Independent versus Collective Expertise*

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Abstract

We consider the problem of a decision-maker who seeks for advice from reputation-concerned experts. The experts have herding incentives when their prior belief about the state of the world is sufficiently strong. We address the following question: Should experts be allowed to exchange their information before providing advice ("collective expertise") or not ("independent expertise")? Allowing for such information exchange modifies the herding incentives in a non-trivial way. The effect is beneficial for the quality of advice when there is low prior uncertainty about the state and detrimental in the opposite case. We also show that independent expertise is more likely to be optimal when the decision-maker has a valuable enough "safe" option with a state-independent payoff. Finally, collective expertise is more likely to be optimal as the number of experts grows.

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1 Introduction

Decision-makers routinely rely on expert advice, and often there are multiple experts available. In this paper we address the following question: Should experts be given an opportunity to share their information before talking to the decision-maker? A peer review process in academic journals is a typical example where experts (referees) cannot

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talk to each other (we call it "independent expertise") as they are simply not aware of each other's identity. At the other extreme, a CEO openly asking her colleagues for advice on the firm's strategy would naturally induce (some) information sharing between the colleagues before they deliver their advice at the next meeting (we call it "collective expertise").

In such and many other examples, experts care about their reputation for being considered smart. These reputation concerns is the key friction in our paper. As was argued in a series of papers by Ottaviani and Sørensen (2001, 2006a, 2006b), reputation concerns can make advisors herd on the prior belief, and, consequently, lead to loss of information for the decision-maker.

We show that, due to aggregation of information *prior* to advice, collective expertise is better at predicting *which* state of the world is more likely. However, it fails to provide the decision maker with the information on individual signals of experts, which is valuable when it is also important to know *how* likely the more likely state is.

We consider a model with two states of the world, 0 and 1, and three potential actions to choose from: 0, 1 and *safe* action. Action i is optimal in state i. Safe action has a state-independent payoff which, in any state, is below that from the optimal action in that state. It can be interpreted as the option to wait until the realization of uncertainty (which involves a cost of delay), costly investment in learning the state now, or implementing a safe project with low return. We assume that the state of nature is revealed at the end of the game regardless of the action chosen.

This payoff structure yields the following optimal decision rule: take the action corresponding to the more likely state only if you are sufficiently confident about the state, otherwise take the safe action.

Experts receive informative *non-verifiable* signals about the state. The informativeness of an expert's signal depends on his/her ability, which is unknown to anyone, including him/herself. The objective of each expert is to maximize the decision-maker's posterior belief about his/her *absolute* ability (i.e., experts do not care about their relative standing in the eyes of the decision-maker). In the baseline model we have two experts with the same expected ability.

The presence of the safe option is important, because it makes the decision-maker care not just about which state is more likely, given the experts' information, but also *how* likely the more likely state is.

We compare two communication schemes. Under "independent expertise", each experts sends a report to the decision-maker without knowing anything about the other expert's signal. Under "collective expertise", the experts share their signals before submitting a joint report. Regardless of the communication scheme, all reports (including reports between the experts) are non-contractible "cheap talk" messages.

The potential benefit of signal-sharing between the experts (provided that they do not lie to each other) is alleviation of the herding-on-the-prior incentives when both experts receive a signal contradicting the prior. Namely, this benefit materializes when each expert's signal is weaker than the prior (so that herd behavior results under independent expertise), but two same signals combined are stronger than the prior.

The potential cost of signal-sharing is that it aggravates herding incentives, when the experts receive opposite signals. Indeed, with two identical experts, two opposite signals just leave the experts' beliefs at the prior, which implies that, in such a case, they will herd on the prior *regardless* of its strength (unless it is exactly 1/2). In fact, we show that, with identical experts, a fully revealing equilibrium never exists under collective expertise, and the partially informative equilibrium that exists for the widest range of priors has the following structure: When both experts have received signals countering the prior, this fact is revealed; all other vectors of signals are pooled.

Therefore, the first main conclusion of our model is that collective expertise is better than independent expertise when there is sufficiently low prior uncertainty about the state (unless the uncertainty becomes so low that herding always occurs even under collective expertise), whereas independent expertise dominates for a sufficiently high prior uncertainty.

We further show that collective expertise is more likely to be preferred when the value of the *safe* action is lower. The intuition is as follows. The advantage of independent expertise is that, conditional on truthful reporting, it provides the decision-maker with the most accurate information about the likelihood of each state, given the experts' information. However, when the safe action has a sufficiently low value, this accuracy is of no use, because the safe action is never taken anyway. In such a case, rough information on just what state is more likely becomes sufficient to take an optimal decision, and collective expertise achieves this for a wider range of the prior beliefs.

Finally, we turn to the following question: What happens if we enlarge the number of experts? Under independent expertise, the incentives of each expert are unaffected by their number. Thus, when the prior is stronger than a single expert's signal, all experts herd and having more experts is of no use. When the prior is weaker than an expert's signal, the experts tell the truth and, thus, having more experts results in more information.

Collective expertise, in contrast becomes more informative both for strong and weak priors. First, a higher number of same signals reduces herding incentives, which allows partially informative communication for higher values of the prior. Second, any loss of information that arises due to partial pooling of vectors of signals is less important, because aggregation of signals gives a more precise information about the state as the number of experts grow.

To be specific, we first show that, in any equilibrium, experts' messages partition the set of all signal profiles into *at most* two ordered subsets, according to the total number of zeroes the experts receive. In particular, there is always an equilibrium in which the experts truthfully communicate one of two messages: "we received at least l zeroes" and "we received at most l zeroes" (with a possible randomization for l zeroes). Now, if we increase the number of experts, these messages, even though staying binary, become more informative about the state. As an extreme example, consider the infinite number of experts. By learning each other's signals, the experts will simply learn the state, by the law of large numbers, and just report whether they received more zeroes or more ones. Each of the two messages will perfectly inform the decision-maker about the state, regardless of the prior.

Thus, if, for some reason we cannot condition the choice of the expertise scheme on the prior or/and the ex-ante quality of experts, the conclusion is that, as the number of experts grows, collective expertise becomes more likely to be optimal.

Our paper joins the literature that explores how information aggregation and decision-making can be improved in the presence of reputation concerns. Ottaviani and Sørensen (2001) examine the role of the order of speech in a public debate among reputation-concerned experts. Prat (2005) studies the effects of transparency of decisions on the actions of a reputation-concerned decision-maker (Levy (2007) addresses a similar question in a committee setting). Catonini and Stepanov (2019) show how the decision-maker can improve extraction of information from reputation-concerned experts by committing to ask for advice only in certain circumstances.

This paper looks at how the adverse effects of reputation concerns can be alleviated by the optimal organization of expertise. In this sense, it is close to the work by Ottaviani and Sørensen (2001). The crucial distinction of our work from Ottaviani and Sørensen (2001) is that in our study the experts exchange their information *privately*, whereas in the latter paper they speak sequentially and *publicly*. The necessity to report publicly changes fundamentally the incentives of both the first speaker and all subsequent speakers. The first speakers' incentives are then determined only by his own information, and not by the reporting behavior of subsequent speakers, contrary to our paper. A subsequent speaker's incentives are affected by earlier speakers' reports, but, differently from our paper, her reporting is constrained by the infeasibility to misrepresent earlier speakers' reports. In fact, in our setup, when all experts are ex-ante identical, public sequential advice is always weakly dominated by independent advice, whereas our "collective expertise" can do strictly better.

There are works on eliciting information from multiple advisors in a Crawford and Sobel (1982) type of setting (e.g., Gilligan and Krehbiel (1989), Krishna and Morgan (2001a,b). Battaglini (2002), Ambrus and Takahashi (2008), McGee and Yang (2013), Wolinsky (2002)). Due to a different nature of communication distortions, this whole literature is largely orthogonal to the "reputational cheap talk" literature. Moreover, most of this literature does not address the central question of our work: Should experts be allowed to talk to each other? (Although some of these models compare sequential and simultaneous communication, see Hori (2006), Li (2010), Li, Rantakari, and Yang (2016)).

The only exception, to our knowledge, is Wolinsky (2002).[1] Wolinsky considers the problem of a decision maker who wants to aggregate decision-relevant information that is disseminated among a number of experts. The decision is binary, and so each expert's piece of information (0 or 1). The experts care about the decision, and both for the experts and for the decision-maker the preferred decision depends on the sum of the experts' pieces of information. However, the experts are biased: For some values of this sum, their preferred decision is 0, while the decision maker's is 1. Because of this, if the decision maker asks each individual expert to reveal his piece of information, the expert will focus on the case when his advice is pivotal and will pretend that his information is 0 also when it is 1 (1 is verifiable but 0 is not). If instead subgroups of experts share their information before providing advice, informative equilibria arise: A subgroup of experts with many 1's will suggest to the decision maker to take decision 1, because the increased weight of their advice on the final decision makes it pivotal also in situations where the experts prefer decision 1.

Thus, the information structure, the nature of distortions in communication, and, most importantly, the channel through which information sharing among experts improves the informativeness of communication all differ with respect to our work. In our model, beliefs about the state is the key determinant of the effect of reputation concerns on the experts' reporting behavior, and information sharing acts through changing these beliefs. In contrast, Wolinsky's paper does not deal with reputation concerns, and belief updating about the state does not play a crucial role in his paper. Instead, information sharing helps the experts to coordinate on disclosing a critical mass of information that is sufficiently influential to be willingly (but coarsely) transmitted to the decision-maker.

Finally, there are works on deliberation in committees (see Austen-Smith and Fed-

[1] Rather than studying ex-post information-sharing, Elliott, Golub, and Kirilenko (2014) consider sharing *technologies* for generating recommendations to the decision-maker in a setup where two experts have different attitude to type I versus type II errors. The authors show that allowing for such sharing can harm the decision-maker, because the resulting expansion in the sets of technologies available to each expert may make the experts switch to suboptimal choices of recommendation-generating procedures from the decision-maker's perspective.

dersen (2009) for a survey). These papers however do not examine whether committee members *should* be allowed to share their information before voting or not.[2] Instead, they are focused on distortions (in both information sharing and voting outcomes) created by divergence of preferences, reputation concerns and strategic voting considerations, and how such distortions can be alleviated through the design of optimal voting rules (Coughlan (2000), Austen-Smith and Feddersen (2005, 2006), Visser and Swank (2007), Gerardi and Yariv (2007)) deliberation rules (Van Weelden (2008)) and transparency regulations (e.g., Meade and Stasavage (2008), Swank and Visser (2013), Fehrler and Hughes (2018))

The rest of the paper is organized as follows. Section 2 sets up a model with two homogeneous experts. Sections 3 and 4 analyze independent and collective expertise, respectively, in this setup. Section 5 deals with the welfare analysis. Section 6 studies the case of more than two experts. Section 7 concludes. The Appendix contains the proofs omitted from the main text, analyzes an extension with heterogeneous experts and discusses robustness of our solution under collective expertise to the communication protocol.

2 Model with two experts

A decision-maker chooses an action from a set consisting of three elements: $\alpha \in \{0, s, 1\}$. Her payoff from an action depends on the unknown state of nature, $\omega \in \{0, 1\}$, in the following way:

$$u_D(a, \omega) = \begin{cases} 1, \text{ if } a = \omega; \\ 0, \text{ if } a = 1 - \omega; \\ k \in (0, 1), \text{ if } a = s, \ \forall \omega \end{cases}$$

That is, the decision-maker wants to match her action to the state, but suffers from making a mistake. In addition, she has a *safe* option, s, with a state-independent payoff which is higher than the payoff from a wrong action but lower than that from the optimal action in any given state. Depending on the real life applications, the safe action can be interpreted as the option to wait until the realization of uncertainty (which involves a cost of delay), costly investment in learning the state now, or implementing a safe project with a low return.

Before taking her decision, the decision-maker can consult with two experts. The experts are ex-ante identical, and each of them can be of two types, Good and Bad with the commonly known prior probability $\Pr(t_i = G) = q \in (0, 1), \ \forall i \in \{1, 2\}$. The experts'

[2] An exception is Ali and Bohren (2018). In their setup, committee members' losses from type I and type II errors are different from those of the principal designing a committee. The authors show that banning deliberation can benefit the principal if she can choose the equilibrium the committee members play at the voting stage or if she can use non-monotone or non-anonymous social choice rules.

types are uncorrelated and unknown to anyone, including the experts themselves. Each expert receives a private non-verifiable signal $\sigma_i \in \{0,1\}$. Independently of the state, an expert's signal is correct with probability either g or $b < g$, depending on his/her type:

$$g := \Pr(\sigma_i = \omega | t_i = G) > b := \Pr(\sigma_i = \omega | t_i = B) \geq 1/2.$$

Conditional on the state, the experts' signals are independent. We denote $\sigma := (\sigma_1, \sigma_2)$.

There is a common prior about the state of nature:

$$p := \Pr(\omega = 0)$$

Without loss of generality, we assume that $p > 1/2$.[3]

Let us denote the expected precision of an expert's signal by

$$\rho := qg + (1-q)b$$

Each expert cares only his/her reputation, which is modelled as the decision-maker's ex-post belief about the expert's type.

The timing of the game is as follows:

1. The nature draws the state ω and the types of the experts.
2. The experts receive their private signals.
3. The experts communicate their information to the decision-maker, according to an *expertise scheme.*
4. The decision-maker takes an action
5. The state is revealed and the players receive their payoffs.

The focus of our work is the expertise scheme employed in stage 3. Under *independent expertise*, each expert sends a non-contractible binary message, $m_i \in \{0,1\}$, to the decision-maker. Under *collective expertise*, expert 2 (he) first sends a non-contractible message $m_2 \in \{0,1\}$ to expert 1 (she), and then expert 1 sends a non-contractible message $m \in \{(0,0),(0,1),(1,0),(1,1)\}$ to the decision-maker. Expert 1 can then be called a *deputy* expert. In the Appendix, Section 8.2 we argue that, with ex-ante identical experts, a particular way in which communication under collective expertise is organized is not important for the results of the model. In particular, we show that our equilibria remain equilibria (even after applying meaningful refinements) in a model that allows *each* expert to send a message to the decision-maker after talking to each other.

[3]We exclude $p = 1/2$ from consideration, as a trivial degenerate case: Under $p = 1/2$, reputation concerns create no misreporting incentives, and there is full information revelation under either expertise scheme.

An expert's payoff is then:

$$u_i(message, \omega) = \Pr(t_i = G|message, \omega), \ \forall i \in \{1, 2\},$$

where $message$ is either m_i or m depending on the expertise scheme.

We will use the term "signal-type (σ_1, m_2)" to call expert 1 who received signal σ_1 and message m_2 from expert 2.

3 Independent expertise

Under independent expertise, an expert's reporting behavior does not depend on the reporting strategy of the other expert. This is because (1) the experts learn nothing about each others' signals prior to reporting, and (2) the state is eventually revealed, thus making the other expert's report redundant in forming the decision-maker's belief about an expert's type.

Hence, each expert behaves as if he/she were a single expert, and we can just apply Lemma 1 from Ottaviani and Sørensen (2001), which deals precisely with the case of a single expert in a setup with two states, two expert types and a binary expert's signal. Given our notation and the assumption that $p > 1/2$, their lemma can be re-formulated as follows:

Lemma 1 *Under independent expertise, the following is true:*
- *When $p \leq \rho$, the experts report their true signals in the most informative equilibrium.*
- *When $p > \rho$, there exists no equilibrium with informative reporting.*

The intuition is simple. An expert wants to maximize the decision-maker's posterior belief that he/she received the signal equal to the state. Since $p > 1/2$, an expert with signal 0 always believes that $\omega = 0$ is more likely. An expert with signal 1 believes that $\omega = 1$ is more likely exactly when $p < \rho$, and considers $\omega = 0$ more likely otherwise. Therefore, when $p < \rho$, reporting the true signal is a natural equilibrium. In contrast, when $p > \rho$, there is a strong temptation to "herd" on the more likely state, which destroys any informative communication.

4 Collective expertise

Suppose expert 2 has truthfully revealed his signal to expert 1. When will the latter truthfully reveal both expert 2's and her own signal, regardless of her information? The striking answer is that, under collective expertise, full information revelation is impossible.

To make sure, with *identical* experts, by "full revelation" we mean truthful reporting of the aggregate number of 0's received by the experts, i.e., we do not require to report who exactly received which signal, because this is immaterial for the decision-maker.

