NATIONAL RESEARCH UNIVERSITY

ADVANCED ECONOMETRICS

Lecture 06.04.16-1

Dynamic Linear Models

Olga Demidova
E-mail:demidova@hse.ru
Department of Applied Economics

An autoregressive panel data models

$$Y_{it} = x_{it}^{'}\beta + \gamma Y_{it-1} + \alpha_i + \varepsilon_{it},\ \varepsilon_{it} \sim iid(0, \sigma_\varepsilon^2), i = 1, \ldots, n, t = 1, \ldots, T$$

$$cor(Y_{it-1}, \alpha_i) \neq 0$$

To eliminate the individual effects α_i,

we take first differences.

The case without endogenous variables:

$$Y_{it} - Y_{it-1} = \gamma(Y_{it-1} - Y_{it-2}) + \varepsilon_{it} - \varepsilon_{it-1}, \quad t = 2, \ldots, T,$$

$$cor(Y_{it-1}, \varepsilon_{it-1}) \neq 0 \Rightarrow IV$$

Instruments

$$Y_{it} - Y_{it-1} = \gamma(Y_{it-1} - Y_{it-2}) + \varepsilon_{it} - \varepsilon_{it-1}, \quad t = 2,\ldots,T,$$

$$Y_{it-2} \quad is \quad correlated \quad with \quad Y_{it-1} - Y_{it-2},$$

$$but \quad not \quad with \quad \varepsilon_{it} - \varepsilon_{it-1} \Rightarrow$$

$$Y_{it-2} \quad is \quad an \quad instrument \quad for \quad Y_{it-1} - Y_{it-2}.$$

Instruments

$$Moment \quad condition:$$

$$E\{(\varepsilon_{it} - \varepsilon_{it-1})Y_{it-2}\} = 0,$$

$$p\lim \frac{1}{N(T-2)} \sum_{i=1}^{N} \sum_{t=2}^{T} (\varepsilon_{it} - \varepsilon_{it-1})Y_{it-2} = 0,$$

$$\hat{\gamma}_{IV}^{(1)} = \frac{\sum_{i=1}^{N} \sum_{t=2}^{T} Y_{it-2}(Y_{it} - Y_{it-1})}{\sum_{i=1}^{N} \sum_{t=2}^{T} Y_{it-2}(Y_{it-1} - Y_{it-2})}$$

Instruments

$$Y_{it} - Y_{it-1} = \gamma(Y_{it-1} - Y_{it-2}) + \varepsilon_{it} - \varepsilon_{it-1}, \quad t = 2, \ldots, T,$$

$$Y_{it-2} - Y_{it-3} \quad \textit{is} \quad \textit{correlated} \quad \textit{with} \quad Y_{it-1} - Y_{it-2},$$

$$\textit{but} \quad \textit{not} \quad \textit{with} \quad \varepsilon_{it} - \varepsilon_{it-1} \Rightarrow$$

$$Y_{it-2} - Y_{it-3} \quad \textit{is} \quad \textit{an} \quad \textit{instrument} \quad \textit{for} \quad Y_{it-1} - Y_{it-2}.$$

Instruments

Moment condition:

$$E\{(\varepsilon_{it} - \varepsilon_{it-1})(Y_{it-2} - Y_{it-3})\} = 0,$$

$$p\lim \frac{1}{N(T-2)} \sum_{i=1}^{N} \sum_{t=3}^{T} (\varepsilon_{it} - \varepsilon_{it-1})(Y_{it-2} - Y_{it-3}) = 0,$$

$$\hat{\gamma}_{IV}^{(2)} = \frac{\sum_{i=1}^{N} \sum_{t=3}^{T} (Y_{it-2} - Y_{it-3})(Y_{it} - Y_{it-1})}{\sum_{i=1}^{N} \sum_{t=3}^{T} (Y_{it-2} - Y_{it-3})(Y_{it-1} - Y_{it-2})}$$

Arellano and Bond Instruments

Imposing more conditions increases the efficiency of the estimators.

Arellano and Bond (1991) suggest special instruments.

Their number vary with t.

