-1} dx,$$ \hspace{1cm} (2)

where $F(x) = \Omega h(x)/V_0$ is the normalized frequency, $l \times b$ are the dimensions of the piezoelectric plate along the axes $x$ and $y$ accordingly, $\epsilon$ is the dielectric permittivity, $k$ is the piezoelectric coupling coefficient, $Z = \rho_l V_1/\rho_0 V_0$ is the relative acoustic impedance, $V_0, V_1$ are the sound velocities and $\rho_0, \rho_l$ are the densities of the transducer and the AO medium respectively.

The total admittance of the non-homogeneous plate is determined as [6]

$$Y = \int_{-l/2}^{l/2} dY = \frac{1}{R(\Omega)} + j \Omega C(\Omega),$$ \hspace{1cm} (3)

where $R$ and $C$ are the resistance and the capacitance in the parallel equivalent scheme of the transducer. The resistance $R$ describes the conversion of the electrical power coming from the HF generator into the acoustic power. The ohmic resistance of electrodes and the resistance of dielectric loses of the piezoelectric, which are responsible for the transducer heating, are usually small and can be neglected. The equivalent parameters $R$ and $C$ depend intrinsically on the acoustic frequency $\Omega$ [16].

Equation (3) allows calculating the voltage applied to the transducer

$$U = \frac{E_0}{1 + Y R_i}.$$ \hspace{1cm} (4)

Figure 1. Excitation of ultrasound by a wedge-shaped transducer.

The acoustic power radiated into the AO medium

$$P_a = \frac{E_0^2 \Re \{Y\}}{|1 + Y R_i|^2}$$ \hspace{1cm} (5)

and the electric-to-acoustic energy conversion coefficient

$$\chi = \frac{P_a}{P_{\text{match}}} = \frac{4 R_i \Re \{Y\}}{|1 + Y R_i|^2}$$ \hspace{1cm} (6)

where $P_{\text{match}}$ is the power given up by the HF generator to the matched load $R = R_i$.

For convenience of numerical calculations, let us introduce the following dimensionless parameters:

$$A = a l/h_0, \quad F_0 = \Omega h_0/V_0, \quad X = x/l \quad \text{and} \quad F(x) = F_0 (1 + A X).$$

Then equation (3) takes the form

$$Y = j \Omega C_0 F_0 \int_{-1/2}^{1/2} \left\{ F(X) - k^2 \frac{Z \sin F(x) + 2j [1 - \cos F(x)]}{Z \cos F(x) + j \sin F(x)} \right\}^{-1} dX,$$ \hspace{1cm} (7)

where $C_0 = \epsilon l b / h_0$ is the static capacitance of the piezoelectric plate.

The calculation of AO interaction characteristics for the case under consideration assumes solving the problem of light diffraction in non-homogeneous acoustic field [10]. The non-homogeneity can be caused by a number of reasons: bad splicing of the transducer with the AO medium, near-field diffraction non-homogeneity, acoustic crystal anisotropy, etc. However in any case such a non-homogeneity can radically change characteristics of AO diffraction. In paper [17], for an ideally homogeneous transducer the influence of the near-field non-homogeneity was analysed, which arose as a consequence of acoustic beam divergence in the presence of a very strong acoustic anisotropy in a paratellurite (TeO2) single crystal near the [110] direction. A similar problem was solved in [18] for a simulated non-homogeneity in the form of a bell-shaped amplitude distribution and a quadratic phase distribution. In this work, in accordance with the aim formulated above, we consider a more realistic situation: AO diffraction of light in a non-homogeneous acoustic field created by a...
wedge-shaped transducer. Following the approach conventional for acousto-optics, let us suppose that a narrow optical beam passes through the AO cell near the transducer and thereafter the acoustic field structure remains constant on the optical beam cross-section. Under these conditions the acoustic strain amplitude can be considered independent of the coordinate z, defining it at z = 0 as [5, 9]

\[
\begin{align*}
\alpha(X) &= -j \frac{E_0 e^{j \Omega}}{\rho_0 V_0} \left[ 1 - \cos F(X) \right] \\
& \cdot \left[ F(X) \sin F(x) - 2k^2 \left[ 1 - \cos F(x) \right] \right] \\
& + jk^2 \sin F(x) - F(X) \cos F(X),
\end{align*}
\]

where \( e \) is the piezoelectric constant. The modulus \( |\alpha(X)| \) describes the amplitude distribution in the acoustic wave along the transducer surface and the argument \( \arg(\alpha(X)) \equiv \Phi(X) \) defines the phase distribution. Thus, the transducer with varying thickness excites the acoustic wave having a complicated amplitude and phase structure. Besides, this structure strongly depends on the acoustic frequency \( \Omega \).

