Reputation and Information Aggregation
Supplemental (Online) Appendix

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I. Other equilibria with information aggregation

Beside the separating, partially separating, and pooling equilibria analyzed in the main body, there may exist other equilibria with truthful reporting by the advisors and positive probability of asking for advice by the decision-maker. Here we analyze alternative equilibrium behavior at the asking/not asking stage, while maintaining the solutions of the decision and advising stages pinned down in Sections 3.1 and 3.2 of the main text.

Note preliminarily that an equilibrium in which only signal-type 0 asks with positive probability and the advisors report truthfully does not exist under A2. There may exist, however, the following equilibria:

- "Bad" partially separating I: signal-type 0 never asks for advice, signal-type 1 randomizes between asking and not asking;
- "Bad" partially separating II: signal-type 0 always asks for advice, signal-type 1 randomizes between asking and not asking;
- "Fully mixed" equilibrium: both signal-types randomize between asking and not asking.

Now we argue that, for any given $\rho \in [\underline{\rho}, \widehat{\rho}]$ in Cases 1 and 2 or $\rho \in [\underline{\widehat{\rho}}, \widehat{\rho}]$ in Case 3, each of these equilibria (if it exists) it is ex-ante worse than the pooling-on-asking equilibrium (if it exists) or the "good" partially separating equilibrium for the same ρ.

First, take a "bad" partially separating equilibrium of type I. With respect to this equilibrium, both signal-types ask with non lower probability in the "good" partially separating equilibrium (as well as in the separating equilibrium with truthful reporting in Cases 1 and 2 if $\rho \leq \overline{\rho}$).

Second, take a "bad" partially separating equilibrium of type II. Signal-type 0 asks with higher probability than signal-type 1. If this still triggers truthful reporting by the advisors, then pooling on asking triggers truthful reporting by the advisors too ((TR) is a fortiori satisfied), and it is an equilibrium by Proposition 3.

Finally, suppose that there exists a "fully mixed" equilibrium. If signal-type 0 asks more frequently than signal-type 1, then pooling on asking must trigger truthful reporting too, and it is an equilibrium. If signal-type 0 asks less frequently than signal-type 1, the "fully mixed" equilibrium is worse than the "good" partially separating equilibrium for the following reason. In order not to be inferior to the "good" partially separating equilibrium, the "fully mixed" equilibrium must yield a higher probability of asking by signal-type 0. This, coupled with a lower than 1 probability of asking by signal-type 1, implies by Lemma 3 (part (i)) that the expected reputation of signal-type 0 after asking is higher than in the "good" partially separating equilibrium. After not asking, if signal-type 1 considers state 1 weakly more likely and hence decides 1, the expected reputation of signal-type 0 is the same in the two equilibria. Else, we have $p > qg + (1-q)b$, so by Lemma 3 (part (i)) the expected reputation of signal-type 0 after not asking is higher in the "good" partially separating equilibrium. Hence, in both cases, in the "fully mixed equilibrium" signal-type 0 would strictly prefer to ask, a contradiction.

II. Proofs for Section 4

Proof of Proposition 5. By inspection of ΔIU's in the proof of Lemma 4, it is easy to observe that the difference in expected instrumental utility between asking and not asking increases when p decreases.

For reputation, suppose first that, as p decreases, $\widehat{S}$ does not change. The difference in expected reputation between asking and not asking for signal-type 0, ΔR_0, reads:

$$\Pr(\widehat{S}|\omega = 0) \Pr(\omega = 0|\sigma = 0)(w - x) + \Pr(\widehat{S}|\omega = 1) \Pr(\omega = 1|\sigma = 0)(v - y).$$

As (i) only $\Pr(\omega|\sigma = 0)$ depends on p, (ii) $\Pr(\omega = 0|\sigma = 0)$ decreases as p decreases, and (iii) $w - x < 0 < v - y$, for a given μ, ΔR_0 increases as p decreases. Moreover, it is straightforward to observe that $\underline{\mu}$ and $\overline{\mu}$ weakly increase as p decreases. By Lemma 3, part (i), an increase in $\nu = \mu$ when signal-type 1 always asks induces an increase in expected reputation of signal-type 0 after asking. Thus, the difference in the overall expected payoff of signal-type 0 between asking and not asking under $\underline{\mu}$, $\overline{\mu}$, and $\mu = 0$ increases as p decreases. Then, since the difference in expected instrumental utility is positive by A1,[1] for signal-type 0 to remain indifferent between asking and not asking as p decreases, $\underline{\widehat{\rho}}$, $\widehat{\rho}$, and $\underline{\rho}$ must increase.