Lemma 2 *Under collective expertise, a fully revealing equilibrium does not exist*

Proof. See the Appendix. A reader can also refer to the proof of Lemma 7 which covers the more general case of $n \geq 2$ identical experts. ■

The intuition behind Lemma 2 is straightforward. Two contradictory signals leave the deputy's belief at the prior, that is, believing that state 0 is more likely. Revealing that $\sigma \in \{(0,1),(1,0)\}$ implies that one of the experts is correct and the other one is wrong irrespective of the realized state. Hence, for any state realization, it induces the belief that the deputy is correct with probability 1/2, whereas a deviation to reporting $(0,0)$ results in "guessing" the state with probability strictly higher than 1. Any partial or full separation of signal-types $(0,1)$ and $(1,0)$ would damage the expected reputation of the deputy expert with $\sigma_1 = 1$ (given that $\sigma_2 = 0$) even more, thus creating an even stronger deviation incentive for her.

Lemma 2 immediately implies that, for $p \leq \rho$, independent expertise *always* provides the decision-maker with more information relative to collective expertise. But what happens for $p > \rho$?

For this we need to explore other informative equilibria. For simplicity (without any effect on the qualitative results), we restrict ourselves to analyzing equilibria in which, provided truthful reporting by expert 2 to expert 1, pairs of signals $(0,1)$ and $(1,0)$ trigger the same distribution over messages ("anonymous" equilibria). This is natural, given that the two pairs of signals generate the same belief about the state.

In this section, we will implicitly assume that expert 2 truthfully reports to expert 1. In the proofs we will show that this is indeed the case. Intuitively, since our equilibria are anonymous, the two experts have identical ex-post reputation, meaning that the incentives of the experts are perfectly aligned, and, thus, expert 2 can gain nothing from lying to expert 1.

First, it can be shown that any equilibrium partitions the set of signal profiles into at most two *ordered* subsets (possibly with a common boundary), equivalent to only two messages being sent (from the deputy expert to the decision-maker). "Ordered" means that any profile of signals in one of the subsets contains a weakly higher number of zero signals than any profile in the other subset. We relegate the proof of this result to Section 6, where we consider the more general n-experts case (Theorem 1). A message can then be interpreted as a statement that the signal profile belongs to a certain element of the bipartition,.with the qualification that a threshold profile can randomize between the two messages.[4] Then, if we consider equilibria without such randomization, there arise two possibilities.

- partition $\{(0,0),(0,1),(1,0)\},\{(1,1)\}$;
- partition $\{(0,0)\},\{(0,1),(1,0),(1,1)\}$.

Let us denote message $\{(0,0),(0,1),(1,0)\}$ by m^0 and message $\{(0,1),(1,0),(1,1)\}$ by m^1.

In addition, there can be equilibria with mixing between partition elements:

- the one in which signal-type $(1,1)$ randomizes between reporting the truth and reporting m^0;
- the one in which signal-type $(0,0)$ randomizes between reporting the truth and reporting m^1;
- the one in which signal-type (i,i) always reports the truth, while signal-types $(0,1)$ and $(1,0)$ mix between reporting $(0,0)$ and reporting $(1,1)$.

Let us first examine the equilibrium $(m^0,(1,1))$.

Lemma 3 *The equilibrium* $(\{(0,0),(0,1),(1,0)\},\{(1,1)\})$ *exists if and only if* $p \leq \overline{p}$, *where* $\overline{p} = \dfrac{\rho^2(2-\rho)}{1-\rho+\rho^2} > \rho$. *Moreover, when* $p = \overline{p}$, $\Pr(\omega = 1|\sigma = (1,1)) > 1/2$.

Proof. See the Appendix. ■

Signal-type $(0,1)$ (or $(1,0)$) would never want to deviate to reporting $(1,1)$: As she believes that $\omega = 0$ is more likely, she would not want to be perceived as having received signal 1.

In contrast, signal-type $(1,1)$ may want to deviate to reporting $\{(0,0),(0,1),(1,0)\}$, if the prior is sufficiently biased to $\omega = 0$. She will clearly do so when the prior is so strong that $\Pr(\omega = 0|\sigma = (1,1)) > 1/2$: As she considers $\omega = 0$ a more likely state, she would not want to be perceived as having received signal 1. When $\Pr(\omega = 0|\sigma = (1,1)) < 1/2$, expert 1 has a trade-off. By revealing her signal, she will essentially "bet" on the more likely state. However, deviating to $\{(0,0),(0,1),(1,0)\}$ does not imply "betting" on the less likely state, because $\{(0,0),(0,1),(1,0)\}$ does not imply that expert 1 necessarily received signal 0. In other words, if $\omega = 1$ is realized, message $\{(0,0),(0,1),(1,0)\}$ results in a lower expected reputational loss compared to the (hypothetical) situation in which the expert is definitely believed to have received signal 0.

As a result, the value of the prior at which expert 1 is indifferent between deviating and not, $p = \overline{p}$, is below p that makes $\Pr(\omega = 0|\sigma = (1,1)) = 1/2$. In other words, at $p = \overline{p}$ signal-type $(1,1)$ still believes that $\omega = 1$ is more likely.

[4] One may wonder whether there can exist multiple *equilibrium* messages that generate exactly the same reputation for the experts in any state of nature ("reputation-equivalent" messages) but provide different information about the state to the decision-maker. It can be shown that, in the setting with two experts, this is impossible: Any two reputation-equivalent messages are also information-equivalent, meaning that they can be considered as the same message. The proof is available upon request.

The crucial thing is that $\overline{p} > \rho$. Two same signals combined are stronger than one. This allows to eliminate herding-on-the-prior incentives of the experts, whenever both signals are 1, for a range of parameters where each expert separately would herd.

Let us now consider the equilibrium $((0,0), m^1)$.

Lemma 4 *The equilibrium* $(\{(0,0)\}, \{(0,1),(1,0),(1,1)\})$ *exists if and only if* $p \leq \frac{1+\rho}{3}$, *which is strictly below* ρ.

Proof. See the Appendix. ■

Here the threshold on p is determined by the incentive compatibility of signal-type $(0,1)$ (or $(1,0)$). Given that the prior is biased to $\omega = 0$, signal-type $(0,0)$ is very confident that $\omega = 0$, and, thus, would never want to lie. In contrast, signal-type $(0,1)$ (or $(1,0)$) has a trade-off similar to the trade-off of signal type $(1,1)$ in the equilibrium of Lemma 3: betting on the more likely state by sending $m = (0,0)$ versus playing a "safer" strategy of staying pooled with the other two signals.

Notice that the threshold on p provided by Lemma 4, $\frac{1+\rho}{3}$, is smaller than ρ.

Finally, let us consider equilibria with mixing between partition elements.

Lemma 5 *Equilibria with mixing between partition elements do not exist for* $p > \overline{p}$

Proof. See the Appendix. ■

Thus, such equilibria do not expand the set of priors where partial information revelation occurs under collective expertise. The analysis of this section implies the following fundamental result:

Proposition 1 *Irrespectively of which informative equilibrium is played under collective expertise (provided an informative equilibrium exists), the following is true. When* $p \leq \rho$, *independent expertise results in more information transmitted to the decision-maker. When* $p \in (\rho, \overline{p}]$, *collective expertise results in more information transmitted to the decision-maker. For* $p > \overline{p}$, *both modes of expertise result in zero information transmission.*

The potential benefit of signal-sharing between the experts is alleviation of the herding-on-the-prior incentives when both experts receive a signal contradicting the prior. This benefit materializes when each expert's signal is weaker than the prior ($\rho < p$, so that herd behavior results under independent expertise), but two same signals combined are sufficiently stronger than the prior (at $p = \overline{p}$ signal-type $(1,1)$ believes that $\omega = 1$ is more likely).

The potential cost of signal-sharing is that it aggravates herding incentives, when the experts receive opposite signals. With two identical experts, two opposite signals just leave the experts' beliefs at the prior, which implies that, in such a case, they will herd on the prior *regardless* of its strength (unless it is exactly $1/2$). Then, at best only partial information revelation is possible under collective expertise.

5 Welfare analysis: Effects of the prior and the value of the safe action

In the previous section, we have seen that, in terms of information provision, independent expertise dominates collective one for $p \leq \rho$, and vice versa for $p \in (\rho, \overline{p}]$. Greater informativeness, however, implies a higher decision-maker's welfare only if it affects his choice of actions – this is the question we turn to in this section.

The important thing to notice is that message $m^0 \equiv \{(0,0),(0,1),(1,0)\}$ pools the signal profiles that, if taken separately, predict that $\omega = 0$ is more likely. At the same time, $\sigma = (1,1)$ implies that $\omega = 1$ is more likely for all $p \leq \overline{p}$ (according to Lemma 3). Hence, if equilibrium $(m^0,(1,1))$ is played, collective expertise correctly predicts which state is more likely conditional on the experts' signals for *any* $p \in [1/2, \overline{p}]$. In contrast, under independent expertise, such information is lost for $p \in (\rho, \overline{p}]$, because no information is transmitted.

Thus, the following lemma is true:

Lemma 6 *Consider the range of p from $1/2$ to $\overline{p}$ and assume that equilibrium $(m^0,(1,1))$ is played under collective expertise. Then, for any pair of the experts' signals, collective expertise correctly predicts which state is more likely for all $p \in [1/2, \overline{p}]$, whereas independent expertise does so only for $p \in [1/2, \rho]$.*

We are now ready to state the key result of this section. In formulating it we will assume that the decision-maker's preferred equilibrium is played under collective expertise.[5] However, as follows intuitively from Proposition 1, the qualitative conclusion will not change if we assume a different equilibrium selection under collective expertise.

Proposition 2 *Irrespective of the value of k, the decision-maker is weakly better off under independent expertise for any $p \in (1/2, \rho]$ and weakly better off under collective expertise for any $p \in (\rho, \overline{p}]$. Moreover, there exist thresholds $\overline{k} < 1$ and $\overline{k}' < \overline{k}$, such that:*

[5]Equilibrium $(m^0,(1,1))$ is not necessarily the best for the decision-maker. Suppose the optimal signal-contingent policy is to take the safe action whenever $\sigma \in \{(0,1),(1,0),(1,1)\}$ and take action 0 otherwise (this could well be the case, because $(0,0)$ generates less uncertainty than $(1,1)$). Then, equilibrium $((0,0),m^1)$ is the best one (provided it exists).

i) when $k \leq 1/2$, the decision-maker is equally well off under both expertise schemes for any $p \in (1/2, \rho]$ and ***strictly*** *better off under collective expertise for any $p \in (\rho, \overline{p}]$;*

ii) when $k \in (1/2, \overline{k}')$, the decision-maker is ***strictly*** *better off under independent expertise for a positive measure subset of $p \in (1/2, \rho]$ and* ***strictly*** *better off under collective expertise for a positive measure subset of $p \in (\rho, \overline{p}]$;*

iii) when $k \in (\overline{k}', \overline{k})$, the decision-maker is ***strictly*** *better off under independent expertise for a positive measure subset of $p \in (1/2, \rho]$ and is equally well off under both expertise schemes for any $p \in (\rho, \overline{p}]$;*

iv) when $k \geq \overline{k}$, the decision-maker is equally well of under both expertise schemes for any p.

Proof. See the Appendix. ■

For very high values of k ($k \geq \overline{k}$), the safe action is so attractive that it is taken for all $p \leq \overline{p}$ regardless of how expertise is organized, hence making the expertise scheme irrelevant (statement (iv)).

When k is below $1/2$, the safe action is never taken because betting on the more likely state is always optimal. In such a case, the only relevant thing is whether an expertise correctly predicts which state is more likely, given the experts' signals. Then, statement (i) of Proposition 2 immediately follows from Lemma 6.

For intermediate values of k, the safe action is sometimes optimal and sometimes not. Thus, not only which state is more likely, but also how likely is the more likely state (conditional on the experts' information) becomes relevant, and only independent expertise is capable of revealing all experts' information for $p \in (1/2, \rho]$. For example, when k exceeds $1/2$ but is sufficiently small, then, for low enough p, the optimal signal-contingent policy is as follows: take the safe action when $\sigma \in \{(0,1),(1,0)\}$, and take the action suggested by the signals otherwise. This can only be achieved under independent expertise. Under collective expertise, $\{(0,1),(1,0)\}$ is always pooled (at least partially) with either $(0,0)$ or $(1,1)$. As a result, the mapping from the experts' signals into the decision-maker's actions will inevitably be suboptimal: either the safe action will sometimes be taken when it should not be taken or vice versa.

The advantage of independent expertise for $p \in (1/2, \rho]$ holds for k up to $\overline{k}$. At the same time, for $p \in (\rho, \overline{p}]$, collective expertise has an advantage as long as different messages result in different actions. This remains to be the case only for k below a certain value, denoted $\overline{k}'$: For $k > \overline{k}'$, the safe action is always taken for all $p \in (\rho, \overline{p}]$. Furthermore, it turns out that $\overline{k}' < \overline{k}$. Hence, for $k \in (\overline{k}', \overline{k})$, collective expertise never dominates independent one, whereas the latter is better than the former for a subset of p belonging to $(1/2, \rho]$.[6]

Proposition 2 implies that, unless the safe action is so attractive that the expertise scheme is irrelevant, a higher value of the safe action makes the independent expertise more likely to be optimal. Of course, for any fixed p and ρ, either one or the other scheme is weakly preferred for all k. Imagine, however, that the decision-maker needs to set up an expertise scheme before he learns the prior or the experts' ex-ante quality or cannot condition the choice of the scheme on these parameters for some reason. (An example of such "institutionalized" scheme is the academic refereeing process). Then the optimal choice of the scheme will depend on k, and Proposition 2 implies the following:

Corollary 1 *Unless the safe option becomes very attractive, the higher its value is, the more likely independent expertise is to be optimal.*

6 More than two experts

We now ask: How does the communication between experts and decision maker change when the number of (identical) experts is higher than two? The good news is that, qualitatively, it does not change: under independent reporting, they will report truthfully if and only if $p \leq \rho$; under collective expertise, roughly speaking, they will only communicate to the decision maker which state is more likely, but this partial information transmission can be achieved also for values of p up to a threshold $\widehat{p} > \rho$, which tends to 1 as the number of experts goes to infinity. Moreover, as the number of experts grows, the aggregate information about the state becomes more and more accurate, and asymptotically the decision maker learns the true state (for any value of p).

Under independent expertise, the behavior of each expert does not depend on the number of other experts and, thus, is fully described by Lemma 1.

Under collective expertise, the non-existence of a fully revealing equilibrium continues to hold:

Lemma 7 *For any number of experts, under collective expertise, a fully revealing equilibrium does not exist*

Proof. See the Appendix. ■

To examine partially revealing equilibria, like in the two homogeneous experts case, we look for equilibria where any two profiles of signals that contain the same number

[6] The fact that independent expertise is capable of delivering signal-contingent actions for some values of k above $\overline{k}'$ roughly follows from the following fact: For p close to ρ, signal $(0, 0)$ is very informative of state 0, which results in taking $a = 0$, rather than the safe action.

of zeros generate the same equilibrium distribution over messages. This is natural given that such two profiles of signals induced the same posterior distribution over the state. In such an anonymous equilibrium, all experts have identical ex-post reputation, therefore the incentives of the deputy expert are perfectly aligned with the incentives of the other experts, who then have the incentive to report truthfully to the deputy. The deputy can be any of the experts, as all experts are ex-ante identical.

We are going to show now (Theorem 1) that any equilibrium under collective expertise is essentially equivalent to the experts reporting just whether they have received more or less than k zeroes (with a possible randomization for k zeroes). To establish this result we need several preliminary steps.

We say that an equilibrium message m is supported by a set of "numbers of zeros" $P \subseteq \{0, ..., n\}$ if m is sent with positive probability if and only if the experts receive a number $k \in P$ of zero signals. For example, if m is sent if and only if the number of zeroes belongs to $\{3, 4, 5\}$, m is supported by the set of numbers of zeroes $\{3, 4, 5\}$.

We say that two messages m, m' are *reputation-equivalent* if, for each $\omega = 0, 1$, $\Pr(G|\omega, m) = \Pr(G|\omega, m')$. Consider now equilibria with the following property: There exist sets of consecutive "numbers of zeros" (i.e., "ordered sets"), whose pairwise intersections have at most one threshold number in common, such that (1) each equilibrium message is supported within one of the sets, (2) each set supports some equilibrium message, and (3) any two messages that are supported within the same set are reputation-equivalent. Since reputation-equivalent messages can always be pooled into one without any effect on the experts' incentives, and since the deputy can randomize between non reputation-equivalent messages only at the thresholds, we call these equilibria "partitional".