Arellano and Bond Instruments

$$T = 4$$

$$Y_{it} - Y_{it-1} = \gamma(Y_{it-1} - Y_{it-2}) + \varepsilon_{it} - \varepsilon_{it-1}, \quad t = 2,3,4,$$

$$t = 2, \quad Y_{i2} - Y_{i1} = \gamma(Y_{i1} - Y_{i0}) + \varepsilon_{i2} - \varepsilon_{i1},$$

$$1 \quad Moment\ \ condition \quad E\{(\varepsilon_{i2} - \varepsilon_{i1})Y_{i0}\} = 0,$$

$$t = 3, \quad Y_{i3} - Y_{i2} = \gamma(Y_{i2} - Y_{i1}) + \varepsilon_{i3} - \varepsilon_{i2},$$

$$2 \quad Moment\ \ conditions \quad E\{(\varepsilon_{i3} - \varepsilon_{i2})Y_{i0}\} = 0, E\{(\varepsilon_{i3} - \varepsilon_{i2})Y_{i1}\} = 0,$$

$$t = 4, \quad Y_{i4} - Y_{i3} = \gamma(Y_{i3} - Y_{i2}) + \varepsilon_{i4} - \varepsilon_{i3},$$

$$3 \quad Moment\ \ conditions \quad E\{(\varepsilon_{i4} - \varepsilon_{i3})Y_{i0}\} = 0, \quad E\{(\varepsilon_{i4} - \varepsilon_{i3})Y_{i1}\} = 0,$$

$$E\{(\varepsilon_{i4} - \varepsilon_{i3})Y_{i2}\} = 0.$$

Arellano and Bond Instruments

GMM framework

$$\Delta\varepsilon_i = \begin{pmatrix} \varepsilon_{i2} - \varepsilon_{i1} \\ \vdots \\ \varepsilon_{iT} - \varepsilon_{iT-1} \end{pmatrix},$$

Matrix of instruments:

$$Z_i = \begin{pmatrix} Y_{i0} & 0 & \dots & 0 \\ 0 & [Y_{i0}, Y_{i1}] & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & [Y_{i0}, \dots, Y_{it-2}] \end{pmatrix}$$

Arellano and Bond Instruments

$$Moment \quad conditions: \quad E\{Z_i^{'}\Delta\varepsilon_i\} = 0,$$

$$1 + 2 + \ldots + T - 1 = \frac{T(T-1)}{2} \quad moment \quad conditions.$$

$$\varepsilon_{it} - \varepsilon_{it\text{-}1} = \Delta Y_{it} - \gamma\Delta Y_{it-1} \Rightarrow$$

$$E\{Z_i^{'}(\Delta Y_i - \gamma\Delta Y_{i,-1})\} = 0.$$

$$The \quad number \quad of \quad moment \quad conditions \quad >$$

$$the \quad number \quad of \quad unknown \quad coefficients \Rightarrow GMM.$$

GMM

$$Moment \quad conditions: \quad E\{Z_i^{'}\Delta\varepsilon_i\} = 0,$$

$$\min_{\gamma}[\frac{1}{N}\sum_{i=1}^{N} Z_i^{'}(\Delta Y_i - \gamma\Delta Y_{i,-1})]' W_N [\frac{1}{N}\sum_{i=1}^{N} Z_i^{'}(\Delta Y_i - \gamma\Delta Y_{i,-1})],$$

$$\hat{\gamma}_{GMM} = [(\sum_{i=1}^{N} \Delta Y_{i,-1} Z_i^{'})]' W_N (\sum_{i=1}^{N} Z_i^{'}\Delta Y_{i,-1})]^{-1} \times$$

$$\times (\sum_{i=1}^{N} \Delta Y_{i,-1}^{'} Z_i) W_N (\sum_{i=1}^{N} Z_i^{'}\Delta Y_{i,-1})$$

GMM

$$\underset{N\to\infty}{p\lim} W_N = \mathrm{var}\{Z_i'\Delta\varepsilon_i\} = E\{Z_i'\Delta\varepsilon_i\Delta\varepsilon_i'Z_i\}^{-1},$$

$$\hat{W}_N^{opt(1)} = \left(\frac{1}{N}\sum_{i=1}^{N} Z_i'\Delta\hat{\varepsilon}_i\Delta\hat{\varepsilon}_i'Z_i\right)^{-1},$$

where $\Delta\hat{\varepsilon}_i$ *is a residual vector from a first step consistent estimator,* *for example* $W_N = I$.