Consider now a change in $\widehat{S}$ as p marginally decreases. Namely, suppose that for some $k \leq n$ and each vector of advices s with $o(s) = k$, some signal-type σ switches from considering $\omega = 0$ to considering $\omega = 1$ more likely. When $\sigma = 0$, were she to still decide 0, the reasoning for the case in which $\widehat{S}$ does not change would hold. By switching to $d = 1$, she improves her expected payoff after asking. When $\sigma = 1$, this means that, after s, signal-type 1 considers $\omega = 0$ and $\omega = 1$ equally likely. Then, if the prior is updated with s but not with $\sigma = 1$, $\omega = 0$ results more likely than $\omega = 1$. Thus, given s, signal-type 0 prefers to be perceived as such rather than pooling with signal-type 1 on $d = 0$. This observation is equivalent to Lemma 3, part (ii), as the probability of $\omega = 0$ conditional on s is higher than $1/2$ like the prior p. Hence, the switch of signal-type 1 to $d = 1$ increases the expected reputation of signal-type 0 after s. Thus, a change in $\widehat{S}$ may only increase the difference in the expected payoff of signal-type 0 between asking and not asking, and this makes $\underline{\rho}$, $\widehat{\rho}$ and $\underline{\widehat{\rho}}$ increase even further.

Finally, consider a switch from $\underline{\rho}$ to $\underline{\widehat{\rho}}$. In Case 3, as $\Pr(\omega = 1|\sigma = 1)$ approaches $hr + l(1-r)$, $\underline{\mu}$ approaches 0. Thus, $\underline{\rho} = \underline{\widehat{\rho}}$ when $\Pr(\omega = 1|\sigma = 1) = hr + l(1-r)$, i.e., as we switch from Case 2 to Case 3. ■

Proof of Proposition 6. By (TR), $\overline{\mu}$, if it is not 1, is defined implicitly by

$$\Pr(\omega = 0|m^1)|_{\mu=\overline{\mu}} = rh + (1-r)l. \tag{1}$$

An increase in $rh + (1-r)l$ allows an increase in $\Pr(\omega = 0|m^1)|_{\mu=\overline{\mu}}$, hence an increase in $\overline{\mu}$.

To see the effect of an increase in the competence of the decision-maker ($\gamma = qg +$

[1] Hence, when signal-type 0 is indifferent between asking and not asking, the difference in expected reputation is negative.

$(1-q)b)$, write $\Pr(\omega = 0|m^1)$ as

$$\frac{[\Pr(m^1|\sigma = 0)\Pr(\sigma = 0|\omega = 0) + \Pr(m^1|\sigma = 1)\Pr(\sigma = 1|\omega = 0)]\Pr(\omega = 0)}{num. + [\Pr(m^1|\sigma = 0)\Pr(\sigma = 0|\omega = 1) + \Pr(m^1|\sigma = 1)\Pr(\sigma = 1|\omega = 1)]\Pr(\omega = 1)}.$$

So we get

$$\Pr(\omega = 0|m^1)|_{\mu=\overline{\mu}} = \frac{(\overline{\mu}\gamma + 1 - \gamma)p}{(\overline{\mu}\gamma + 1 - \gamma)p + (\overline{\mu}(1-\gamma) + \gamma)(1-p)}.$$

As γ goes up, when $\overline{\mu} < 1$, $(\overline{\mu}\gamma + 1 - \gamma)$ goes down and $(\overline{\mu}(1-\gamma) + \gamma)$ goes up. (Thus, as expected, $\Pr(\omega = 0|m^1)|_{\mu=\overline{\mu}}$ goes down). Then, to restore equality (1), $\overline{\mu}$ must go up, so that, by $\gamma > 1/2$ (informative signals), $(\overline{\mu}\gamma + 1 - \gamma)$ increases more than $(\overline{\mu}(1-\gamma) + \gamma)$.
■

Proof of Proposition 7.

1) Greater advisors' competence.

Higher prior competence of the advisors impacts on the decision-maker's asking/not asking incentives in two ways. The most straightforward effect is a higher incentive to ask for advice due to more valuable advisors' information.