Calculations of AO diffraction spectra are usually made with the help of the coupled mode equations termed in acousto-optics the Raman-Nath equations [10, 11]. These equations can be generalized for the non-homogeneous acoustic field. In the case of the Bragg diffraction regime we have

\[
\begin{align*}
2 \frac{dC_0}{dX} &= \beta_0 \xi(X) C_1 \exp \left[ j(\eta X - \Phi(X)) \right], \\
2 \frac{dC_1}{dX} &= -\beta_0 \xi(X) C_0 \exp \left[ -j(\eta X - \Phi(X)) \right],
\end{align*}
\]

where \( C_0 \) and \( C_1 \) are the relative amplitudes of the diffracted waves in the zero and first diffraction orders respectively, \( q \) is the Raman-Nath parameter defined for \( A = 0, \eta = (\Omega/\nu)/(\theta_0 - \theta_0) \) is the phase mismatch parameter, \( \theta_0 \) is the light incidence angle and \( \theta_0 \) is the Bragg angle for the homogeneous acoustic beam. The angles \( \theta_0 \) and \( \theta_0 \) are measured from the transducer surface \( z = 0 \).

The function \( \xi(X) \) proportional to the acoustic wave amplitude \( |\alpha(X)| \) describes the amplitude non-homogeneity of the acoustic field and the function \( \Phi(X) \) characterizes the phase non-homogeneity. System (9) has to be integrated in the range \(-1/2 \leq X \leq 1/2\) with the following boundary conditions: \( C_0(-1/2) = 1 \) and \( C_1(-1/2) = 0 \).

2. Simulation results

Numerical calculations have been carried out for the X-cut lithium niobate (LiNbO₃) transducer attached to the 0°-cut paratellurite cell. For this variant, \( k = 68\% \), \( Z = 0.166 \). The length of the piezoelectric plate \( l \) has been chosen equal to 5.3 mm, its thickness \( h_0 \) has been taken equal to 26 µm; in this case the magnitude \( F_0 = \pi \) corresponds to the frequency \( f = \Omega/2\pi = 92 \text{ MHz} \).

Commonly, transducers work in the vicinity of its fundamental frequency (first harmonic). Nevertheless, it is known that they can be also excited at odd harmonics [16]. However, this kind of excitation is applied rarely because increasing the harmonic order in homogeneous transducers is accompanied by narrowing the frequency band and decreasing the electric-to-acoustic conversion coefficient. Our research has shown that in the case of wedge-shaped transducers these negative effects can be substantially eliminated.

Figure 2 demonstrates the conversion coefficient \( \chi \) as a function of the normalized acoustic frequency \( F_0 \).

![Figure 2. Conversion coefficient \( \chi \) as a function of the normalized acoustic frequency \( F_0 \).](image-url)
shifted to the left thinner edge, because this edge has a higher resonant frequency. In a similar manner, with decreasing the frequency, the right thicker edge of the plate is excited stronger. The difference in the maximum values of $a(X)$ is explained by varying the transducer impedance with frequency.

The phase characteristics (Figure 3b) actually visualize the form of the ultrasonic wave front in the plane $z = 0$. They show that the direction of the wave normal changes along the transducer surface. For the angle $\gamma$ defining the wave normal direction one can derive the following relationship:

$$\gamma = \frac{V_t}{f \Omega} \frac{d\Phi}{dX} = \frac{V_t h_0}{V_0 F_0} \frac{d\Phi}{dX} = \frac{\varphi \, d\Phi}{2\pi dX},$$

(10)

where $\varphi = V_t / f$ is the divergence angle of the homogeneous acoustic beam. The peak value of the derivative $[d\Phi/dX]$ in Figure 3b is equal to 26.8. Thus, the variations of the angle $\gamma$ exceed the angle $\varphi$ by the factor 4.3. The comparison with the acoustic divergence angle is not accidental because all the AO devices operate within the range $\varphi$ [10].