Definition 1 *We say that an equilibrium is partitional when there exist non-empty sets* $P^1, ..., P^J \subseteq \{0, ..., n\}$ *with* $\cup_{j=1,...J} P^j = \{0, ..., n\}$ *and* $\max P^{j-1} \in \{\min P^j - 1, \min P^j\}$ *for each* $j = 2, ..., J$ *if* $J \geq 2$*, such that:*

1. *for each equilibrium message* m*, there exists* $j(m) \in \{1, ..., J\}$ *such that* m *is supported by some* $P \subseteq P^{j(m)}$*;*

2. *for each* $j = 1, ..., J$*, there exists an equilibrium message* m *such that* $j(m) = j$*;*

3. *any two messages* m *and* m' *with* $j(m) = j(m')$ *are reputation-equivalent.*

A simple argument shows that all equilibria are partitional.

Lemma 8 *Every equilibrium is partitional.*

Proof. See the Appendix. ■

The proposition basically says that equilibrium messages are "ordered": It cannot be the case that an equilibrium message is sent under two different numbers of zeroes, k' and k'', and a different, *not reputationally equivalent*, message is sent for some k between k' and k''. The intuition is as follows. Take two *equilibrium* messages, m^1 and m^2, which are not reputationally equivalent. Suppose m^1 generates a higher reputation than m^2 if $\omega = 0$ is revealed. This automatically means that m^1 results in a lower reputation than m^2 if the revealed state is $\omega = 1$, otherwise m^2 would never be sent in equilibrium. Consequently, the attractiveness of m^1 relative to m^2 grows with the perceived probability of $\omega = 0$. Now, suppose m^1 is weakly preferred to m^2 by experts with k zeroes. Then, it will be strictly preferred to m^2 for *any* higher number of zeroes, because of a higher likelihood of $\omega = 0$.

As already mentioned, two reputation-equivalent messages can obviously be coalesced into one message that gives exactly the same reputation as the original two under each state, thus that satisfies the experts' equilibrium incentives as well. Hence, for any equilibrium in which more than one reputation-equivalent message is sent within some partition elements, there exists an equilibrium with *exactly the same* partition in which only one message is sent within each partition element (with the qualification that threshold types belonging to two adjacent sets randomize between the "neighboring" messages). Therefore, we can focus on these "*almost pure*" equilibria to prove the following result: no equilibrium can be based on a partition with more than two elements. We call the equilibria that satisfy Definition 1 with $J = 2$ "bipartitional"; we say that an equilibrium is "at most bipartitional" if it satisfies Definition 1 with $J = 1$ or $J = 2$. We need two preparatory lemmas.

Lemma 9 *Suppose message m is believed to be sent if and only if the experts received between l and r zeros (inclusive), where $0 \leq l < r \leq n$, with the exception that the threshold signal-types may randomize between sending m and a corresponding neighboring message. Then, at least one of the following holds:*

1. *experts who received r zeros weakly prefer revealing it to sending m, and then they consider state 0 strictly more likely;*

2. *experts who received l zeros weakly prefer revealing it to sending m, and then they consider state 1 strictly more likely.*

Proof. See the Appendix. ■

Lemma 10 *Suppose the experts received exactly k signals equal to $\overline{\omega}$ and consider state $\overline{\omega}$ strictly more likely. If there is an equilibrium message m which is never sent when the* *number of signals $\overline{\omega}$ in the profile is below k and is sent with a positive probability when it is above k, they strictly prefer sending m to revealing the true number of signals $\overline{\omega}$ they received.*

Proof. See the Appendix. ■

Theorem 1 *Every equilibrium is at most bipartitional.*

Proof. By Lemma 8, each equilibrium is partitional. Suppose by contradiction that the equilibrium has three ordered messages. Take the intermediate message, i.e. a message interpreted as "the experts have received between l and r zeros". Then, by Lemma 9, either the experts with l zeros or the experts with r zeros would prefer to reveal their exact number of zeros rather than sending the message and consider state 1 or state 0 (respectively) strictly more likely. But then, by Lemma 10, such profile of experts would deviate to the neighbouring message. ■

It is interesting to observe the contrast with cheap talk à la Crawford and Sobel (1982), where the friction is not reputation concerns but a misalignment of preferences between the sender and the receiver over the receiver's actions. In that model, the equilibrium messages yield an ordered partition of the sender's information space that can be finer than a bipartition. The intuition behind this difference is the following. In Crawford and Sobel's cheap talk game, the high threshold type of an intermediate message would like to pretend to be a higher type. But letting the higher message correspond to a sufficiently larger set of types than the intermediate message, the impression this threshold type would give by sending the higher message becomes "excessively high" even for his bias. On the contrary, in our model, a deputy who considers state ω strictly more likely prefers to pretend that every expert has received signal ω rather than revealing the true signals profile. However, one could hope to compensate this with a larger intermediate message, that does not reveal precisely the experts' signals profile. The experts draw indeed an advantage from larger messages, because the ambiguity regarding their signals profile induces the decision maker to skew her belief towards a higher number of signals that correspond to the true state, once she learns it. Yet, as the size of the hypothetical intermediate message grows, the confidence of at least one threshold type in one of the states grows quicker than the benefit of the ambiguity (see Lemma 9), inducing a deviation towards the neighbouring message (see Lemma 10).

Interestingly, if we substitute the group of n experts, each receiving one signal, with one expert receiving n signals, there can be an equilibrium with an intermediate message. The intermediate message can be interpreted as the expert "abstaining" about the true state, and is more likely to be sustained in equilibrium when (i) the prior probability of being a good expert is high, and (ii) good experts nevertheless make mistakes with a sufficiently high probability. In our n experts model, the expected reputation of an individual expert under each truthfully revealed signals profile is a simple lottery over the same two values — the reputations for a surely correct or incorrect guess — where the probabilities, in turn, are the expected shares of correct and incorrect signals across the two states. Therefore, the expert just wants to minimize the expected number of mistakes, which leads to a "corner solution": betting on the more likely state. In the one expert with n signals case, all mistakes are certainly to attributed to the expert, and their detrimental effect on reputation is non linear: under suitable values of the parameters, the downfall of reputation in case of many mistakes dominates the potential increase in reputation in case of many correct guesses. An example of one expert receiving two signals and "abstaining" in equilibrium when the signals are conflicting is available upon request.

Now we will show that a bipartitional equilibrium actually exists up to a value of p that would preclude informative communication under independent reporting. We also argue that this threshold asymptotically goes to 1. Also in this case, we need two lemmas.

Lemma 11 *There exists $\overline{p} \in (\rho, 1)$ tending to 1 as $n \to \infty$ such that the experts with all ones weakly prefer to reveal themselves rather than sending the complementary message if and only if $p \leq \overline{p}$.*

Proof. See the Appendix. ■

Lemma 12 *Let $\overline{\overline{p}}$ be the value of p such that the experts with all ones consider the two states equally likely. There exists $\widehat{p} \in \left[\overline{p}, \overline{\overline{p}}\right]$ such that a bipartitional equilibrium exists if and only if $p \in (1/2, \widehat{p}]$.*

Proof. See the Appendix. ■

Theorem 2 *For every $n \geq 2$, there exists $\widehat{p} > \rho$ tending to 1 as $n \to \infty$ such that a bipartitional equilibrium exists if and only if $p \in \left(\frac{1}{2}, \widehat{p}\right]$.*

Proof. Immediate from Lemma 12 and Lemma 11. ■

Theorem 2 extends the results of the two experts case by showing that (i) for every number of experts, there are values of p where collective expertise dominates independent expertise, and (ii) this region expands asymptotically with the number of experts.

How informative is an equilibrium bipartition? Although monotonicity of the equilibrium informativeness with respect to n is generally not guaranteed for all n, we show that any given "precision of communication" can be achieved by picking a large enough n, and asymptotically the decision-maker learns the true state in equilibrium.

Theorem 3 *For every* $p \in \left(\frac{1}{2}, 1\right)$ *and* $\mu \in \left[\frac{1}{2}, 1\right)$*, there exists* $\overline{n} > 1$ *such that for every* $n \geq \overline{n}$*, in any bipartitional equilibrium* (m, m')*,* $\Pr(\omega = 1|m) > \mu$*,* $\Pr(\omega = 0|m') > \mu$*.*

Proof. Let n go to infinity and consider any sequence of bipartitional equilibria $(m(n), m'(n))$. Since $\Pr(\sigma_i = 0|\omega = 0) = \Pr(\sigma_i = 1|\omega = 1)$, $\Pr(\omega|\sigma)$ depends on σ only through the difference between the number of zeroes and the number of ones in σ. Thus, if k is the number of zeroes in σ, $\Pr(\omega|\sigma)$ can be rewritten as $\Pr(\omega|k - (n - k)) = \Pr(\omega|2k - n)$. Suppose $k(n)$ is some sequence of k depending on n. Clearly, if $2k(n) - n \to \infty$, then $\lim_{n\to\infty} \Pr(\omega = 0|2k(n) - n) = 1$ (likewise for $\Pr(\omega = 1|n - 2k(n))$ when $n - 2k(n) \to \infty$). Hence, in order to keep the threshold type(s) from deviating to the neighboring message, the difference between the number of zeroes and the number of ones must stay bounded for these types. Then

$$\lim_{n\to\infty} \Pr(m'(n)|\omega = 1) = 0,$$

because, conditional on $\omega = 1$, by the law of large numbers, the proportion of ones $\frac{n-k(n)}{n} \xrightarrow{P} \rho$, whereas it stays bounded away from ρ as $n \to \infty$ even for the type with the lowest number of zeroes sending m'.

This means that

$$\Pr(\omega = 0|m'(n)) = \frac{\Pr(m'(n)|\omega = 0) \cdot p}{\Pr(m'(n)|\omega = 0) \cdot p + \Pr(m'(n)|\omega = 1) \cdot (1 - p)} \to 1 \text{ as } n \to \infty$$

By similar reasoning, $\Pr(\omega = 1|m) \to 1$ as $n \to \infty$. ■

Theorems 2 and 3 imply that, as the number of expert grows large enough, the advantage of collective expertise over independent expertise for $p > \rho$ increases, whereas its disadvantage for $p \leq \rho$ shrinks, as the loss of information under collective expertise diminishes and tends to zero at the limit.

Hence, we can formulate the following proposition:

Proposition 3 *Collective expertise is more likely to be preferred to independent expertise when the number of experts is larger.*

7 Conclusion

In this paper we have studied optimal organization of expertise with multiple experts. The only friction in the model was the experts' reputation concerns, which generated incentives to herd on the state suggested by the prior. Our key question was: shall the experts be allowed to talk to each other before providing advice to the decision-maker? Information-sharing between the experts alleviates their herding-on-the-prior incentives when their receive similar signals. However, it aggravates herding when the experts receive signals opposing each other, as disagreement tends to leave their beliefs close to the prior. As a result, the experts tend to hide disagreement and herd on the prior instead. Thus, some information is inevitably lost (for the decision-maker) under collective expertise.

As a result, collective expertise is beneficial when the prior uncertainty is not too high, so that independent reporting would lead to complete herding on the prior. However, when the prior uncertainty becomes very high, independent reporting becomes fully informative. In such a case, it is better to keep the experts unaware of their potential disagreement (by not allowing them to talk) in order to prevent them from herding on the prior.

Although some information is always lost under collective expertise, it correctly predicts the more likely state, conditional on the experts' information, for a wider range of parameters, compared to independent expertise. Therefore, if the decision-maker just needs to know which state is more likely, collective expertise is always weakly better than the independent one. However, if the decision-maker also needs to know how likely is the more likely state, independent expertise is better, provided it induces no herding (i.e., when the prior uncertainty is sufficiently large). Thus, if the decision-maker, in addition to "betting on the more likely state", has a valuable enough "safe" option, which is optimal to choose whenever there is a high enough residual uncertainty about the state, independent expertise is more likely to be optimal.

Finally, collective expertise is more likely to be optimal as the number of experts grows. This is because any loss of information arising under collective expertise becomes less important (as the experts' aggregate information becomes more precise), whereas the set of parameters under which collective expertise results in information transmission expands.

8 Appendix

8.1 Heterogeneous experts case

Suppose the two experts have different prior abilities: ρ_1 and $\rho_2 \leq \rho_1$. We will show that, unless the heterogeneity is too high, all qualitative results of the model with identical experts hold through. However, the difference between independent and collective expertise diminishes as the heterogeneity grows. At the end we will argue that it is weakly optimal to make the stronger expert the "deputy".

Under independent expertise, since the strategies of the experts are nor related to each other, the solution is obviously as follows:

- for $p \leq \rho_2$ there is full information revelation

- for $p \in (\rho_2, \rho_1]$ only expert 1 reveals her signal
- for $p > \rho_1$ no information is revealed.

Let is now turn to collective expertise and keep assuming that expert 1 is the deputy expert, for now. Rather that performing a full equilibrium analysis, we will focus on how the heterogeneity between the experts affects the existence of a fully revealing equilibrium and whether the equilibrium $(m^0, (1,1))$ becomes easier or more difficult to sustain.

Let us start from the latter question.

Lemma 13 *The equilibrium* $(\{(0,0),(0,1),(1,0)\},\{(1,1)\})$ *exists if and only if* $p \in [\max\{1/2, \underline{p}'\}, \overline{p}]$, *where* $\overline{p}$ *is determined by the incentive compatibility constraint of signal-type (1,1) and* $\underline{p}'$ *– by the condition that expert 2 with* $\sigma_2 = 0$ *is willing to reveal his signal to expert 1. The differences* $\overline{p} - \rho_1$ *and* $\overline{p} - \underline{p}'$ *are decreasing in* ρ_1 *and increasing in* ρ_2 *and turn zero for sufficiently large* ρ_1 *or sufficiently small* ρ_2.

Proof. *See the Appendix (proof of Lemmas 3 and 13).* ■

As in Section 4, threshold $\overline{p}$ results from the no-deviation condition for signal-type $(1,1)$. In contrast to the identical experts case, signal-type $(1,0)$ now may want to deviate, as $\Pr(\omega = 1|\sigma = (1,0))$ may exceed 1/2. Her incentive compatibility constraint would yield threshold $\underline{p}$ (see formula (5) in the proof). However, this condition turns to be never binding, because there appears a stronger constraint: the no-lying condition of expert 2 who received $\sigma_2 = 0$, yielding threshold $\underline{p}' > \underline{p}$.

The logic is as follows. If $\sigma_1 = 1$ and $\sigma_2 = 0$, expert 2, being the weaker expert, suffers more from message m^0 compared to the strong expert: For any realization of the state, the decision maker will rationally assign a higher probability to the weak expert receiving a wrong signal, compared to the strong one – we label this effect "shifting the blame". Therefore, expert 2 has a higher temptation to induce deviation by expert 1 to reporting $(1,1)$ compared to the temptation of expert 1 herself to deviate to reporting $(1,1)$. This can be achieved by misreporting σ_2 when $\sigma_2 = 0$. Notice that such misreporting will induce the deputy to report $(1,1)$ if and only if $\sigma_1 = 1$, for if $\sigma_1 = 0$ the deputy will report m^0 irrespective of expert 2's signal (as $\sigma = (0,1)$ make the deputy believe that $\omega = 0$ is more likely).

There are two effects of ρ_1 and ρ_2 on the incentive compatibility constraint of signal-type $(1,1)$, i.e., on $\overline{p}$. One effect is through confidence about the state: higher either ρ_1 or ρ_2 raises the deputy's belief that $\omega = 1$. The other effect is the familiar "shifting the blame" effect: Similarly to raising the temptation of expert 2 to deviate from m^0 to $(1,1)$, a rise in ρ_1 or a decline in ρ_2 increases the temptation of expert 1 to deviate from $(1,1)$ to m^0. Formally, $\Pr(\sigma_1 = 0|\omega = 0, \sigma \in m^0)$ and $\Pr(\sigma_1 = 1|\omega = 1, \sigma \in m^0)$ both go up with ρ_1 and go down with ρ_2 (the expressions can be found in the proof).