The less obvious effect is a possible discontinuous decrease in the expected reputation of signal-type 0 from asking (hence, a lower incentive to ask). It can arise because, with higher advisors' competence, there is a lower chance for signal-type 0 to separate and reveal her signal *after* asking (for instance, in the extreme case of the advisors receiving perfect signals, both signal-types will always take the same decision after asking). Suppose we are in Case 2 and consider a situation in which a certain profile of advices makes signal-type 1 believe that $\omega = 1$ is just marginally more likely than $\omega = 0$. Then, under this profile of advices, the two signal-types separate with the decision, but a marginal increase in the prior quality of the advisors will make signal-type 1 switch to $d = 0$. This induces a discrete fall in signal-type 0's expected reputation after asking, by the same argument as in the proof of Proposition 5.

Consider now $\rho = \underline{\rho}$ and suppose that a marginal improvement in the prior quality of advisors does not cause the second effect. Then such an improvement makes signal-type 0 strictly prefer to ask, which is going to destroy the advisors' truthtelling.[2] Effectively, $\underline{\rho}$

[2] As argued, better advisors' competence also widens the set of beliefs about the state for which they report truthfully, but since we consider a marginal improvement in competence, this effect will be marginal, whereas the change in the asking/not asking behavior of signal-type 0 is discrete (and actually extreme).

moves up and there is no equilibrium with information aggregation at the initial value of $\underline{\rho}$. In this case, higher advisors' competence harms through provoking excessive advice-seeking.

Consider now $\rho = \widehat{\rho}$ and suppose we are exactly at the point where a marginal increase in the advisors' competence is going to cause the second effect. Then, such an increase leads to a discrete fall in $\widehat{\rho}$. At the initial value of $\widehat{\rho}$, this results in the failure of not only the second- or first-best equilibrium, but also of any hypothetical equilibrium sufficiently close to the second- or first best (in terms of the probabilities of asking). Thus, the fall in information aggregation will be discontinuous. In such a case, a higher advisors' competence harms through provoking excessive advice-avoidance.

In both cases, while the improvement of advisors' competence is marginal, the fall in information aggregation is discrete, meaning a reduction in the efficiency of decisions.

2) Greater decision-maker's competence.

Consider the pooling-on-asking equilibrium at $\widehat{\rho}$. In this equilibrium, signal-type 0 is indifferent between asking and not asking. Consider a marginal increase in the competence of the decision-maker. The increase in signal-type 0's confidence reduces her expected instrumental utility benefit from asking and can obviously increase her expected reputational gain from not asking. Then, signal-type 0 will strictly prefer not to ask when both signal-types are always expected to ask. Therefore, the pooling equilibrium cannot be sustained anymore at the old value of $\widehat{\rho}$. The same applies to any hypothetical equilibrium in the neighborhood of the pooling equilibrium, whenever signal-type 1 strictly prefers to ask in the pooling equilibrium under the initial level of competence. Indeed, by continuity, signal-type 1 would strictly prefer to ask in such an equilibrium, which implies $\nu \equiv \Pr(m^1|\sigma = 0)/\Pr(m^1|\sigma = 1) \leq 1$. But since, due to Lemma 3 (part (i)), the expected reputation of signal type 0 from asking is increasing in ν, deviation for $\nu = 1$ implies deviation for any $\nu \leq 1$.

Thus, the fall in information aggregation will be discrete. Since the increase in the decision-maker's competence is marginal, this implies a reduction in the efficiency of decisions. ■

III. Robustness of results to different modeling assumptions

This section complements Section 5 of the main body of the text.

III.A. Asking a subset of advisors

In this section, we consider equilibria in which only a proper subset of advisors is asked and argue that they are qualitatively the same as the equilibria of the baseline model.

First, suppose there is an equilibrium in which signal-type 0, with positive probability, asks a subset of advisors Ω_0 different from any subset approached by signal-type 1. Then, due to A2, asking Ω_0 leads to herding by the advisors and, thus, it is equivalent to not asking at all.

Pooling equilibria with a proper subset of advisors being asked are possible, although they are obviously dominated by the pooling equilibrium in which all advisors are asked. In any case, an analogue of Proposition 3 clearly holds with respect to any such pooling equilibrium: Once ρ becomes too high, signal-type 0 would want to deviate to not asking.[3]

It is also clear that separating and partially-separating equilibria in which signal-type 1 asks a proper subset of advisors are similar to their counterparts of the baseline model: The trade-offs and, hence, the incentive compatibility constraints of both signal-types remain qualitatively the same. Essentially, these equilibria are just equilibria of the baseline game with a reduced number of advisors, with deviations to asking more advisors being ruled out by picking appropriate off-the-path beliefs.[4] Therefore, the analysis of Section 3.4.1 of the main text holds for any given subset of advisors being asked.

