Seminar Theoretical Computer science. April 9th, 2018.

Exercise 1. What is the error in the following fallacious argument that $\mathsf{P} \neq \mathsf{NP}$? Assume that $\mathsf{P} = \mathsf{NP}$, then $\mathrm{SAT} \in \mathsf{P}$ and so for some k, $\mathrm{SAT} \in \mathsf{TIME}(n^k)$. Therefore NP-completeness of SAT implies $\mathsf{P} = \mathsf{NP} \subseteq \mathrm{TIME}(n^k)$, But by the time hierarchy theorem, $\mathsf{TIME}(n^{k+1})$ contains a language not in $\mathsf{TIME}(n^k)$, which contradicts $\mathsf{P} \subseteq \mathsf{TIME}(n^k)$. Therefore $\mathsf{P} \neq \mathsf{NP}$.

Exercise 2. Show that $\mathsf{PSPACE}(n^3) \not\subseteq \mathsf{NPSPACE}(n)$.

Exercise 3. Let A^{c} be the complement of language A. Show that $\mathsf{P}^A = \mathsf{P}^{A^{\mathsf{c}}}$ and $\mathsf{NP}^A = \mathsf{NP}^{A^{\mathsf{c}}}$.

Exercise 4. Define the USAT problem to be

$$\{\langle \phi \rangle : \phi \text{ is a Boolean formula that has a single satisfying assignment}\}$$

Show that $\mathrm{USAT} \in \mathsf{P}^{\mathrm{SAT}}$.

Exercise 5. Show that if $\mathsf{P}^{\mathrm{SAT}} = \mathsf{NP}$ then $\mathsf{NP} = \mathrm{co}\mathsf{NP}$.

Exercise 6. Let MIN-FORMULA be the language defined in the previous seminar. Show that the complement of this language is in $\mathsf{NP}^{\mathrm{SAT}}$.

Exercise 7. Show that $\mathsf{P}^{\mathrm{TQBF}} = \mathsf{NP}^{\mathrm{TQBF}}$.

Exercise 8. Imagine you are given two oracles and one of them is the set TQBF. You don't know which one. Design an algorithm that can access the two oracles A and B and that decides TQBF in polynomial time.