If we increase ρ_1 and decrease ρ_2 in such a way that $\Pr(\omega = 1|\sigma = (1,1))$ stays constant, only the "shifting the blame" effect remains, and, thus, $\overline{p}$ goes down. In general, if ρ_1 increases sufficiently fast relative to a decrease in ρ_2 (so that $\Pr(\omega = 1|\sigma = (1,1))$ rises), $\overline{p}$ may go up. However, we also need to look at the other threshold, $\underline{p}'$. Naturally, it rises as ρ_1 increases or ρ_2 decreases. This is because such changes in ρs: (1) increase $\Pr(\omega = 1|\sigma = (1,0))$ and (2) enhance the "shifting the blame" effect.

To be sure, $\underline{p}'$ may be below $1/2$, but when ρ_1 becomes sufficiently large or ρ_2 becomes sufficiently small, it exceeds $1/2$ and eventually hits $\overline{p}$, at which point the equilibrium of the lemma ceases to exist. This dynamics is shown in Figure 2.

Note also that $\overline{p} - \rho_1$ also decreases as ρ_1 goes up or ρ_2 goes down, which is illustrated in Figure 1. Thus, even if segment $\left[\underline{p}', \overline{p}\right]$ is non-empty, the range of parameters where collective expertise unambiguously dominates independent expertise shrinks.

Consider now the fully revealing equilibrium under collective reporting. In contrast to the identical experts case, it becomes possible because, for sufficiently low values of p, expert 1's signal determines what state is more likely regardless of the signal of expert 2. Therefore, she has an incentive to reveal her signal truthfully independently of the weak expert's information. In turn, expert 2, not knowing expert 1's signal, will tell the truth to the deputy, provided that the prior is sufficiently close to $1/2$.

Lemma 14 *Under collective expertise, a fully revealing equilibrium exists if and only if* $p \leq \min\{p_{FR}, \rho_2\}$, *where* $p_{FR} = \dfrac{\rho_1(1-\rho_2)}{\rho_1(1-\rho_2) + \rho_2(1-\rho_1)}$. *The value of* p_{FR} *is increasing in* ρ_1 *and decreasing in* ρ_2, *takes value* $1/2$ *for* $\rho_1 = \rho_2$ *and hits* ρ_2 *for sufficiently high* ρ_1 *or sufficiently small* ρ_2.

Proof. The value of p_{FR} is determined by the condition $\Pr(\omega = 0|\sigma = (1,0)) = 1/2$, from which it is straightforward to derive the explicit expression for p_{FR}. For $p > p_{FR}$, $\sigma_2 = 0$ makes expert 1 believe that $\omega = 0$ is more likely even when she has got $\sigma_1 = 1$, and, hence, truthtelling by expert 1 is destroyed – she would deviate to pretending that $\sigma_1 = 0$.

For $p \leq p_{FR}$, expert 1's own signal always determines which state she believes is more likely. Then, following Ottaviani and Sørensen (2001), it is straightforward to show expert 1 always prefers to be perceived as having received the signal corresponding to the state she considers more likely rather than the opposite signal. Thus, she will always truthfully reveal her signal (in the most informative equilibrium). In addition, she does not lose anything from disclosing the weak expert's message as well. Thus, if the latter tells the truth to expert 1, full information will occur in equilibrium. Since expert 2 does not observe the signal of expert 1 when sending his message, his truthtelling incentives (anticipating that his message will be disclosed) are the same as under independent reporting, i.e., expert 2 tells the truth iff $p \leq \rho_2$.

It is straightforward to verify that p_{FR} is increasing in ρ_1 and decreasing in ρ_2, takes value 1/2 for $\rho_1 = \rho_2$ and becomes equal to ρ_2 when. $\frac{\rho_2^2}{(1-\rho_2)^2} = \frac{\rho_1}{1-\rho_1}$. ■

Naturally, an increase in the competence of expert 1 or a decrease in the competence of expert 2 expands the set of p ($p \leq p_{FR}$) for which expert 1's signal alone determines what state is more likely, i.e., for which $\Pr(\omega = 0|\sigma = (1,0)) \leq 1/2$. As a result, truthtelling becomes possible for a wider range of priors under collective expertise, and thus, independent expertise gradually loses its advantage for $\rho < \rho_2$ until segment $[p_{FR}, \rho_2]$ shrinks to zero, see Figure 1.

Figures 1 and 2 summarize the effects of the experts' heterogeneity, presented in Lemmas 13 and 14 Start with $\rho_1 = \rho_2$ and gradually decrease ρ_2 and/or increase ρ_1. The zone $(\rho_1, \overline{p}]$, where collective expertise dominates, shrinks (though $\overline{p}$ does not necessarily have to decrease as depicted), whereas p_{FR} increases. At some point, $\overline{p}$ hits ρ_1 and p_{FR} hits ρ_2 (one can show that both things happen simultaneously). At this point, collective expertise completely loses its advantage for $\rho \geq \rho_1$, and it also loses its *dis*advantage for $\rho < \rho_2$.

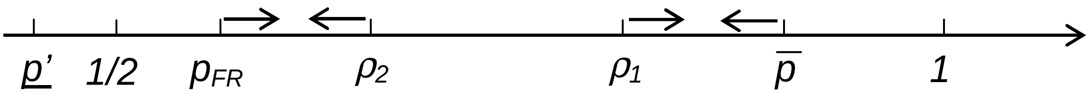

Figure 1. Effects of heterogeneity, when the difference in the abilities is not too high.

Collective expertise, however, may still be preferred for $[\max\{\underline{p}', p_{FR}\}, \overline{p}]$, as equilibrium $(m^0, (1,1))$ may be preferred to just expert 1 revealing her signal (which de facto corresponds to partition $(\{(0,0),(0,1)\},\{(1,0),(1,1)\})$. (To be sure, for $\rho \in [\rho_2, \rho_1]$, collective expertise can never do worse than the independent one, because, under collective expertise, there is always an equilibrium in which expert 2 babbles to expert 1, and expert 1 truthfully reports her own signal).

However, according to Lemma 13, a further decrease in ρ_2 and/or increase in ρ_1 reduces the zone where equilibrium $(m^0, (1,1))$ exists (i.e., segment $[\underline{p}', \overline{p}]$; one can show that, for $\overline{p} < \rho_1$, $\underline{p}'$ exceeds 1/2.) until, at some point, it disappears completely (when $\underline{p}'$ and $\overline{p}$ become equal) – see Figure 2. Moreover, as the gap between ρ_2 and ρ_1 widens, it becomes less likely that $(m^0, (1,1))$ is preferred to $(\{(0,0),(0,1)\},\{(1,0),(1,1)\})$, because knowing expert 1's signal becomes "on average" more important relative to learning whether both experts received signals 1 or not.

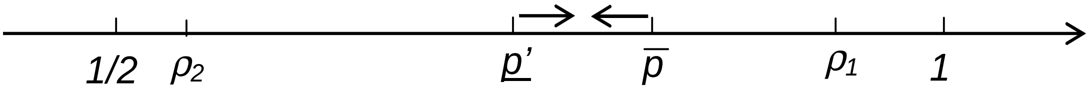

Figure 2. Effects of heterogeneity, when the difference in the abilities is very high.

Overall, the above analysis implies the following proposition:

Proposition 4 *When heterogeneity between the experts is small enough, all the qualitative results of the model with ex-ante identical experts hold through. However, as the heterogeneity grows, the choice of the expertise scheme becomes less and less relevant, as collective expertise loses its advantage for high priors, and independent expertise loses its advantage for low priors.*

Remark on the optimality of expert 1 as the deputy: Suppose the roles of the experts are inverted: expert 2 is the deputy, and expert 1 has to report to expert 2. It is easy to show that Lemma 13 remains intact. This is because the no-lying conditions of a non-deputy expert are equivalent to his/her incentive compatibility conditions once he/she becomes a deputy. For example, expert 2, being a non-deputy, can influence the message of expert 1 to the decision-maker only if the latter received $\sigma_1 = 1$. Thus, by considering whether to lie to expert 1 or not, he considers a deviation from m^0 to $(1,1)$ when $\sigma = (1,0)$ and a deviation from $(1,1)$ to m^0 when $\sigma = (1,1)$, as if he actually were the deputy. The same is true for expert 1. Thus, regardless of who is assigned the role of the deputy, the same four constraints are the necessary and sufficient conditions for the equilibrium $(m^0, (1,1))$ to exist: for $\forall i \in \{1,2\}$, expert i must not be willing to deviate to $(1,1)$ when $\sigma = (1,0)$ and to m^0 when $\sigma = (1,1)$

However, two of these conditions are always stronger than the other two: the no-deviation condition of expert 2 when $\sigma = (1,0)$ (yielding $\underline{p}'$) and the no-deviation condition of expert 1 when $\sigma = (1,1)$ (yielding $\overline{p}$).

In contrast, the fully revealing equilibrium never exists when expert 2 is the deputy. Since expert 1's signal is stronger than that of expert 2, there cannot be a situation in which expert 2's signal determines which state is more likely regardless of expert 1's signal. In particular, if expert 1 reveals $\sigma_1 = 0$ to expert 2, the latter will fail to reveal $\sigma_2 = 1$ to the decision-maker, as he believes that $\omega = 0$ is more likely.

Thus, making expert 2 the deputy is weakly suboptimal.

8.2 Robustness to the communication protocol

All our equilibria for the collective expertise scenario rely on the equilibrium selection hypothesis that the experts share their signals truthfully. How is this robust to different assumptions about the nature of their interaction or equilibrium refinement arguments? For simplicity, we consider a setting with two experts.

If experts' signals were verifiable among them, truthful revelation to each other would be the only feasible option for the experts. Note that signals verifiability among experts is not at odds with being unverifiable for the decision maker, signals typically need specialized expertise to be understood.

What if we keep signals unverifiable and we let both experts speak to the decision-maker? Let us consider an alternative game in which both experts send a private message to the decision maker after talking simultaneously with each other. We are going to show that when our equilibrium of interest exists, i.e. the one where the deputy communicates $(1, 1)$ or m^0, it exists also in the alternative game, in the sense that the experts report truthfully to each other and both communicate to the decision maker $(1, 1)$ or m^0.

Moreover, it also survives an adaptation of neologism-proofness and of a strengthening of the intuitive criterion to this double-cheap-talk game with multiple senders whenever independent expertise does not generate (full) information transmission.

Let us consider a candidate equilibrium where the experts report truthfully to each other, both report truthfully to the decision maker whether their declared profile is $(1, 1)$ or not, and the decision maker believes that the true signal profile is $(1, 1)$ whenever at least one of the two experts reports $(1, 1)$, and has the other equilibrium belief (i.e. that the experts' signals are $(0, 0)$, $(0, 1)$ or $(1, 0)$) otherwise.

It is immediate to check that this is an equilibrium. But is it plausible that the decision maker believes that signals are $(1, 1)$ when just one expert, say expert 2, reports so? It could also be the case that expert 1 received 0 but lied to expert 2: if expert 2 receives signal 0, the lie is inconsequential; if expert 2 receives signal 1, expert 2 is tricked into reporting $(1, 1)$ and expert 1 can "admit the lie" with the decision maker to convince her that he got signal 0. Provided that the decision-maker is convinced by expert 1, it can be shown that such "tricking strategy" will indeed be a profitable deviation for expert 1 with signal 0 whenever $p > p_{cheat}$, where $p_{cheat} \in (1/2, \rho)$. That is, for such values of p, he prefers revealing his signal 0 rather than sending m^0, regardless of expert 2's signal.

Net of obvious out-of-the-model considerations that make such admission of lie unattractive, we are going to check whether this alternative interpretation of incongruent reports by the decision maker is the only reasonable one according to neologism proofness, and if our equilibrium interpretation is reasonable according to the spirit of the intuitive criterion.

Since for $p \leq p_{cheat}$ the "tricking strategy" is unprofitable anyway, we will only consider the case $p > p_{cheat}$.

Let us start from neologism-proofness. In neologism-proofness, deviations are interpreted under the view that the sender believes that equilibrium messages are interpreted as expected, while neologisms are expected to be taken at face value. We are going to apply this idea as well. But since we have two senders in our model, a further question arises: do we want to force expert 1 to believe that expert 2 never lies to him? Such a restriction seems unjustified: An expert who schemes as above can also conceive that the other expert schemes in the same way.

So, we first formulate the following adaptation of neologism proofness (we specify it without loss of generality for the case above):

> When the decision maker receives message $(1, 1)$ from expert 2 and a different message from expert 1, she must conclude that expert 1 received signal 0 if the following condition is satisfied. Expert 1 may prefer to send the alternative message when he truly received signal 0 but not when he received signal 1 under the following hypotheses:
>
> (i) he expects the alternative message to be interpreted as evidence that he got signal 0;
>
> (ii) he expects $(1, 1)$ to be interpreted as evidence that he received signal 1;
>
> (iii) he expects expert 2 to behave rationally under (i) and (ii) as well.

Let us suppose that expert 1 got signal 1, reports signal 1 to and hears 1 from expert 2, but thinks that the expert 2 always reports 1 even when his true signal is 0, to then admit his lie in front of the decision maker when he hears 1 from expert 1. As depicted for expert 1 with signal 0 above, this is a rational behavior for expert 2, so (iii) is satisfied for $p > p_{cheat}$. If expert 1, conditional on his signal 1, considers state 1 more likely, he will not change his mind after hearing 1 from expert 2, which he regards as babbling. So, for $p < \rho$, expert 1 prefers not to send $(1, 1)$ under hypotheses (i) and (ii) only when he truly gets signal 0. Thus, adapted neologism proofness has bite and our equilibrium of interest does not survive this refinement. But, if expert 1, conditional on his signal 1, still considers state 0 more likely, he will prefer not to send $(1, 1)$ as well, given that hearing 1 from expert 2 does not change his mind. Therefore, for $p \geq \rho$, the decision maker needs not conclude that expert 1 truly received 0. So, adapted neologism proofness has no bite.

Now, we take the opposite perspective, which is typical of the intuitive criterion. Is our off-path belief reasonable under the following view: The sender would never make a deviation that, for any belief, cannot yield at least the expected payoff from the equilibrium messages under the equilibrium interpretation. In cheap talk games, the original intuitive criterion has no bite because messages are costless. To obtain refinement power, we impose that senders would never deviate if they do not expect a *strict* improvement of their payoff. In our case of interest, this can be expressed as follows:

> When the decision maker receives message $(1,1)$ from expert 2 and a different message from expert 1, she must *exclude* that expert 1 received signal 1 if the following condition is satisfied. Expert 1 may *strictly* prefer to send the alternative message when he received signal 0 but not when he received signal 1 under the following hypotheses:
>
> (i) he expects $(1,1)$ to be interpreted as evidence that he received signal 1;
>
> (ii) he expects expert 2 to behave rationally under (i).

When expert 1 with signal 1 believes that also expert 2 has received signal 1, he cannot improve his expected payoff by sending a message different from $(1,1)$. Therefore, any other messages shall be interpreted as belief that, instead, expert 1 considers state 0 strictly more likely. But where can this belief come from? It could as well come from the belief that expert 2, despite declaring having received 1, has actually received 0 but is trying to deceive expert 1, exactly as discussed for neologism-proofness. Then, for $p > \rho$, expert 1 with signal 1 will still consider state 0 more likely. Under this interpretation, the decision maker can believe that expert 1 actually received signal 1. Therefore, our off-path belief is not ruled out by this adaptation of the intuitive criterion to our multiple senders context.