In principle, asking a proper subset of advisors can extend the set of ρ where some information aggregation is possible: Lowering the number of advisors that are asked can reduce the incentive of signal-type 0 to ask and, thus, lower $\underline{\rho}$. Thus, once we go below $\underline{\rho}$ of the baseline model, we can still sustain *some* information aggregation by reducing the equilibrium subset of asked advisors. However, it is rather obvious that, as ρ moves

[3] In addition, such equilibria, even though they formally exist, look implausible for sufficiently low ρ, in the sense of Grossman and Perry (1986). As any signal-type would be happy to ask the full set of advisors to improve her instrumental payoff, such a deviation should naturally keep the advisors' belief about the decision-maker's signal unchanged, thus making the deviation indeed profitable for both signal-types.

[4] For example, we can impose that asking more than the equilibrium subset makes the advisors believe that $\sigma = 0$, which results in herding.

down, information aggregation eventually deteriorates due a lower and lower number of advisors being asked. Hence, our qualitative result that a too low weight on reputation is detrimental to information aggregation still holds.

III.B. Decision-maker's statements after asking for advice

Suppose that the decision-maker can make a non-verifiable statement about her signal after asking for advice. We argue that this additional cheap talk stage does not change substantially the results of the model.

Consider an equilibrium of the modified game in which both signal-types ask with positive probability, make different and informative statements τ' and τ'', and both statements trigger truthful reporting (otherwise one would be clearly equivalent to not asking)

First, suppose that $\Pr(m^1, \tau|\sigma = 0)/\Pr(m^1, \tau|\sigma = 1) < 1$ for $\tau = \tau', \tau''$. Hence, signal-type 0 does not always ask. Since the two statements are different and signal-type 0 makes both less frequently than signal-type 1, by Lemma 3 (part (i)) signal-type 0 strictly prefers and makes only one of the two statements, say τ'. Then, signal-type 1 would strictly prefer τ' to τ'' if she would consider state 0 more likely. Since sometimes she states τ'', it must be that she considers state 1 more likely. Thus, signal-type 0 is perceived as such after not asking (for signal-type 1 decides 1). Moreover, since she plays both τ' and not asking, she must be indifferent between the two. But then, since expected reputation depends only on relative probabilities, there also exists (and aggregates more information) our "good" partially separating equilibrium, where signal-type 0 asks with probability $\Pr(m^1, \tau'|\sigma = 0)/\Pr(m^1, \tau'|\sigma = 1)$ (instead of $\Pr(m^1, \tau'|\sigma = 0)$ like here).

Second, suppose that after one statement, say τ', $\Pr(m^1, \tau'|\sigma = 0)/\Pr(m^1, \tau'|\sigma = 1) \geq 1$. Then, also pooling on asking triggers truthful reporting and can be implemented in equilibrium without statements.

When the pooling equilibrium of our baseline model exists, an equilibrium where signal-type 0 always plays (m^1, τ') and signal-type 1 randomizes between (m^1, τ') and (m^1, τ'') may exist above $\widehat{\rho}$. So, it is true that under some restrictive conditions on the parameters the additional cheap talk stage extends the implementation of the first best above $\widehat{\rho}$. However, as shown, the introduction of the non-verifiable statements does not affect at all our results for the intermediate values of ρ we are interested in, and it only confirms the message that intermediate values ρ are generally optimal, while too high or too low values of ρ harm information aggregation.

III.C. Impossibility of not asking for advice

In some real-life contexts, it could be impossible to prevent an advisor from expressing his opinion by not asking. In such cases, "not asking" essentially becomes unfeasible, and the decision-maker can only make one of two non-verifiable statements about her signal prior to receiving advice. Such a modification does not affect substantially the results. First, the separating, partially separating, and pooling equilibria exist and have the same characteristics as in the baseline model for the same values of ρ. To see this, simply note that in any of these equilibria not asking is played only by signal-type 0. Then, we can substitute not asking with statement τ' without any effect, because, due to A2, the advisors will herd after τ'. Second, any novel equilibrium of the modified game has exactly the same features as pooling on asking with subsequent statements τ' and τ'' in the game with statements after asking. So, the argument and the conclusions of Section III.B apply here as well.