It should be noticed that the effect of the wave front rotation inherent in the wedge-shaped transducer has quite another nature than refraction of an acoustic wave during its passing through an elastic wedge-like plate. This is evident even from the facts that the function $\Phi(X)$ is nonlinear, it depends on the frequency, and that the greatest wave front rotation takes place just in the area of the most efficient ultrasound excitation. The effect is caused by the phase shift between the voltage applied to the transducer and the acoustic strain at its surface.

The phase non-homogeneity of the acoustic field affects the phase matching condition at AO interaction. In this case, the conventional definition of the Bragg angle loses its meaning because the wave front is curved and the phase mismatch $(\eta X - \Phi$ in Equations 9) differs from zero at any incidence angle of light $\theta_0$. Nevertheless, there exists an optimal angle of incidence $\theta_{opt}$ which provides achieving maximum diffraction efficiency.

As follows from Figure 2, broadening the frequency band $\Delta F^{(3)}$ with $a$ is accompanied by noticeable decreasing the conversion coefficient $\chi$ (by the factor 7.3 for $A = 0.5$). However our calculations have shown that the wedge-like form of the transducer results in much smaller variations of the capacitance $C(\Omega)$ close to the third harmonic compared to the fundamental harmonic. This peculiarity can be used for enhancing the conversion coefficient. For this purpose, a compensative inductance should be connected in parallel to the transducer so that an oscillatory circuit would form with the transducer capacitance tuned to the third harmonic frequency. This method will eliminate the transducer reactivity and lead to more effective conversion of electric power into acoustic one.

Figure 4 presents frequency characteristics of the transducer in the area of the third harmonic with the additional inductance matched according to the method expounded. It is seen an essential increase of the conversion coefficient $\chi$ (by the factor 7.3 for $A = 0.5$). However our calculations have shown that the wedge-like form of the transducer results in much smaller variations of the capacitance $C(\Omega)$ close to the third harmonic compared to the fundamental harmonic. This peculiarity can be used for enhancing the conversion coefficient. For this purpose, a compensative inductance should be connected in parallel to the transducer so that an oscillatory circuit would form with the transducer capacitance tuned to the third harmonic frequency. This method will eliminate the transducer reactivity and lead to more effective conversion of electric power into acoustic one.
sentential because it can be easily increased with choosing additional matching elements; it is important retaining the broad band of ultrasound excitation. The wider the initial (without matching) frequency band $\Delta f$, the easier to provide broadband ultrasound excitation with the help of matching circuit.

The amplitude and phase non-homogeneities of the acoustic field influence significantly characteristics of AO interaction. Figure 5 shows the angular characteristics as the dependence of the diffraction efficiency $\zeta = |C_1|^2$ on the mismatch coefficient $\eta$ which is proportional to deviation of the incidence angle $\theta_0$ from the Bragg angle $\theta_B$. The simulations are fulfilled for the frequency $F_0 = F_0^{(3)} = 9.46$ and the regime of small diffraction efficiency, when the angular characteristic of AO interaction represents the Fourier transform of the function $a(X)$ [10]. Curve 1 refers to the variant of the homogeneous transducer; in this case the dependency $\zeta(\eta)$ is described by the $\text{sinc}^2$-function which peaks at the point of phase matching $\eta = 0$.

One can note the following peculiarities of the angular characteristics conditioned by the wedge-like structure of the transducer. Firstly, the optimal incidence angle of light $\theta_{opt}$ which provides maximum diffraction efficiency differs from the Bragg angle $\theta_B$. The reason lies in the effect of rotation of the acoustic wave front relative to the transducer plane $z = 0$. In the case of isotropic diffraction [10, 11, 12], the normalized deviation of the angle $\theta_{opt}$ from $\theta_B$ is connected with the optimal mismatch value $\eta_{opt}$ by the formula

$$\delta = \frac{\theta_{opt} - \theta_B}{\theta_B} = \frac{2}{Q} \eta_{opt}. \quad (11)$$

where $Q = 2\pi n t f^2/n_i V_t^2$ is the Klein-Cook parameter, $\lambda$ is the optical wavelength in vacuum, $n_i$ is the refractive index of AO medium. The effect of the optimal angle shift is rather large: for example, $\eta_{opt} = 6$ for curve 4 ($A = 0.5$). This value should be compared with the condition $|\eta| \leq 0.89\pi$ which determines the range of AO interaction at 3 dB level [10]. Due to this effect the experimental dependence $\theta_{opt}(f)$ can significantly differ from the classical frequency dependence of the Bragg angle, as it was revealed in our experiment [1].