8.3 Preliminaries to proofs. The case of identical experts.

Denote:

$$x_i := \Pr(t = G|\sigma_i = \omega); \;\; y_i := \Pr(t = G|\sigma_i \neq \omega)$$

– the expected reputation of expert i conditional on having received correct and incorrect signal respectively. Since the experts are identical, we can drop subscript i at x and y. Using the Bayes Rule, one can easily derive:

$$x = \frac{qg}{\rho}, \;\; y = \frac{q(1-g)}{(1-\rho)}$$

Let m be the message sent by the "deputy" expert to the decision-maker and I – the information available to her prior to reporting to the decision-maker. Then, expert i's expected reputation from m (conditional on I) is:

$$R_i(m,I) = \Pr(\omega=0|I)[\Pr(\sigma_i=0|\omega=0,m)\cdot x + \Pr(\sigma_i=1|\omega=0,m)\cdot y]+$$
$$+\Pr(\omega=1|I)[\Pr(\sigma_i=0|\omega=1,m)\cdot y + \Pr(\sigma_i=1|\omega=1,m)\cdot x] =$$
$$= \alpha(m,I)\cdot x + \beta(m,I)\cdot y,$$

where

$$\alpha(m,I) := \Pr(\omega=0|I)\Pr(\sigma_i=0|\omega=0,m) + \Pr(\omega=1|I)\Pr(\sigma_i=1|\omega=1,m)$$
$$\beta(m,I) := \Pr(\omega=0|I)\Pr(\sigma_i=1|\omega=0,m) + \Pr(\omega=1|I)\Pr(\sigma_i=0|\omega=1,m)$$

It is easy to see that $\alpha+\beta=1$. It is also straightforward to derive that $x>y$. Therefore, all comparisons of expected reputations are equivalent to comparing values of $\alpha(m,I)$:

$$R_i(m',I) > R_i(m'',I) \Leftrightarrow \alpha(m',I) > \alpha(m'',I), \text{ for any } I \text{ and any } m' \text{ and } m'' \quad (1)$$

8.4 Proofs

Proof of Lemma 2. Suppose expert 2 has truthfully revealed his signal to expert 1. Let message m' be a message sent with a *positive* probability by signal-type $(1,0)$. Full revelation requires that, for any vector of signals, the number of zeroes is truthfully revealed. This implies that, in a fully revealing equilibrium, message m' can only be sent by either signal-type $(1,0)$ or $(0,1)$.

We would like to show that it is profitable for signal-type $(1,0)$ to deviate to reporting $(0,0)$, that is, $\alpha(m=(0,0),\sigma=(1,0)) > \alpha(m=m',\sigma=(1,0))$.

Since $\Pr(\sigma_1=1|m=(0,0))=0$ in a fully revealing equilibrium, irrespective of the realized state, we have

$$\alpha(m=(0,0),\sigma=(1,0)) = p$$

Now compute $\alpha(m=m',\sigma=(1,0))$.

Denote:

$$\mu \;:\; = \Pr(m=m'|\sigma=(1,0))$$
$$\nu \;:\; = \Pr(m=m'|\sigma=(0,1))$$

Using the fact that message m' is never sent by signal-types $(0,0)$ and $(1,1)$, we can derive:

$$\Pr(\sigma_1 = 0|\omega = 0, m = m') = \frac{\Pr(\sigma_1 = 0 \cap m = m'|\omega = 0)}{num. + \Pr(\sigma_1 = 1 \cap m = m'|\omega = 0)} =$$
$$= \frac{\Pr(\sigma = (0,1) \cap m = m'|\omega = 0)}{num. + \Pr(\sigma = (1,0) \cap m = m'|\omega = 0)} = \frac{\rho(1-\rho)\nu}{\rho(1-\rho)\nu + (1-\rho)\rho\mu} = \frac{\nu}{\nu + \mu}$$

$$\Pr(\sigma_1 = 1|\omega = 1, m = m') = \frac{\Pr(\sigma_1 = 1 \cap m = m'|\omega = 1)}{num. + \Pr(\sigma_1 = 0 \cap m = m'|\omega = 1)} =$$
$$= \frac{\Pr(\sigma = (1,0) \cap m = m'|\omega = 1)}{num. + \Pr(\sigma = (0,1) \cap m = m'|\omega = 1)} = \frac{\rho(1-\rho)\mu}{\rho(1-\rho)\mu + (1-\rho)\rho\nu} = \frac{\mu}{\nu + \mu}$$

Hence,

$$\alpha(m = m', \sigma = (1,0)) =$$
$$= p \cdot \frac{\nu}{\nu + \mu} + (1-p) \cdot \frac{\mu}{\nu + \mu}$$

Since, by assumption, $\mu > 0$, $\alpha(m = m', \sigma = (1,0)) < p = \alpha(m = (0,0), \sigma = (1,0))$ ■

Proof of Lemmas 3 and 13. As Lemma 13 covers the more general case, in this proof we allow the experts to have ex-ante different expected abilities, ρ_1 and ρ_2. Assuming $\rho_1 > \rho_2$, we need to check the incentive compatibility constraints of signal-types $(1,1)$ and $(1,0)$. There is no need to check those for signal-types $(0,1)$ and $(0,0)$, for if either of them wants to deviate to $(1,1)$, then $(1,0)$ definitely wants to deviate, as she assigns a higher probability to $\omega = 1$ compared to the other two signal-types. We will first assume truthful reporting by expert 2 to expert 1. Then we will verify that expert 2 will not want to deviate from telling the truth in equilibrium.

Incentive compatibility of signal-type $(1,1)$:

First, compute α of signal-type $(1,1)$ if she does not deviate.

$$\begin{aligned} \Pr(\omega &= 0|\sigma = (1,1)) = \frac{\Pr(\sigma = (1,1)|\omega = 0)\Pr(\omega = 0)}{num. + \Pr(\sigma = (1,1)|\omega = 1)\Pr(\omega = 1)} = \\ &= \frac{(1-\rho_1)(1-\rho_2)p}{(1-\rho_1)(1-\rho_2)p + \rho_1\rho_2(1-p)} \end{aligned} \tag{2}$$

$$\Pr(\sigma_1 = 0|\omega = 0, \sigma \in (1,1)) = \Pr(\sigma_1 = 0|\omega = 1, \sigma \in (1,1)) = 0$$

Thus,

$$\begin{aligned}\alpha(m &= (1,1), \sigma=(1,1)) = \\ &= \Pr(\omega=0|\sigma=(1,1))\cdot 0 + \Pr(\omega=1|\sigma=(1,1))\cdot 1 = \\ &= \frac{\rho_1\rho_2(1-p)}{(1-\rho_1)(1-\rho_2)p+\rho_1\rho_2(1-p)} \end{aligned} \tag{3}$$

Now, compute α of signal-type $(1,1)$ if she deviates to m^0.

$$\Pr(\sigma_1=0|\omega=0,\sigma\in m^0) = \frac{\Pr(\sigma_1=0\cap\sigma\in m^0|\omega=0)}{\Pr(\sigma\in m^0|\omega=0)} =$$
$$= \frac{\Pr(\sigma_1=0\cap\sigma\in(0,0)|\omega=0)+\Pr(\sigma_1=0\cap\sigma\in(0,1)|\omega=0)+\Pr(\sigma_1=0\cap\sigma\in(1,0)|\omega=0)}{\Pr(\sigma\in m^0|\omega=0)} =$$
$$= \frac{\rho_1\rho_2+\rho_1(1-\rho_2)}{\rho_1\rho_2+\rho_1(1-\rho_2)+(1-\rho_1)\rho_2} = \frac{\rho_1}{(1-\rho_1)\rho_2+\rho_1}$$

$$\Pr(\sigma_1=1|\omega=1,\sigma\in m^0) = \frac{\Pr(\sigma_1=1\cap\sigma\in m^0|\omega=1)}{\Pr(\sigma\in m^0|\omega=1)} =$$
$$= \frac{\Pr(\sigma_1=1\cap\sigma\in(0,0)|\omega=1)+\Pr(\sigma_1=1\cap\sigma\in(0,1)|\omega=1)+\Pr(\sigma_1=1\cap\sigma\in(1,0)|\omega=1)}{\Pr(\sigma\in m^0|\omega=1)} =$$
$$= \frac{\rho_1(1-\rho_2)}{(1-\rho_1)(1-\rho_2)+(1-\rho_1)\rho_2+\rho_1(1-\rho_2)} = \frac{\rho_1(1-\rho_2)}{1-\rho_1\rho_2}$$

Thus,

$$\begin{aligned}\alpha(m &= m_0, \sigma=(1,1)) = \\ &= \Pr(\omega=0|\sigma=(1,1))\cdot\frac{\rho_1}{(1-\rho_1)\rho_2+\rho_1} + \Pr(\omega=1|\sigma=(1,1))\cdot\frac{\rho_1(1-\rho_2)}{1-\rho_1\rho_2} = \\ &= \frac{(1-\rho_1)(1-\rho_2)p}{(1-\rho_1)(1-\rho_2)p+\rho_1\rho_2(1-p)}\cdot\frac{\rho_1}{(1-\rho_1)\rho_2+\rho_1} + \\ &\quad + \frac{\rho_1\rho_2(1-p)}{(1-\rho_1)(1-\rho_2)p+\rho_1\rho_2(1-p)}\cdot\frac{\rho_1(1-\rho_2)}{1-\rho_1\rho_2}\end{aligned}$$

The expert will not deviate whenever

$$\alpha(m=(1,1),\sigma=(1,1)) \geq \alpha(m=m^0,\sigma=(1,1)),$$

which yields

$$p \leq \frac{\rho_2[(1-\rho_1)\rho_2+\rho_1]}{1-\rho_2+\rho_2^2} =: \overline{p}. \tag{4}$$

For $\rho_1=\rho_2=\rho$,

$$\overline{p} = \frac{\rho^2(2-\rho)}{1-\rho+\rho^2}$$

It is straightforward to show that $\overline{p} > \rho$, given that $\rho > 1/2$.

Let us show now that at $p = \overline{p}$, $\Pr(\omega = 1|\sigma = (1,1)) > 1/2$

$$\Pr(\omega = 1|\sigma = (1,1)) = \frac{\Pr(\sigma = (1,1)|\omega = 1) \cdot (1-p)}{\Pr(\sigma = (1,1))} = \frac{\rho^2(1-p)}{\rho^2(1-p) + (1-\rho)^2 p}$$

Setting this expression equal to 1/2 yields $p = \dfrac{\rho^2}{\rho^2 + (1-\rho)^2}$. Simple algebra shows that this is greater than $\overline{p}$. Hence, it must be that $\Pr(\omega = 1|\sigma = (1,1)) > 1/2$ at $p = \overline{p}$.

Incentive compatibility of signal-type $(\mathbf{1},\mathbf{0})$:

First, compute α of signal-type $(1,0)$ if she does not deviate.

$$\Pr(\omega = 0|\sigma = (1,0)) = \frac{\Pr(\sigma = (1,0)|\omega = 0)\Pr(\omega = 0)}{num. + \Pr(\sigma = (1,0)|\omega = 1)\Pr(\omega = 1)} = \frac{(1-\rho_1)\rho_2 p}{(1-\rho_1)\rho_2 p + \rho_1(1-\rho_2)(1-p)}$$

Using the expressions for $\Pr(\sigma_1 = 0|\omega = 0, \sigma \in m^0)$ and $\Pr(\sigma_1 = 1|\omega = 1, \sigma \in m^0)$ derived above, we obtain:

$$\begin{aligned}\alpha(m &= m^0, \sigma = (1,0)) = \frac{(1-\rho_1)\rho_2 p}{(1-\rho_1)\rho_2 p + \rho_1(1-\rho_2)(1-p)} \cdot \frac{\rho_1}{(1-\rho_1)\rho_2 + \rho_1} + \\ &+ \frac{\rho_1(1-\rho_2)(1-p)}{(1-\rho_1)\rho_2 p + \rho_1(1-\rho_2)(1-p)} \cdot \frac{\rho_1(1-\rho_2)}{1-\rho_1\rho_2}\end{aligned}$$

Now, compute α of signal-type $(1,0)$ if she deviates to $(1,1)$.

$$\Pr(\sigma_1 = 0|\omega = 0, \sigma \in (1,1)) = \Pr(\sigma_1 = 0|\omega = 1, \sigma \in (1,1)) = 0$$

Thus,

$$\begin{aligned}\alpha(m &= (1,1), \sigma = (1,0)) = \Pr(\omega = 0|\sigma = (1,0)) \cdot 0 + \Pr(\omega = 1|\sigma = (1,0)) \cdot 1 = \\ &= \frac{\rho_1(1-\rho_2)(1-p)}{(1-\rho_1)\rho_2 p + \rho_1(1-\rho_2)(1-p)}\end{aligned}$$

The expert will not deviate whenever $\alpha(m = m^0, \sigma = (1,0)) \geq \alpha(m = (1,1), \sigma = (1,1))$, which yields

$$(1-\rho_1\rho_2)\rho_2 p \geq (1-\rho_2)[(1-\rho_1)\rho_2 + \rho_1](1-p)$$

or

$$p \geq \frac{(1-\rho_2)[(1-\rho_1)\rho_2 + \rho_1]}{\rho_2(1-\rho_1\rho_2) + (1-\rho_2)[(1-\rho_1)\rho_2 + \rho_1]} =: \underline{p} \tag{5}$$

For $\rho_1 = \rho_2 = \rho$, the condition becomes

$$p \geq \frac{2-\rho}{3}$$

As $\rho > 1/2$, the left-hand side is always below 1/2. Thus, for $\rho_1 = \rho_2 = \rho$, the incentive compatibility condition of signal-type $(1,0)$ is always satisfied.

Truthtelling by expert 2 to expert 1

Let us finally show that, when the experts are ex-ante identical, expert 2 will indeed truthfully reveal his signal to expert 1, if the incentive compatibility conditions of the latter (derived above) hold.

Expert 2 can influence the message of expert 1 only if the latter received $\sigma_1 = 1$. Thus, his incentives to misreport should be considered given $\sigma_1 = 1$. Then, given that the experts are identical, if expert 2 has received 0, his incentives to misreport are identical to that of expert 1 who knows that $\sigma = (1,0)$ and considers a deviation to reporting $(1,1)$. Analogously, if expert 2 has received 1, his incentives to misreport are identical to that of expert 1 who knows that $\sigma = (1,1)$ and considers a deviation to reporting m^0.

Thus, the incentive compatibility conditions of expert 2 are identical to those of expert 1 derived above and, thus, can be ignored.

If $\rho_1 > \rho_2$, then the following is easy to show: The no-lying incentives of expert 2 are equivalent to *his* incentives not to deviate if he were the deputy. The reason is that expert 2, being a non-deputy, can influence the message of expert 1 to the decision-maker only if the latter received $\sigma_1 = 1$. Thus, by considering whether to lie to expert 1 or not, he considers a deviation from m^0 to $(1,1)$ when $\sigma = (1,0)$ and a deviation from $(1,1)$ to m^0 when $\sigma = (1,1)$, as if he actually were the deputy.

Analyzing the former deviation yields condition $p \geq \underline{p}'$, where $\underline{p}'$ is derived analogously to $\underline{p}$ and equals:

$$\underline{p}' \equiv \frac{\rho_1(1-\rho_2)^2[(1-\rho_1)\rho_2 + \rho_1]}{(1-\rho_1)\rho_2^2(1-\rho_1\rho_2) + \rho_1(1-\rho_2)^2[(1-\rho_1)\rho_2+\rho_1]}$$

Analyzing the latter deviation yields condition $p \leq \overline{p}'$, where $\overline{p}'$ is derived analogously to $\overline{p}$ and equals:

$$\overline{p}' \equiv \frac{\rho_1[(1-\rho_1)\rho_2 + \rho_1]}{1-\rho_1+\rho_1^2}$$

It is easy to show that $\underline{p}' > \underline{p}$ and $\overline{p}' > \overline{p}$. Thus, when the experts are heterogeneous, there are two, rather than four, relevant no-deviation conditions: $p \geq \underline{p}'$ and $p \leq \overline{p}$.

Not surprisingly, $\underline{p}' = \underline{p} = \frac{2-\rho}{3}$ when $\rho_1 = \rho_2$.

Simple algebra shows that the differences $\overline{p} - \rho_1$ and $\overline{p} - \underline{p}'$ are decreasing in ρ_1 and increasing in ρ_2. Finally, it is straightforward to derive that, for $\rho_1 = 1$ or $\rho_2 = 1/2$, $\overline{p} < \rho_1$ and $\overline{p} < \underline{p}'$. ■

Proof of Lemma 4. Since the equilibrium is symmetric to $(m^0, (1,1))$, the incentive compatibility conditions are exactly the same as in the proof of the previous lemma, with the only difference that p has to be substituted with $1-p$.

Thus, the no-deviation condition of signal-type $(0,0)$ is

$$p \geq 1 - \frac{\rho_2[(1-\rho_1)\rho_2 + \rho_1]}{1 - \rho_2 + \rho_2^2} \equiv 1 - \overline{p}$$

Since $\overline{p} > \rho > 1/2$, the right-hand side is smaller than $1/2$, and, hence, the condition is always satisfied.

The no-deviation condition of signal-type $(0,1)$ is

$$p \leq 1 - \frac{(1-\rho_2)[(1-\rho_1)\rho_2 + \rho_1]}{\rho_2(1-\rho_1\rho_2) + (1-\rho_2)[(1-\rho_1)\rho_2 + \rho_1]}$$

For $\rho_1 = \rho_2 = \rho$, the condition becomes

$$p \leq \frac{1+\rho}{3},$$

which is smaller than ρ, as $\rho > 1/2$.