III.D. Possibility of unobserved advice-seeking

Suppose now that the decision-maker was given the additional opportunity to ask for advice without being observed by the observer. Obviously, this can happen only when the observer is not the advisors. Both when the decision-maker "secretly" asks for advice and when she does not ask for advice at all, the observer observes only the final decision. Then, the reputation of the decision-maker must be the same in the two situations, given the same decision.

It is straightforward to note that the separating, partially separating, pooling-on-asking equilibria of the baseline model have equivalent counterparts in the modified game. If only signal-type 0 asks for advice secretly, she will not receive truthful advice, and this sustains the separating and partially separating equilibria. To sustain the pooling equilibrium with public asking, it is enough that when the advisors are asked for advice secretly, they assign probability 1 to signal-type 0 of the decision-maker (and so does the observer, when no advice-seeking is observed).

Suppose now instead that both signal-types ask for advice secretly, with probabilities that induce the advisors to report truthfully. Then, by A1, the decision-maker strictly prefers to ask secretly rather than not asking. So, whenever the decision-maker does not ask for advice publicly, it is clear to the observer that the decision-maker is asking for advice secretly. Thus, the situation is analogous to the one where asking and not asking are substituted by two different statements: asking publicly and asking "secretly". The only difference is the following: After asking secretly, the observer does not learn the advice that the decision-maker has received. Thus, for each of the two decisions and states of the world, the reputation of the decision-maker will be the same regardless of the unobserved vector of advices. This may eliminate separation at the decision stage in some contingencies where the two signal-types of the decision-maker do consider different states more likely (because a deviation does not entail being perceived as the opposite signal-type anymore). However, as we have already mentioned in Section 3.1 of the main text, different equilibrium choices at the decision stage do not affect qualitatively the results. Hence, all the observations of Section III.C apply here too.

III.E. Privately known decision-maker's type. Effect of p.

Assume the decision-maker knows her competence-type. There will be now four privately known competence-signal-types (call them just "types"), as each of the competence-types $\{G, B\}$ can receive either $\sigma = 0$ or $\sigma = 1$: $G0$, $G1$, $B0$, $B1$.

For $p \leq rh + (1-r)l$, pooling on asking generates truthful reporting by the advisors. Hence reputation concerns do not matter as long as they are not so high that $G0$ prefers to signal her competence-type by not asking[5].

When $p > rh+(1-r)l$, the first best cannot be achieved and, similarly to the baseline model, all informative equilibria will, roughly speaking, have the following feature: signal-types 0 will refrain from asking more often than signal-types 1.

Let us focus, for simplicity, on equilibria in pure strategies. As an example, consider the following equilibrium: $G0$ does not ask for advice, while $G1$, $B0$ and $B1$ ask, and the advisors report truthfully.[6] Such an equilibrium must exist for a range of parameters.

[5] Assuming the natural off-the-path belief that not asking followed by $d = 0$ makes the observer believe that $\theta = G$.

[6] Another possible equilibrium is the one in which $G0$ and $B0$ always refrain from asking, while $G1$ and $B1$ always ask and receive truthful advice. From the point of view of the advisors, the strategy of the decision-maker conveys the same information as in the separating equilibrium of the baseline model. From the point of view of the decision-maker, since asking and not asking are unable to signal the competence-type directly, the trade-off is qualitatively the same as in the baseline model, and it is solved through a similar single-crossing argument. (Of course, since the competence-types are privately

Provided that p it is neither too high nor too low relative to the precision of the good competence-type, g, the advisors' belief after being asked will be sufficiently close to $1/2$ so that (TR) holds (if the proportion of bad competence-types is high enough, then p should just not be too high).

Thus we will have the familiar trade-off between having a higher instrumental utility from asking and higher reputational payoff from not asking. Since $G0$ is most confident about the state of all types, her expected instrumental utility from asking is smaller compared to the other three. For simplicity (not crucial), we can assume that not asking followed by $d = 1$ yields an (off-the-path) belief that the decision-maker is $G1$ (a version of A3 for the privately known types setup). Then not asking always yields the belief that $\theta = G$. Then, naturally, there will be thresholds $\underline{\rho}'$ and $\overline{\rho}'$ such the that the equilibrium under consideration exists if and only if $\rho \in [\underline{\rho}', \overline{\rho}']$ Threshold $\underline{\rho}'$ will be determined by the incentive compatibility of $G0$: when $\rho < \underline{\rho}'$, the reputation concerns are so low that $G0$ will want to deviate to asking for advice. Threshold $\overline{\rho}'$ will be determined by the incentive compatibility of either $B0$ or $G1$ (the deviation incentive of $B1$ is obviously weaker than that of $B0$): when $\rho > \overline{\rho}'$, high reputation concerns will make either of these types deviate to not asking.