GMM

$$\mathrm{var}[\varepsilon] = \sigma_\varepsilon^2 I_N \Rightarrow$$

The absence of autocorrelation:

$$E\{\Delta\varepsilon_i \Delta\varepsilon_i^{'}\} = \sigma_\varepsilon^2 G = \sigma_\varepsilon^2 \begin{pmatrix} 2 & -1 & 0 & \ldots 0 \\ -1 & 2 & -1 & \ldots 0 \\ \vdots & & \ddots & \vdots \\ 0 & \ldots & 0 & -1 2 \end{pmatrix},$$

$$\hat{W}_N^{opt(2)} = \left(\frac{1}{N} \sum_{i=1}^{N} Z_i^{'} G_i Z_i \right)^{-1},$$

does not involve unknown parameters $\Rightarrow$

optimal GMM estimator can be computed in one step.

Dynamic models with exogenous variables

$$Y_{it} = x_{it}^{'}\beta + \gamma Y_{it-1} + \alpha_i + \varepsilon_{it}, \; \varepsilon_{it} \sim iid(0, \sigma_{\varepsilon}^{2}), i = 1, \ldots, n, t = 1, \ldots, T$$

$x_{it}^{'}$ *are strictly exogenous* $\Rightarrow$

$$E\{x_{is}\Delta\varepsilon_{it}\} = 0 \quad \forall s, t$$

$\Rightarrow x_{i1}, \cdots, x_{iT}$ *can be added to the instrument list*

$\Rightarrow$ *number of rows in Z is too large.*

Δx_{it} *are instruments for all* $x_{is}, \quad s = 1, \ldots, T$

$$E\{\Delta x_{it}\Delta\varepsilon_{it}\} = 0 \quad \forall t$$

Dynamic models with exogenous variables

$$Matrix \quad of \quad instruments:$$

$$Z_i = \begin{pmatrix} [Y_{i0}, \Delta x_{i2}^{'}] & 0 & \dots & 0 \\ 0 & [Y_{i0}, Y_{i1}, \Delta x_{i3}^{'}] & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & [Y_{i0}, \dots, Y_{it-2}, \Delta x_{iT}^{'}] \end{pmatrix}$$

Dynamic models with exogenous variables

$$If \quad x_{it}^{'} \quad are \quad not \quad strictly \quad exogenous, \quad but \quad predetermined$$

$$\Rightarrow E\{x_{it}\varepsilon_{is}\} = 0 \quad \forall s \geq t$$

$$\Rightarrow x_{it-1}, \cdots, x_{i1} \quad are \quad valid \quad instruments \quad for \quad \Delta\varepsilon_{it}.$$

$$E\{x_{it-j}\Delta\varepsilon_{it}\} = 0 \quad \forall j = 1, \ldots, t-1$$

Overidentifying restriction test (Sargan test)

$$H_0: overidentifying \quad restrictions \quad are \quad valid$$
$$H_1: overidentifying \quad restrictions \quad are \quad not \quad valid$$

$$Test \quad statistics:$$
$$\sum_{i=1}^{N}(\hat{\varepsilon}_i' Z_i)' \hat{W}_N^{opt} \sum_{i=1}^{N} Z_i' \hat{\varepsilon}_i \sim \chi^2 (number \quad of \quad instruments -$$
$$number \quad of \quad parameters)$$

Test that the errors be serially uncorrelated (AB test)

$$H_0 : \operatorname{cov}(\Delta\varepsilon_{it}, \Delta\varepsilon_{it-k}) = 0, \quad k \geq 2$$

$$H_1 : \exists k \geq 2 : \operatorname{cov}(\Delta\varepsilon_{it}, \Delta\varepsilon_{it-k}) \neq 0,$$

$$\varepsilon_{it} \sim iid \Rightarrow \operatorname{cov}(\Delta\varepsilon_{it}, \Delta\varepsilon_{it-1}) = \operatorname{cov}(\varepsilon_{it} - \varepsilon_{it-1}, \varepsilon_{it-1}, \varepsilon_{it-2}) \neq 0.$$

AB test based on the correlation of the fitted residuals $\Delta\hat{\varepsilon}_{it}$.