Analogously to the previous lemma, these conditions also ensure truthtelling by expert 2 to expert 1, when the experts are identical. ■

Proof of Lemma 5. First, consider the equilibrium in which signal-type $(1,1)$ randomizes between reporting the truth (with probability μ) and reporting m^0 (with probability $1-\mu$). It must be the case that $R_1(m = (1,1), \sigma = (1,1)) = R_1(m = m_0, \sigma = (1,1))$ or, equivalently (from (1)), $\alpha(m = (1,1), \sigma = (1,1)) = \alpha(m = m_0, \sigma = (1,1))$.

Since, in such an equilibrium, $m = (1,1)$ implies $\sigma = (1,1)$ with certainty, we can use (3) to obtain

$$\alpha(m = (1,1), \sigma = (1,1)) = \frac{\rho^2(1-p)}{(1-\rho)^2 p + \rho^2(1-p)}$$

Let us now compute

$$\begin{aligned} &\alpha(m = m^0, \sigma = (1,1)) \equiv \\ &\equiv \Pr(\omega = 0 | \sigma = (1,1)) \cdot \Pr(\sigma_1 = 0 | \omega = 0, m = m^0) + \\ &+ \Pr(\omega = 1 | \sigma = (1,1)) \cdot \Pr(\sigma_1 = 1 | \omega = 1, m = m^0) \end{aligned}$$

From (2), for the case of identical ρs, we have

$$\Pr(\omega = 0|\sigma = (1,1)) = \frac{(1-\rho)^2 p}{(1-\rho)^2 p + \rho^2(1-p)}$$

Next,

$$\Pr(\sigma_1 = 0|\omega = 0, m = m^0) = \frac{\Pr(m = m^0|\sigma_1 = 0, \omega = 0)\Pr(\sigma_1 = 0|\omega = 0)}{num. + \Pr(m = m^0|\sigma_1 = 1, \omega = 0)\Pr(\sigma_1 = 1|\omega = 0)}$$

Since m^0 is always sent when $\sigma_1 = 0$,

$$\Pr(m = m^0|\sigma_1 = 0, \omega = 0) = 1$$

$$\begin{aligned}
&\Pr(m = m^0|\sigma_1 = 1, \omega = 0) = \\
&= \Pr(\sigma = (0,0)|\sigma_1 = 1, \omega = 0) + \Pr(\sigma \in \{(0,1),(1,0)\}|\sigma_1 = 1, \omega = 0) + \\
&+ \Pr(\sigma = (1,1)|\sigma_1 = 1, \omega = 0)(1-\mu) = \\
&= \rho + (1-\rho)(1-\mu)
\end{aligned}$$

Thus,

$$\Pr(\sigma_1 = 0|\omega = 0, m = m^0) = \frac{\rho}{\rho + [\rho + (1-\rho)(1-\mu)](1-\rho)}$$

Analogously,

$$\Pr(\sigma_1 = 1|\omega = 1, m = m^0) = \frac{\Pr(m = m^0|\sigma_1 = 1, \omega = 1)\Pr(\sigma_1 = 1|\omega = 1)}{num. + \Pr(m = m^0|\sigma_1 = 0, \omega = 1)\Pr(\sigma_1 = 0|\omega = 1)},$$

$$\Pr(m = m^0|\sigma_1 = 0, \omega = 1) = 1,$$

$$\Pr(m = m^0|\sigma_1 = 1, \omega = 1) = 1 - \rho + \rho(1-\mu),$$

yielding

$$\Pr(\sigma_1 = 1|\omega = 1, m = m^0) = \frac{[1-\rho+\rho(1-\mu)]\rho}{[1-\rho+\rho(1-\mu)]\rho + 1 - \rho}$$

Thus,

$$\begin{aligned}
\alpha(m \;\; = \;\; m_0, \sigma = (1,1)) = \frac{(1-\rho)^2 p}{(1-\rho)^2 p + \rho^2(1-p)} \cdot \frac{\rho}{\rho + [\rho + (1-\rho)(1-\mu)](1-\rho)} + \\
+ \frac{\rho^2(1-p)}{(1-\rho)^2 p + \rho^2(1-p)} \cdot \frac{[1-\rho+\rho(1-\mu)]\rho}{[1-\rho+\rho(1-\mu)]\rho + 1 - \rho}
\end{aligned}$$

Solving $\alpha(m = (1,1), \sigma = (1,1)) = \alpha(m = m_0, \sigma = (1,1))$ yields

$$p = \frac{\rho[1-(1-\rho)^2\mu]}{1-\rho\mu(1-\rho)}$$

The right-hand side is increasing in μ and takes values ρ and $\overline{p}$ at $\mu = 0$ and $\mu = 1$ respectively. Thus, the equilibrium in which signal-type $(1,1)$ randomizes between reporting the truth and reporting m^0 does not exist for $p > \overline{p}$.

The equilibrium in which signal-type $(0,0)$ randomizes between reporting the truth and reporting m^1 is symmetric. The indifference condition for signal-type $(0,0)$ thus yields

$$p = 1 - \frac{\rho[1-(1-\rho)^2\mu]}{1-\rho\mu(1-\rho)},$$

which ranges from $1-\overline{p}$ at $\mu = 1$ to $1-\rho$ at $\mu = 0$. Both values are below $1/2$, meaning that such an equilibrium does not exist in our setup.

Finally, consider the equilibrium in which signal-types $(0,0)$ and $(1,1)$ always report the truth, while signal-type $\{(0,1),(1,0)\}$ mixes between reporting $(0,0)$ (with probability μ) and reporting $(1,1)$ (with probability $1-\mu$).

$$\begin{gathered}\alpha(m = (0,0), \sigma = (0,1)) \equiv \\ \equiv \Pr(\omega = 0|\sigma = (0,1)) \cdot \Pr(\sigma_1 = 0|\omega = 0, m = (0,0)) + \\ + \Pr(\omega = 1|\sigma = (0,1)) \cdot \Pr(\sigma_1 = 1|\omega = 1, m = (0,0))\end{gathered}$$

$$\Pr(\omega = 0|\sigma = (0,1)) = p$$

$$\Pr(\sigma_1 = 0|\omega = 0, m = (0,0)) = \frac{\Pr(m = (0,0)|\sigma_1 = 0, \omega = 0)\Pr(\sigma_1 = 0|\omega = 0)}{num. + \Pr(m = (0,0)|\sigma_1 = 1, \omega = 0)\Pr(\sigma_1 = 1|\omega = 0)}$$

$$\begin{aligned}\Pr(m &= (0,0)|\sigma_1 = 0, \omega = 0) = \Pr(\sigma = (0,0)|\sigma_1 = 0, \omega = 0) + \Pr(\sigma = (0,1)|\sigma_1 = 0, \omega = 0)\mu = \\ &= \rho + (1-\rho)\mu \\ \Pr(m &= (0,0)|\sigma_1 = 1, \omega = 0) = \Pr(\sigma = (1,0)|\sigma_1 = 1, \omega = 0)\mu = \rho\mu\end{aligned}$$

Thus,

$$\Pr(\sigma_1 = 0|\omega = 0, m = (0,0)) = \frac{[\rho + (1-\rho)\mu]\rho}{[\rho + (1-\rho)\mu]\rho + \rho\mu(1-\rho)} = \frac{\rho + (1-\rho)\mu}{\rho + 2(1-\rho)\mu}$$

Analogously, it is straightforward to derive that

$$\begin{aligned}\Pr(\sigma_1 &= 1|\omega=1, m=(0,0)) = \frac{\Pr(m=(0,0)|\sigma_1=1,\omega=1)\Pr(\sigma_1=1|\omega=1)}{num. + \Pr(m=(0,0)|\sigma_1=0,\omega=1)\Pr(\sigma_1=0|\omega=1)} = \\ &= \frac{(1-\rho)\mu\rho}{(1-\rho)\mu\rho + [(1-\rho)+\rho\mu](1-\rho)} = \frac{\rho\mu}{2\rho\mu+1-\rho}\end{aligned}$$

Hence,

$$\alpha(m=(0,0),\sigma=(0,1)) = p\cdot\frac{\rho+(1-\rho)\mu}{\rho+2(1-\rho)\mu} + (1-p)\cdot\frac{\rho\mu}{2\rho\mu+1-\rho}$$

By symmetry, to obtain $\alpha(m=(1,1),\sigma=(0,1))$ we just need to replace μ with $1-\mu$ and p with $1-p$:

$$\alpha(m=(1,1),\sigma=(0,1)) = (1-p)\cdot\frac{\rho+(1-\rho)(1-\mu)}{\rho+2(1-\rho)(1-\mu)} + p\cdot\frac{\rho(1-\mu)}{2\rho(1-\mu)+1-\rho}$$

Equation $\alpha(m=(0,0),\sigma=(0,1)) = \alpha(m=(1,1),\sigma=(0,1))$ can be rewritten as

$$\frac{p}{1-p} = \frac{f(\mu)}{f(1-\mu)},$$

where $f(\cdot)$ can be shown to be a decreasing function. Thus, p decreases with μ. It is then straightforward to derive that p ranges from $\frac{2-\rho}{3}$ for $\mu=1$ to $\frac{1+\rho}{3}$ for $\mu=0$. Hence, the equilibrium under consideration does not exist for $p > \frac{1+\rho}{3}$, which is below ρ. ■

Proof of Proposition 2. The first sentence of the proposition is an obvious consequence of Proposition 1.

When $k \leq 1/2$, betting on the more likely state is always better than the safe action. Then, statement (i) of the proposition immediately follows from Lemma 6.

Denote by $\overline{k}$ the minimum value of k for which the safe action is *always* taken under *independent* expertise for *all* $p \in (1/2, \rho)$. This value must make the decision-maker indifferent between taking the safe action and betting on the more likely state under the strongest possible belief about a state that can arise under independent expertise for $p \in (1/2, \rho)$:

$$\overline{k} := \Pr(\omega=0|\sigma=(0,0))|_{p=\rho}$$

It is obvious that for all $k > \overline{k}$ and $p \in (1/2, \rho)$ the safe action is always taken under collective expertise as well.

Similarly, denote by $\overline{k}'$ the minimum value of k for which the safe action is *always* taken under *collective* expertise for *all* $p \in (\rho, \overline{p})$. From Lemmas 3, 4 and the proof of Lemma 5, only two types of informative equilibria exist for $p \in (\rho, \overline{p})$: $(m^0, (1,1))$ and the one in which $(1,1)$ randomizes between reporting m^0 and $(1,1)$. The former is unambiguously more informative than the latter. Moreover, it is easy to show that $\Pr(\omega = 0|\sigma \in \{(0,0),(0,1),(1,0)\}) > \Pr(\omega = 1|\sigma = (1,1))$ for any $p \geq \rho$. Thus, $\overline{k}'$ is determined by

$$\overline{k}' := \Pr(\omega = 0|\sigma \in \{(0,0),(0,1),(1,0)\})|_{p=\overline{p}}$$

It is obvious that for all $k > \overline{k}'$ and $p \in (\rho, \overline{p})$ the safe action is always taken under independent expertise as well: Since the safe action is preferred to betting on $\omega = 0$ conditional on $\sigma \in \{(0,0),(0,1),(1,0)\}$, so it is unconditionally, because $p < \Pr(\omega = 0|\sigma \in \{(0,0),(0,1),(1,0)\})$.

Suppose $\overline{k} > \overline{k}'$, later we will show that this is indeed the case. Then, for all $k > \overline{k}$ and $p \in (1/2, \overline{p})$ the safe action is always taken under either type of expertise, which proves statement (iv) of the proposition.

Consider now $k \in (1/2, \overline{k})$ and $p \in (1/2, \rho)$. From the definition of $\overline{k}$ it follows that $\Pr(\omega = 0|\sigma = (0,0))|_{p=\rho} > k$ for any $k < \overline{k}$. Thus, by continuity, for any given $k < \overline{k}$ there exists a set (p', ρ) such that, for all p belonging to this set, $\Pr(\omega = 0|\sigma = (0,0)) > k$, implying that taking $a = 0$ is optimal for such values of p if $\sigma = (0,0)$.

Now, fix $k < \overline{k}$. There are two possible cases. Suppose first that $\Pr(\omega = 0|\sigma \in \{(0,1),(1,0)\})|_{p=\rho} \leq k$, which means that, for any $p < \rho$, the safe action is optimal if $\sigma \in \{(0,1),(1,0)\}$. Then, when $p \in (p', \rho)$, collective expertise can achieve the optimal signal-contingent policy only if equilibrium $((0,0), m^1)$ is realized. But, due to Lemma 4, it exists only for $p \leq \frac{1+\rho}{3} < \rho$. Hence, for $p \in \left(\max\left\{\frac{1+\rho}{3}, p'\right\}, \rho\right)$, collective expertise cannot achieve the optimal signal-contingent policy, whereas independent expertise can.

Suppose now $\Pr(\omega = 0|\sigma \in \{(0,1),(1,0)\})|_{p=\rho} > k$. Since $\Pr(\omega = 0|\sigma \in \{(0,1),(1,0)\})|_{p=\rho} = \rho$ and $\Pr(\omega = 0|\sigma = (0,0))|_{p=1/2} > \rho$, we have $\Pr(\omega = 0|\sigma = (0,0))|_{p=1/2} > k$. However, for p sufficiently close to 1/2, the safe action is optimal if and only if $\sigma \in \{(0,1),(1,0)\}$ (as $k < \overline{k}$). But this can be achieved only under independent expertise, as collective expertise inevitably pools (fully or partially) $\{(0,1),(1,0)\}$ with either $(0,0)$ or $(1,1)$ or both, depending on the equilibrium.

Thus, for all $k \in (1/2, \overline{k})$, there exists a positive measure subset of p belonging to $(1/2, \rho)$, in which independent expertise strictly dominates collective expertise. Thus, we have proven statements (iii) and (iv) in relation to $p \in (1/2, \rho]$.

Consider now $p \in (\rho, \overline{p})$. For $k \in (\overline{k}', \overline{k})$, the safe action is taken for any $p \in (\rho, \overline{p})$ under either expertise scheme. In contrast, by definition of $\overline{k}'$ and continuity, for $k \in (1/2, \overline{k}')$ there must be a positive measure subset of $p \in (\rho, \overline{p})$ on which the safe action is not taken under collective expertise. Moreover, the optimal action will be messagecontingent there, as $\Pr(\omega = 1|\sigma = (1,1))|_{p=\overline{p}} > 1/2$. Thus, when $k \in (1/2, \overline{k}')$, collective expertise strictly dominates independent one for a positive measure subset of $p \in (\rho, \overline{p})$. This completes the proof of statements (iii) and (iv).

Finally, let us show that $\overline{k} > \overline{k}'$, as we conjectured. Preliminarily, notice that, for any collection of independent signals $\sigma_1, \sigma_2, ...\sigma_n$, the following is true:

$$\Pr(\omega = 0|\sigma_1, ...\sigma_n)|_{p=p'} = \Pr(\omega = 0|\sigma_{m+1}, ...\sigma_n)|_{p=p''},$$
$$\text{where } p'' = \Pr(\omega = 0|\sigma_1, ...\sigma_m)|_{p=p'}$$

It is straightforward to show that $\Pr(\omega = 0|\sigma = 0)|_{p=1/2} = \rho$. It follows then that $\overline{k} \equiv \Pr(\omega = 0|\sigma = (0,0))|_{p=\rho} = \Pr(\omega = 0|\sigma = (0,0,0))|_{p=1/2}$.

Next, since $\Pr(\omega = 1|\sigma = (1,1))|_{p=\overline{p}} > 1/2$ (according to Lemma 3), we have $\Pr(\omega = 1|\sigma = (1,1))|_{p=1/2} > \overline{p}$, and, by symmetry, $\Pr(\omega = 0|\sigma = (0,0))|_{p=1/2} > \overline{p}$ This implies that $\Pr(\omega = 0|\sigma = 0)|_{p=\overline{p}} < \Pr(\omega = 0|\sigma = (0,0,0))|_{p=1/2}$. Thus, the following chain of relationships is true:

$$\overline{k}' \equiv \Pr(\omega = 0|\sigma \in \{(0,0),(0,1),(1,0)\})|_{p=\overline{p}} < \Pr(\omega = 0|\sigma \in \{(0,0),(0,1)\})|_{p=\overline{p}} =$$
$$= \Pr(\omega = 0|\sigma_1 = 0)|_{p=\overline{p}} < \Pr(\omega = 0|\sigma = (0,0,0))|_{p=1/2} = \overline{k}$$

■

Proof of Lemma 7. Suppose all non-deputy experts have truthfully revealed their signals to the deputy. Let message m' be a message sent with a *positive* probability by the deputy when she is the only one with signal 1. Full revelation requires that, for any vector of signals, the number of zeroes is truthfully revealed. This implies that, in a fully revealing equilibrium, message m' can only be sent when the number of zeroes is $n-1$.