As p grows, type $G0$ becomes more confident about the state and, thus, less tempted to ask for advice. Therefore, a lower level of reputation concerns becomes enough for her to refrain from asking, i.e., $\underline{\rho}'$ decreases. Type $B0$ also becomes more confident that $\omega = 0$, which makes her less willing to ask. Consequently, a lower level of reputation concerns is needed to keep $B0$ asking. If $\overline{\rho}'$ is determined by the incentive compatibility of $B0$, this means that $\overline{\rho}'$ goes down. If $\overline{\rho}'$ is determined by the incentive compatibility of $G1$, it must be that $G1$ believes that $\omega = 1$ is more likely. Then, a higher p results in higher willingness to ask by $G1$, meaning an increase in $\overline{\rho}'$. It is clear, however, that at some point $\overline{\rho}'$ becomes determined by the incentive compatibility of $B0$, and, thus, eventually goes down.

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known, the relevant incentive compatibility constraints will not be exactly the same as in the baseline model). Provided that $G1$ does not have a too strong belief in $\omega = 1$, she will prefer pooling with $B1$ instead of not asking and choosing $d = 1$.

Finally, when p is sufficiently close to $1/2$, there can potentially exist equilibria in which G-types never ask and take the decisions corresponding to their signals (the behavior of B-types is likely to vary depending on the equilibrium).

0.1 III.F. Costly information acquisition setup

Consider an alternative setup in which advisors have no reputation concerns and care about the quality of decisions, but need to incur a cost of acquiring a signal. Assume, for simplicity, there is only one advisor, whose ex-post payoff is

$$\begin{cases} 1 \text{ if } d = \omega; \\ 0, \text{ if } d \neq \omega. \end{cases}$$

At stage 2 the advisor has no signal. At stage 4 (if asked for advise at stage 3) by incurring a fixed cost c the advisor can acquire a binary signal $s \in \{0, 1\}$ of precision $\alpha := \Pr(s = \omega)$ for any ω. If he does not invest in information acquisition, no signal is acquired. The advisor then reports whether he invested in information and which signal he received. Otherwise the model is the same as in our baseline setup.

Notice that there is absolutely no reason for the advisor to lie, so let us assume he will always reveal the truth.

Denote $\lambda := qg + (1 - q)b$.

To avoid uninteresting cases, let us make the following two assumptions. First, assume

$$\Pr(\omega = s|\sigma, s) > 1/2 \text{ for any } \sigma \text{ and } s.$$

That is, like in our baseline model (assumption A1), advice is always potentially useful for both signal-types. The decision-maker, thus, will *always* follow the advice. This means that the advisor's update about the decision-maker's signal after being asked does not affect his expected payoff: regardless of the state, the decision will be correct with probability α – the expected quality of the advisor's signal. Thus, if the advisor decides to invest in information acquisition, his expected payoff (after he is asked for advice) will be $\alpha - c$.

Second, assume that, in the case when $\Pr(\omega = 0|\sigma = 1) > 1/2$, learning that $\sigma = 1$ always results in information acquisition (this is analogous to the second part of A2 for $\Pr(\omega = 0|\sigma = 1) > 1/2$). If signal-type 1 believes that state 0 is more likely, absent any advice she will take $d = 0$, and the advisor's expected payoff will be $\Pr(\omega = 0|\sigma = 1)$. Thus, the advisor will acquire information after learning that $\sigma = 1$ (assuming information acquisition in case of indifference) if and only if

$$\alpha - c \geq \Pr(\omega = 0|\sigma = 1).$$

Additionally, to simplify matters, assume that, if signal-type 1 believes that $\omega = 1$ is more likely, the advisor prefers to invest in information when both signal-types always ask for advice (in fact, this is equivalent to assuming that at $p = 1/2$ learning σ triggers information acquisition). In this case, if the advisor does not acquire information, the decision-maker will always take $d = \sigma$, which implies the advisor's expected payoff of λ – the ex-ante probability that σ coincides with ω. Then, the necessary and sufficient condition for information acquisition is