Example

```
. xtabond WAGE  EXPER EXPER2 UNION MAR PUB, twostep artest(3) lags(1) noconst

Arellano-Bond dynamic panel-data estimation  Number of obs         =      3270
Group variable: NR                           Number of groups      =       545
Time variable: YEAR
                                             Obs per group:    min =         6
                                                               avg =         6
                                                               max =         6

Number of instruments =     26               Wald chi2(6)          =    421.28
                                             Prob > chi2           =    0.0000
Two-step results
```

WAGE	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
WAGE						
L1.	.1316536	.0291677	4.51	0.000	.074486	.1888211
EXPER	.0597001	.0130915	4.56	0.000	.0340413	.085359
EXPER2	-.001074	.0008341	-1.29	0.198	-.0027087	.0005608
UNION	.014931	.0232595	0.64	0.521	-.0306568	.0605188
MAR	.0435347	.0215447	2.02	0.043	.0013079	.0857614
PUB	.0425532	.0389307	1.09	0.274	-.0337495	.1188559

```
Warning: gmm two-step standard errors are biased; robust standard
         errors are recommended.
Instruments for differenced equation
        GMM-type: L(2/.).WAGE
        Standard: D.EXPER D.EXPER2 D.UNION D.MAR D.PUB
```

Example

```
. estat sargan
Sargan test of overidentifying restrictions
        H0: overidentifying restrictions are valid

        chi2(20)      =  31.80804
        Prob > chi2   =    0.0454

. estat abond

Arellano-Bond test for zero autocorrelation in first-differenced errors
```

Order	z	Prob > z
1	-6.531	0.0000
2	1.8709	0.0614
3	-.00783	0.9938

```
   H0: no autocorrelation

.
```

Example

```
. xtabond WAGE  EXPER EXPER2 UNION MAR PUB, twostep artest(3) lags(2) noconst

Arellano-Bond dynamic panel-data estimation  Number of obs          =      2725
Group variable: NR                           Number of groups       =       545
Time variable: YEAR
                                             Obs per group:     min =         5
                                                                avg =         5
                                                                max =         5

Number of instruments =     25               Wald chi2(7)           =    354.27
                                             Prob > chi2            =    0.0000
Two-step results
------------------------------------------------------------------------------
        WAGE |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        WAGE |
         L1. |   .2388887   .0397615     6.01   0.000     .1609575    .3168198
         L2. |   .0968376   .0210594     4.60   0.000      .055562    .1381132
             |
       EXPER |   .0519849   .0173472     3.00   0.003      .017985    .0859849
      EXPER2 |  -.0010869   .0010184    -1.07   0.286     -.003083    .0009092
       UNION |    .002034   .0251797     0.08   0.936    -.0473173    .0513852
         MAR |   .0298813   .0212814     1.40   0.160    -.0118294     .071592
         PUB |   .0460213   .0444743     1.03   0.301    -.0411467    .1331893
------------------------------------------------------------------------------
Warning: gmm two-step standard errors are biased; robust standard
         errors are recommended.
Instruments for differenced equation
        GMM-type: L(2/.).WAGE
        Standard: D.EXPER D.EXPER2 D.UNION D.MAR D.PUB
```

Example

```
. estat sargan
Sargan test of overidentifying restrictions
        H0: overidentifying restrictions are valid

        chi2(18)     =  23.94911
        Prob > chi2  =    0.1567

. estat abond

Arellano-Bond test for zero autocorrelation in first-differenced errors
```

Order	z	Prob > z
1	-5.7144	0.0000
2	-.64004	0.5221
3	1.2148	0.2245

```
H0: no autocorrelation
```

NATIONAL RESEARCH UNIVERSITY

Thank you for your attention!

20, Myasnitskaya str., Moscow, Russia, 101000
Tel.: +7 (495) 628-8829, Fax: +7 (495) 628-7931
www.hse.ru