We would like to show that it is profitable for signal-type $(1, 0...0)$ (i.e., the deputy when she is the only one with signal 1) to deviate to reporting $(0...0)$, that is, $\alpha(m = (0, ...0), \sigma = (1, 0...0)) > \alpha(m = m', \sigma = (1, 0...0))$.

Denote $\Pr(\omega = 0|\sigma = (1, 0...0)) := \pi > 1/2$.

Since $\Pr(\sigma_1 = 1|m = (0...0)) = 0$ in a fully revealing equilibrium, irrespective of the realized state, we have

$$\alpha(m = (0, ...0), \sigma = (1, 0...0)) = \pi$$

Now compute $\alpha(m = m', \sigma = (1, 0...0))$.

Denote the set of all vectors of the experts' signals containing only one signal 1 by Σ.

Denote:

$$\begin{aligned}\mu &:= \Pr(m = m'|\sigma = (1, 0...0)) \\ \nu &:= \Pr(m = m'|\sigma \in \Sigma\backslash(1, 0...0))\end{aligned}$$

Then, using the facts that, in a fully revealing equilibrium, message m' can only be sent by signal-types from Σ, all vectors in Σ are equally likely, and there are n vectors in Σ, we can derive:

$$\Pr(\sigma_1 = 0|\omega = 0, m = m') = \frac{\Pr(\sigma_1 = 0 \cap m = m'|\omega = 0)}{num. + \Pr(\sigma_1 = 1 \cap m = m'|\omega = 0)} =$$
$$= \frac{\nu(n-1)\rho^{n-1}(1-\rho)}{\nu(n-1)\rho^{n-1}(1-\rho) + \mu(1-\rho)\rho^{n-1}} = \frac{\nu(n-1)}{\nu(n-1)+\mu} =: \gamma$$

$$\Pr(\sigma_1 = 1|\omega = 1, m = m') = \frac{\Pr(\sigma_1 = 1 \cap m = m'|\omega = 1)}{num. + \Pr(\sigma_1 = 0 \cap m = m'|\omega = 1)} =$$
$$= \frac{\mu\rho(1-\rho)^{n-1}}{\mu\rho(1-\rho)^{n-1} + \nu(n-1)(1-\rho)^{n-1}\rho} = \frac{\mu}{\nu(n-1)+\mu} =$$
$$= 1 - \gamma$$

Hence,

$$\alpha(m = m', \sigma = (1, 0...0)) = \pi\gamma + (1-\pi)(1-\gamma)$$

Since, by assumption, $\mu > 0$, $1 - \gamma > 0$, and, thus, $\alpha(m = m', \sigma = (1, 0...0)) < \pi$. ■

Proof of Lemma 8. From the definition of the partitional equilibrium it follows that the equilibrium is partitional if and only if the following holds: For any two equilibrium messages which are not reputation-equivalent, there exists a number of zeroes k that separates them, meaning that one of them is never sent when the experts have received less than k zeroes and the other one is never sent when the experts have received more than k zeroes.

Consider two non-reputation-equivalent equilibrium messages, m^1 and m^2. Assume, without loss of generality, that $\Pr(G|\omega = 0, m^1) > \Pr(G|\omega = 0, m^2)$. This implies that $\Pr(G|\omega = 1, m^2) > \Pr(G|\omega = 1, m^1)$, otherwise the experts would always strictly prefer to send m^1 instead of m^2. Let k' be the smallest number of zeroes for which the experts send m^1 with positive probability. It must be that

$$\begin{aligned}\Pr(G|\omega &= 0, m^1)\Pr(\omega = 0|k') + \Pr(G|\omega = 1, m^1)\Pr(\omega = 1|k') \geq \\ &\geq \Pr(G|\omega = 0, m^2)\Pr(\omega = 0|k') + \Pr(G|\omega = 1, m^2)\Pr(\omega = 1|k').\end{aligned}$$

or, rearranging terms,

$$\begin{aligned}\Pr(\omega &= 0|k')[\Pr(G|\omega=0,m^1)-\Pr(G|\omega=0,m^2)] \geq \\ &\geq \Pr(\omega=1|k')[\Pr(G|\omega=1,m^2)-\Pr(G|\omega=1,m^1)].\end{aligned}$$

For any $k'' > k'$ we have $\Pr(\omega = 0|k'') > \Pr(\omega = 0|k')$. Hence,

$$\begin{aligned}\Pr(\omega &= 0|k'')[\Pr(G|\omega=0,m^1)-\Pr(G|\omega=0,m^2)] > \\ &> \Pr(\omega=1|k'')[\Pr(G|\omega=1,m^2)-\Pr(G|\omega=1,m^1)].\end{aligned}$$

Thus, the experts with $k'' > k'$ zeros never send m^2. Recall that, by definition of k', the experts with $k'' < k'$ zeros never send m^1. Hence, k' is the desired k. ■

Proof of Lemma 9. We will first consider the case when signal-types l and r send m with probability 1; then we will generalize the argument by allowing for randomization by the threshold types.

For notational convenience, a profile of signals will be identified with the number of zeros it contains and a set of profiles will be identified with a message that communicates it. As we have shown in Preliminaries, comparing expected reputations boils down to comparing $\alpha(m, I)$. When the experts have received k zeros and the deputy sends message m, for each expert i we have:

$$\alpha(m,k) = \Pr(\omega = 0|k)\Pr(\sigma_i = 0|\omega = 0, m) + \Pr(\omega = 1|k)\Pr(\sigma_i = 1|\omega = 1, m)$$

We will first consider how the expected reputation of the expert changes when the experts have received $k = r$ or $k = l$ zeros and the opposite boundary of the message is "marginally" increased from l to $l + 1$ in the first case and decreased from r to $r - 1$ in the second case. Then we will argue that if an expert's expected reputation weakly rises after a given marginal change, it will weakly increase following the next marginal change, up to when the message coincides with revealing r in the first case and revealing l in the second case.

Consider an alternative message m' which is interpreted as "the experts have received between $l + 1$ and r zeros" or "the experts have received between l and $r - 1$ zeros." Denote by t the number of zeroes left out by a given marginal cut, and let k be the intact boundary. Then, $t = l$ and $k = r$ in the first case, and $t = r$ and $k = l$ in the second case. Hence, for both cases, $\alpha(m,k)$ can be rewritten as:

$$\begin{aligned}
\Pr(\omega &= 0|k)(\Pr(\sigma_i=0|t)\Pr(t|\omega=0,m)+\Pr(\sigma_i=0|\omega=0,m')\Pr(m'|\omega=0,m))+\\
\Pr(\omega &= 1|k)(\Pr(\sigma_i=1|t)\Pr(t|\omega=1,m)+\Pr(\sigma_i=1|\omega=1,m')\Pr(m'|\omega=1,m))=\\
\Pr(\omega &= 0|k)(\Pr(\sigma_i=0|t)\Pr(t|\omega=0,m)+\Pr(\sigma_i=0|\omega=0,m')(1-\Pr(t|\omega=0,m))+\\
\Pr(\omega &= 1|k)(\Pr(\sigma_i=1|t)\Pr(t|\omega=1,m)+\Pr(\sigma_i=1|\omega=1,m')(1-\Pr(t|\omega=1,m))=\\
\Pr(\omega &= 0|k)\Pr(t|\omega=0,m)(\Pr(\sigma_i=0|t)-\Pr(\sigma_i=0|\omega=0,m'))+\\
\Pr(\omega &= 1|k)\Pr(t|\omega=1,m)(\Pr(\sigma_i=1|t)-\Pr(\sigma_i=1|\omega=1,m'))+\alpha(m',k).
\end{aligned}$$

Hence, we have $\alpha(m,k)\le\alpha(m',k)$ if and only if

$$\begin{aligned}
\Pr(\omega &= 0|k)\Pr(t|\omega=0,m)(\Pr(\sigma_i=0|t)-\Pr(\sigma_i=0|\omega=0,m'))+\\
\Pr(\omega &= 1|k)\Pr(t|\omega=1,m)(\Pr(\sigma_i=1|t)-\Pr(\sigma_i=1|\omega=1,m'))\le 0.
\end{aligned}$$

First, note that in the first case

$$\begin{aligned}
\Pr(\sigma_i &= 0|l)-\Pr(\sigma_i=0|\omega=0,m')=1-\Pr(\sigma_i=1|l)-\Pr(\sigma_i=0|\omega=0,m')\le\\
&\le 1-\Pr(\sigma_i=1|l)-(1-\Pr(\sigma_i=1|\omega=1,m'))=\Pr(\sigma_i=1|\omega=1,m')-\Pr(\sigma_i=1|l)<0,
\end{aligned}$$

and in the second case

$$\begin{aligned}
\Pr(\sigma_i &= 1|r)-\Pr(\sigma_i=1|\omega=1,m')=1-\Pr(\sigma_i=0|r)-\Pr(\sigma_i=1|\omega=1,m')\le\\
&\le 1-\Pr(\sigma_i=0|r)-(1-\Pr(\sigma_i=0|\omega=0,m'))=\Pr(\sigma_i=0|\omega=0,m')-\Pr(\sigma_i=0|r)<0,
\end{aligned}$$

So, in the first case, we have $\alpha(m,k)\le\alpha(m',k)$ if

$$\begin{gathered}
\Pr(\omega=0|r)\Pr(l|\omega=0,m)\ge\Pr(\omega=1|r)\Pr(l|\omega=1,m)\Leftrightarrow\\
\frac{\Pr(r|\omega=0)\Pr(\omega=0)}{\Pr(r)}\frac{\Pr(l|\omega=0)}{\Pr(m|\omega=0)}\ge\frac{\Pr(r|\omega=1)\Pr(\omega=1)}{\Pr(r)}\frac{\Pr(l|\omega=1)}{\Pr(m|\omega=1)}\Leftrightarrow\\
\frac{\Pr(m|\omega=1)}{\Pr(m|\omega=0)}\ge\frac{\Pr(l|\omega=1)\Pr(r|\omega=1)\Pr(\omega=1)}{\Pr(l|\omega=0)\Pr(r|\omega=0)\Pr(\omega=0)}
\end{gathered}\tag{6}$$

In the second case, we have $\alpha(m,k)\le\alpha(m',k)$ if

$$\begin{gathered}
\Pr(\omega=0|l)\Pr(r|\omega=0,m)\le\Pr(\omega=1|l)\Pr(r|\omega=1,m)\Leftrightarrow\\
\frac{\Pr(l|\omega=0)\Pr(\omega=0)}{\Pr(l)}\frac{\Pr(r|\omega=0)}{\Pr(m|\omega=0)}\le\frac{\Pr(l|\omega=1)\Pr(\omega=1)}{\Pr(l)}\frac{\Pr(r|\omega=1)}{\Pr(m|\omega=1)}\Leftrightarrow\\
\frac{\Pr(m|\omega=1)}{\Pr(m|\omega=0)}\le\frac{\Pr(l|\omega=1)\Pr(r|\omega=1)\Pr(\omega=1)}{\Pr(l|\omega=0)\Pr(r|\omega=0)\Pr(\omega=0)}
\end{gathered}\tag{7}$$

Clearly, at least one of the two must be true. Without loss of generality, suppose now that the first is true, i.e. that

$$\frac{\Pr(m|\omega=1)\Pr(l|\omega=0)}{\Pr(m|\omega=0)\Pr(l|\omega=1)} \geq \frac{\Pr(r|\omega=1)\Pr(\omega=1)}{\Pr(r|\omega=0)\Pr(\omega=0)}.$$

Now we want to show that if the passage from l to $l+1$ weakly increases expert i's expected reputation, so does the passage from $l+1$ to $l+2$. By induction, this will imply that so does *any* increase in l. So, we want to show that $\alpha(m',k) \leq \alpha(m'',k)$, where m'' is the message which is interpreted as "the experts have received between $l+2$ and r zeroes." We just need to show that the above inequality holds for $l+1$. Note that

$$\frac{\Pr(m|\omega=1)\Pr(l|\omega=0)}{\Pr(m|\omega=0)\Pr(l|\omega=1)} = \frac{\sum_{t=l}^{r}\frac{\Pr(t|\omega=1)}{\Pr(l|\omega=1)}}{\sum_{t=l}^{r}\frac{\Pr(t|\omega=0)}{\Pr(l|\omega=0)}} = \frac{\sum_{t=l}^{r}\left(\frac{1-\rho}{\rho}\right)^{t-l}\frac{\binom{n}{t}}{\binom{n}{l}}}{\sum_{t=l}^{r}\left(\frac{\rho}{1-\rho}\right)^{t-l}\frac{\binom{n}{t}}{\binom{n}{l}}} = \frac{\sum_{t=l}^{r}\left(\frac{1-\rho}{\rho}\right)^{t-l}\binom{n}{t}}{\sum_{t=l}^{r}\left(\frac{\rho}{1-\rho}\right)^{t-l}\binom{n}{t}}. \tag{8}$$

We just need to show that raising l by 1 increases the above ratio. This can be shown through the following chain of relations.

$$\frac{\sum_{t=l+1}^{r}\left(\frac{1-\rho}{\rho}\right)^{t-l-1}\binom{n}{t}}{\sum_{t=l+1}^{r}\left(\frac{\rho}{1-\rho}\right)^{t-l-1}\binom{n}{t}} = \frac{\sum_{t=l}^{r-1}\left(\frac{1-\rho}{\rho}\right)^{t-l}\binom{n}{t+1}}{\sum_{t=l}^{r-1}\left(\frac{\rho}{1-\rho}\right)^{t-l}\binom{n}{t+1}} = \frac{\sum_{t=l}^{r-1}\left(\frac{1-\rho}{\rho}\right)^{t-l}\binom{n}{t}\frac{n-t}{t+1}}{\sum_{t=l}^{r-1}\left(\frac{\rho}{1-\rho}\right)^{t-l}\binom{n}{t}\frac{n-t}{t+1}} = \tag{9}$$

$$= \frac{\sum_{t=l}^{r-1}\left(\frac{1-\rho}{\rho}\right)^{t-l}\binom{n}{t}\frac{n-t}{t+1} + \left(\frac{1-\rho}{\rho}\right)^{r-l}\cdot 0}{\sum_{t=l}^{r-1}\left(\frac{\rho}{1-\rho}\right)^{t-l}\binom{n}{t}\frac{n-t}{t+1} + \left(\frac{\rho}{1-\rho}\right)^{r-l}\cdot 0} > \frac{\sum_{t=l}^{r}\left(\frac{1-\rho}{\rho}\right)^{t-l}\binom{n}{t}}{\sum_{t=l}^{r}\left(\frac{\rho}{1-\rho}\right)^{t-l}\binom{n}{t}}$$

The equalities in this formula are obvious, while the inequality in the second line is due to the following argument. Consider a class of ratios of the form

$$\frac{a_1x_1 + a_2x_2 + ... + a_mx_m}{a_1y_1 + a_2y_2 + ... + a_my_m}, \tag{10}$$

such that

$$x_1 > x_2 > ... > x_m \text{ and } y_1 < y_2 < ... < y_m.$$

The left-hand side ratio in the inequality belongs to this class, with $x_t = \left(\frac{1-\rho}{\rho}\right)^{t-l}$, $y_t = \left(\frac{\rho}{1-\rho}\right)^{t-l}$, $a_t = \begin{pmatrix} n \\ t \end{pmatrix} \frac{n-t}{t+1}$ for $t = l, ...r-1$, $a_r = 0$, and $m = r$.

Consider a transformation to coefficients a_t such that each subsequent coefficient is multiplied by a higher number. Let us call the new coefficients b_t; b_t/a_t grows with t. The transformed ratio is

$$\frac{b_1x_1 + b_2x_2 + ... + b_mx_m}{b_1y_1 + b_2y_2 + ... + b_my_m},$$

The right-hand side ratio in the inequality is obtained through exactly this type of transformation: for $t = l, ...r-1$ each term in the left-hand side is multiplied by $\dfrac{t+1}{n-t}$, which grows with t, and the last term is multiplied by ∞.