Secondly, the form of the angular characteristic varies; it becomes asymmetrical. And thirdly, the width of the characteristic increases (by the factor 2.2 for curve 4). These two effects are conditioned by the fact that the modification of the acoustic beam structure is not confined to the wave front rotation only, but, as seen in Figure 3b, focusing or defocusing of the beam at the areas of quadratic changing the phase takes place as well.

The differences between maximal values of the diffraction efficiency for different $A$ are caused primarily by the differences in the conversion coefficients (Figure 4). In this connection, the question of maximum values $\zeta_{max}$ and the corresponding acoustic power is of great importance.

The frequency dependence $\eta_{opt}(F_0)$ for different wedge angles $A$ is shown in Figure 6a. Horizontal line 1 refers to the homogeneous transducer. It is seen that the deviation $\delta$ takes large values right in the area of most efficient excitation of ultrasound. The value $\delta$ increases with the wedge angle. This result is in a good agreement with our measurements presented in [1].

In the homogeneous acoustic field, the diffractions efficiency reaches 100% when the incidence angle is equal to the Bragg angle and the acoustic power provides...
the Raman-Nath \( q = \pi \) [10, 11, 12]. The phase non-homogeneity of the acoustic field does not make it possible to satisfy the phase matching condition. Therefore the question of maximum diffraction efficiency is a topical problem. Our calculations have shown that in the case \( \theta_0 = \theta_{opt} \) it is possible to reach the diffraction efficiency near 100%. However, this requires greater acoustic power than for the case of the homogeneous acoustic field. The analogous result was obtained in [17, 18].

The influence of the acoustic field non-homogeneity on the diffraction efficiency is illustrated by Figure 6b. Here the frequency dependencies \( \theta_{opt}(F) \) are shown, where \( \theta_{opt} \) is the value of the Raman-Nath parameter that ensures nearly 100% diffraction efficiency at \( \theta_0 = \theta_{opt} \). The simulations are fulfilled in assumption of equal acoustic power in cases \( A = 0 \) and \( A \neq 0 \). Thereby the difference in transducer impedances is excluded in the calculations, the plot demonstrates the impact of the acoustic field non-homogeneity solely. Line 1 refers to the case of the homogeneous transducer for which \( \theta_{opt} = \pi \) independently of frequency. One can see that the greater the wedge angle, the more is the effect of the acoustic non-homogeneity which causes the enhancement of the acoustic power required for complete light transfer into the diffraction order.

3. Conclusion

In this work, characteristics of AO cells with wedge-shaped piezoelectric transducers are studied theoretically. It is shown that these transducers allow essential broadening the operating frequency band without a noticeable deterioration of the electric-to-acoustic conversion coefficient. The excited acoustic beam has a complicated structure with both amplitude and phase non-homogeneity. It is important to note that this structure changes with ultrasound frequency. Therefore AO interaction characteristics differ noticeably from that for AO cells with the invariable thickness. It is established that the phase non-homogeneity influences both the diffraction efficiency and the optimal incidence angle of light. As a consequence, the frequency dependencies of the diffraction efficiency and the optimal incidence angle, which are of great importance in AO devices, can differ substantially from the case of the homogeneous acoustic field. If this effect is considered as negative which prevents from getting AO characteristics specified, then our calculations permit estimating a necessary precision of transducer making. On the other hand, these non-conventional regularities can be useful for improving AO devices parameters.

The advantages of the wedge-shaped transducers show themselves more brightly when they are excited at high harmonics, in particular, at the third harmonic where the frequency band broadening can exceed 10 times. Qualitatively, this effect can be explained as follows. The impact of the wedge-like form of the piezoelectric plate is defined by the relation between the difference in the plate thickness and the ultrasonic wavelength. Therefore, the higher the excitation frequency, the stronger the influence of the thickness non-homogeneity on the transducer frequency characteristic.

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