Our argument then consists of two steps:

1. Any such transformation of the whole ratio can be achieved by a sequence of transformations of the form

$$\frac{a_1x_1 + ... + a_sx_s + c(a_{s+1}x_{s+1} + ... + a_mx_m)}{a_1y_1 + ... + a_sy_s + c(a_{s+1}y_{s+1} + ... + a_my_m)}, \tag{11}$$

where $c > 1$, and c and s are different for each transformation.

2. Any such intermediate transformation reduces the ratio. Hence, the overall transformation reduces the ratio as well.

The first step is trivial. We first need to multiply all coefficients by b_1/a_1 to make the first coefficient b_1, then all coefficients starting from the second one by $\dfrac{b_2a_1}{b_1a_2}$ to make the second coefficient b_2, and so on.

To prove the second statement, differentiate (11) with respect to c. We obtain:

$$\frac{(a_{s+1}x_{s+1} + ... + a_mx_m) \cdot denom - (a_{s+1}y_{s+1} + ... + a_my_m) \cdot num}{denom^2} =$$

$$= \frac{(a_{s+1}x_{s+1} + ... + a_mx_m)(a_1y_1 + ... + a_sy_s) - (a_{s+1}y_{s+1} + ... + a_my_m)(a_1x_1 + ... + a_jx_j)}{denom^2}$$

The numerator is the sum of the following terms: $a_ia_j(x_iy_j - x_jy_i)$, where $i > j$. Thus, by the properties of the sequences of x_i and y_i, each of the terms is negative. Thus, the derivative is negative, meaning that the considered multiplication by $c > 1$ reduces the ratio. Since, after each intermediate transformation, the ratio preserves the form of (10), the statement holds for all intermediate transformations.

Let us show now that if the expert's expected reputation when the experts received r zeros is higher by revealing it (message "r") than by sending m, then the expert considers state 0 strictly more likely; and likewise for l with state 1. Suppose not; that is, suppose that $\Pr(\omega = 0|r) \leq 1/2$. Note that

$$\begin{aligned}
\Pr(\sigma_i &= 0|\omega = 0, m) + \Pr(\sigma_i = 1|\omega = 1, m) > \Pr(\sigma_i = 0|\omega = 0, r) + \Pr(\sigma_i = 1|\omega = 1, r) = 1; \\
\Pr(\sigma_i &= 0|\omega = 0, m) < \Pr(\sigma_i = 0|\omega = 0, r); \\
\Pr(\sigma_i &= 1|\omega = 1, m) > \Pr(\sigma_i = 1|\omega = 1, r).
\end{aligned}$$

Then, $\alpha(m, r) < \alpha(r, r)$, a contradiction.

Finally, let us show that the above proof generalizes to the case when signal-types l and/or r are allowed to send m with probability below 1. First, notice that until formula (6) no derivation relies on r or l playing a pure strategy (m' keeps the meaning of the set of all profiles with the number of zeroes between $l+1$ and r or between l and $r-1$, inclusive).

Derivations in (6) need to be slightly modified:

$$\begin{gathered}
\Pr(\omega = 0|r) \Pr(l|\omega = 0, m) \geq \Pr(\omega = 1|r) \Pr(l|\omega = 1, m) \Leftrightarrow \\
\frac{\Pr(r|\omega = 0) \Pr(\omega = 0)}{\Pr(r)} \frac{\Pr(m|\omega = 0, l) \Pr(l|\omega = 0)}{\Pr(m|\omega = 0)} \geq \\
\geq \frac{\Pr(r|\omega = 1) \Pr(\omega = 1)}{\Pr(r)} \frac{\Pr(m|\omega = 1, l) \Pr(l|\omega = 1)}{\Pr(m|\omega = 1)}
\end{gathered}$$

Since $\Pr(m|\omega, l)$ does not depend on ω, the new terms that appeared in both sides cancel out and, thus, we obtain the same condition as before:

$$\frac{\Pr(m|\omega = 1)}{\Pr(m|\omega = 0)} \geq \frac{\Pr(l|\omega = 1) \Pr(r|\omega = 1) \Pr(\omega = 1)}{\Pr(l|\omega = 0) \Pr(r|\omega = 0) \Pr(\omega = 0)}$$

The same is true for (7).

Formula (8) changes in the following way. In the summations $\sum_{t=l}^{r} \Pr(t|\omega = 1)$ and $\sum_{t=l}^{r} \Pr(t|\omega = 0)$ the first and the last term have to be multiplied by $\Pr(m|l)$ and $\Pr(m|r)$ respectively. Equivalently, $\begin{pmatrix} n \\ t \end{pmatrix}$ has to be multiplied by these terms for $t = l$ and $t = r$ in both the numerator and the denominator.

Correspondingly, in all summations in (9) the last term has to be just multiplied by a constant ($\Pr(m|r)$). Additionally, the first term in the summations in the right-hand side of the inequality has to be multiplied by $\Pr(m|l)$. Overall, these modifications can be considered as affecting only coefficients a_t and b_t and not affecting x_t and y_t. Moreover, it is easy to check that the desired property of the transformation of a_t's into b_t's is preserved. Thus, the argument goes through. ■

Proof of Lemma 10. The value of α from revealing k signals equal to $\overline{\omega}$ is

$$\begin{aligned}\alpha(k,k) &= \Pr(\omega=\overline{\omega}|k)\Pr(\sigma_i=\overline{\omega}|\omega=\overline{\omega},k)+\Pr(\omega\neq\overline{\omega}|k)\Pr(\sigma_i\neq\overline{\omega}|\omega\neq\overline{\omega},k)= \\ &= \Pr(\omega=\overline{\omega}|k)\frac{k}{n}+\Pr(\omega\neq\overline{\omega}|k)\frac{n-k}{n}=\end{aligned}$$

If, instead, m is sent,

$$\alpha(m,k)=\Pr(\omega=\overline{\omega}|k)\Pr(\sigma_i=\overline{\omega}|\omega=\overline{\omega},m)+\Pr(\omega\neq\overline{\omega}|k)\Pr(\sigma_i\neq\overline{\omega}|\omega\neq\overline{\omega},m)$$

Since $\Pr(\omega=\overline{\omega}|k)>\Pr(\omega\neq\overline{\omega}|k)$, $\Pr(\sigma_i=\overline{\omega}|\omega=\overline{\omega},m)>\frac{k}{n}$ and $\Pr(\sigma_i=\overline{\omega}|\omega=\overline{\omega},m)+\Pr(\sigma_i\neq\overline{\omega}|\omega\neq\overline{\omega},m)\geq 1=\frac{k}{n}+\frac{n-k}{n}$, we have

$$\alpha(m,k)>\alpha(k,k)$$

■

Proof of Lemma 11.

For every $p\leq\rho$, the expected reputation of the experts with all ones from revealing themselves is strictly greater than q:

$$\Pr(G|\sigma=\overrightarrow{1})=\Pr(\omega=1|\sigma=\overrightarrow{1})x+\Pr(\omega=0|\sigma=\overrightarrow{1})y>\rho x+(1-\rho)y=q,$$

because, by $p\leq\rho$ and $n>1$,

$$\begin{aligned}\Pr(\omega &= 1|\sigma=\overrightarrow{1})=\frac{\rho^n(1-p)}{\rho^n(1-p)+(1-\rho)^n p}= \\ &= \rho\left(\rho+(1-\rho)\frac{p}{(1-p)}\left(\frac{1-\rho}{\rho}\right)^{n-1}\right)^{-1}>\rho\left(\rho+(1-\rho)\right)^{-1}=\rho.\end{aligned}$$

Instead, the expected reputation from the complementary message, m^0, conditional on having received all ones is lower than q. To see this, notice first that the unconditional expected reputation can be written as

$$q=\Pr(\sigma\neq\overrightarrow{1})\Pr(G|\sigma\neq\overrightarrow{1})+\Pr(\sigma=\overrightarrow{1})\Pr(G|\sigma=\overrightarrow{1})$$

Since, as we have just shown, $\Pr(G|\sigma=\overrightarrow{1})>q$, it must be that $\Pr(G|\sigma\neq\overrightarrow{1})<q$.

Now, the expected reputation from message m^0, conditional on having received all ones, is

$$\begin{aligned}
&R_i(m^0, \sigma = \overrightarrow{1}) = \\
&\quad = \Pr(\omega = 1|\sigma = \overrightarrow{1}) \Pr(G|\omega = 1, \sigma \neq \overrightarrow{1}) + \Pr(\omega = 0|\sigma = \overrightarrow{1}) \Pr(G|\omega = 0, \sigma \neq \overrightarrow{1}) < \\
&\quad < \Pr(\omega = 1|\sigma \neq \overrightarrow{1}) \Pr(G|\omega = 1, \sigma \neq \overrightarrow{1}) + \Pr(\omega = 0|\sigma \neq \overrightarrow{1}) \Pr(G|\omega = 0, \sigma \neq \overrightarrow{1}) = \\
&\quad = R_i(m^0, \sigma \neq \overrightarrow{1}) \equiv \Pr(G|\sigma \neq \overrightarrow{1}),
\end{aligned}$$

where the inequality follows from the observation that $\Pr(\omega = 0|\sigma \neq \overrightarrow{1}) > \Pr(\omega = 0|\sigma = \overrightarrow{1})$ and $\Pr(G|\omega = 0, \sigma \neq \overrightarrow{1}) > \Pr(G|\omega = 1, \sigma \neq \overrightarrow{1})$.

Thus, $R_i(m^0, \sigma = \overrightarrow{1}) < q$. This implies that the incentive compatibility constraint of the experts with all ones, $\Pr(G|\sigma = \overrightarrow{1}) \geq R_i(m^0, \sigma = \overrightarrow{1})$, holds as a *strict* inequality when $p = \rho$, and, by continuity, for some $p > \rho$.

The incentive compatibility constraint can be rewritten as

$$\Pr(\omega = 1|\sigma = \overrightarrow{1})(x - \Pr(G|\omega = 1, \sigma \neq \overrightarrow{1})) \geq \Pr(\omega = 0|\sigma = \overrightarrow{1})(\Pr(G|\omega = 0, \sigma \neq \overrightarrow{1}) - y)$$

Obviously, $x > \Pr(G|\omega = 1, \sigma \neq \overrightarrow{1})$ and $\Pr(G|\omega = 0, \sigma \neq \overrightarrow{1}) > y$. Thus, the left-hand side is decreasing in p, whereas the right-hand side is increasing in p. Furthermore, the condition is violated when $p = 1$, because then $\Pr(\omega = 1|\sigma = \overrightarrow{1}) = 0$. Hence, the existence of $\overline{p} > \rho$ follows; it is determined by $\Pr(G|\sigma = \overrightarrow{1}) = R_i(m^0, \sigma = \overrightarrow{1})$.

For given p, when $n \to \infty$, $\Pr(\omega = 1|\sigma = \overrightarrow{1}) \to 1$, $\Pr(\omega = 0|\sigma = \overrightarrow{1}) \to 0$ and $\Pr(G|\omega = 1, \sigma \neq \overrightarrow{1}) \to q$. From the definition of x and y, $y < q < x$. Thus, when $n \to \infty$, $\overline{p} \to 1$. ■

Proof of Lemma 12. Without loss of generality, we focus on "almost pure" equilibria, i.e., those in which there are no different reputation-equivalent messages. Fix $k \in \{1, ..., n\}$. Let m denote the message "at most $k - 1$ zeros" and m' the message "at least k zeros". The two incentive compatibility constraints of the threshold profiles read:

$$\begin{aligned}
\Pr(\omega &= 1|k-1) \Pr(G|\omega = 1, m) + \Pr(\omega = 0|k-1) \Pr(G|\omega = 0, m) \geq \\
\Pr(\omega &= 1|k-1) \Pr(G|\omega = 1, m') + \Pr(\omega = 0|k-1) \Pr(G|\omega = 0, m')
\end{aligned}$$

and

$$\begin{aligned}
\Pr(\omega &= 1|k) \Pr(G|\omega = 1, m') + \Pr(\omega = 0|k) \Pr(G|\omega = 0, m') \geq \\
\Pr(\omega &= 1|k) \Pr(G|\omega = 1, m) + \Pr(\omega = 0|k) \Pr(G|\omega = 0, m).
\end{aligned}$$

Note that by $\Pr(\omega = 1|k-1) > \Pr(\omega = 1|k)$ and $\Pr(G|\omega = 1, m) + \Pr(G|\omega = 0, m') > \Pr(G|\omega = 1, m') + \Pr(G|\omega = 0, m)$, the following holds:

$$\begin{aligned}\Pr(\omega &= 1|k-1)[\Pr(G|\omega = 1, m) + \Pr(G|\omega = 0, m') - \Pr(G|\omega = 1, m') - \Pr(G|\omega = 0, m)] \geq \\ &\geq \Pr(\omega = 1|k)[\Pr(G|\omega = 1, m) + \Pr(G|\omega = 0, m') - \Pr(G|\omega = 1, m') - \Pr(G|\omega = 0, m)],\end{aligned}$$

which can be rewritten as the sum of the two IC constraints:

$$\begin{aligned}\Pr(\omega &= 1|k-1)\Pr(G|\omega = 1, m) + \Pr(\omega = 0|k-1)\Pr(G|\omega = 0, m) + \\ \Pr(\omega &= 1|k)\Pr(G|\omega = 1, m') + \Pr(\omega = 0|k)\Pr(G|\omega = 0, m') \geq \\ &\geq \Pr(\omega = 1|k-1)\Pr(G|\omega = 1, m') + \Pr(\omega = 0|k-1)\Pr(G|\omega = 0, m') + \\ \Pr(\omega &= 1|k)\Pr(G|\omega = 1, m) + \Pr(\omega = 0|k)\Pr(G|\omega = 0, m).\end{aligned}$$

Hence, at least one of the two IC must be satisfied. If for some k both IC are satisfied, (m, m') is the desired (pure) equilibrium. Otherwise, proceed as follows. First, note that for every $p > 1/2$, the experts with n zeros prefer to reveal themselves rather than sending the complementary message. This comes from the fact that, by Lemma 11, the experts with n ones would prefer to reveal themselves for $p < 1/2$ (if $p < 1/2$ were allowed by the assumptions of the model), and the problem of "n zeroes" for $p > 1/2$ is identical to that of "n ones" for $p < 1/2$. So, for $k = n$ the second IC is satisfied.

Second, *suppose* that there is $k' > 0$ such that the first IC is satisfied; we will argue later that this supposition is correct up to the desired threshold $\widehat{p}$. Then, there must be $k \in \{1, ..., k'\}$ where the inversion happens: only the first is satisfied for k and only the second is satisfied for $k+1$. This means that the experts with k zeros prefer both the message "at most $k-1$ zeros" to the message "at least k zeros" and the message "at least $k+1$ zeros" to the message "at most k zeros". So, calling m and m' two messages sent with probabilities η and $1-\eta$ by profile k and with probability 1 by the experts with, respectively, less and more than k zeros, we have $m \succ m'$ for $\eta = 0$ and $m' \succ m$ for $\eta = 1$. Therefore, by continuity, there exists η such that profile k is indifferent between m and m', and then the other profiles strictly prefer the message they are supposed to send. So we have the desired equilibrium.

Now we argue the existence of $\widehat{p} \in \left[\overline{p}, \overline{\overline{p}}\right]$ such that there is k where the first IC is satisfied if and only if $p \in (1/2, \widehat{p}]$. By Lemma 11, for each $p \in (1/2, \overline{p}]$ the first IC is satisfied for $k = 1$. Now, fix $p > \overline{\overline{p}}$. Fix k and let m denote the message "at most $k-1$ zeros". Since the experts always consider state 0 strictly more likely, by Lemma 9 the experts will all ones cannot prefer revealing themselves to sending m, but then the experts with $k-1$ zeros will. Then, by Lemma 10, they strictly prefer to send m', and thus the first IC is violated. To show that there exists a precise $\widehat{p}$ that separates the values of p for which the first IC is satisfied for some k and violated for every k, note that, for each k, if for some p the first IC is violated, so it will be for any $p' > p$.

To conclude: For every $p \in (1/2, \widehat{p}]$ we have a bipartitional equilibrium by the argument above; for every $p > \widehat{p}$, since the first IC is violated for every k, no bipartitional equilibrium exists. ■

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