$$\alpha - c \geq \lambda.$$

The above two inequalities can be combined into[7]

$$\alpha - c \geq \max\left\{\Pr(\omega = 0|\sigma = 1), \lambda\right\}.$$

Let us first analyze the second-best solution. Like in the baseline model, signal-type 1 must always ask in the second best, as more asking by signal-type 1 never hurts the advisor's incentives. Indeed, it raises the advisor's belief that the decision-maker is less confident about the state (as signal-type 1 is always less confident), which can only encourage information acquisition.

Let μ be the probability that signal-type 0 asks for advice.

Consider first the case $p > \lambda$, that is, the case when signal-type 1 believes that $\omega = 0$ is more likely.

If the advisor does not acquire information, the decision-maker will always take $d = 0$. Thus, the advisor's expected payoff will then be just $\Pr(\omega = 0|m^1)$, and he will invest in information if and only if

$$\alpha - c \geq \Pr(\omega = 0|m^1).$$

Clearly, $\Pr(\omega = 0|m^1) < p$ for all $\mu < 1$, increases with μ and reaches p at $\mu = 1$.

[7] $\Pr(\omega = 0|\sigma = 1) = \frac{(1-\lambda)p}{(1-\lambda)p+\lambda(1-p)}$, which can be either higher or smaller than λ depending on p. Hence, neither inequality is redundant.

Start with p large enough so that $\alpha - c < p$. The second-best μ, i.e., $\overline{\mu}$, is determined by $\alpha - c = \Pr(\omega = 0|m^1)$; it exists thanks to our assumption that $\alpha - c \geq \Pr(\omega = 0|\sigma = 1)$. As we decrease p, $\Pr(\omega = 0|m^1)$ declines for given μ. Hence, $\overline{\mu}$ has to go up to keep the equality satisfied. If p becomes equal to $\alpha - c$ while we are still in the case $p > \lambda$, $\overline{\mu}$ hits 1. Then the second best becomes the first best, and $\mu = 1$ achieves information provision for any lower value of $p > \lambda$ as well.

If $p \leq \lambda$, signal-type 1 believes that $\omega = 1$ is weakly more likely. Then, thanks to our assumption that $\alpha - c \geq \lambda$, $\mu = 1$ generates information acquisition.

Thus, the entire dynamics of $\overline{\mu}$ is the same as in the baseline model.

Let us turn now to the equilibrium asking/not asking behavior. For the decision-maker's incentives, it is immaterial why exactly advisors provide truthful information; it only matters whether, for a given asking/non-asking behavior, asking triggers information provision. Hence, the trade-offs of the two signal-types remain qualitatively the same as in the baseline model.

Given the dynamics of $\overline{\mu}$, the equilibrium structure, thus, also remains the same. When $\alpha - c < p$, pooling on asking does not trigger information acquisition. The existence of $\underline{\rho}$, below which no information provision occurs in the case is straightforward: Under too low reputation concerns, signal-type 0 would deviate from any (partially or fully) separating equilibrium and ask for advice. The existence of $\widehat{\rho}$ (corresponding to $\overline{\mu}$) is straightforward as well. Given that $\overline{\mu}$ is decreasing in p, we can apply the arguments of the baseline model to show that both $\underline{\rho}$ and $\widehat{\rho}$ increase with the degree of prior uncertainty.

IV. Numerical Example

We conclude the Supplemental Appendix with a numerical example, which shows how the ρ-thresholds for equilibria are determined and change with the prior uncertainty.

Fix the following values of the parameters:

$$q = r = \frac{1}{2};\ g = h = \frac{7}{9};\ b = l = \frac{5}{9};\ n = 3.$$

We leave the prior uncertainty p free, to study how it influences the effect of reputation concerns on information aggregation. Note that the average signal precision, i.e. the ex-ante probability that a state generates the corresponding signal, is the same for the decision-maker and for the advisors (2/3). This has two implications. First, if both signal-types of the decision-maker always ask, each signal-type of the advisor has the same posterior over the state of the world as the decision-maker of the same signal-type. Second, the posterior over the state of the world of the decision-maker depends only on the total number of signals of each kind that she learns, *including her own*. Note that this is not a knife-edge case, in the sense that whether the decision-maker is on average better informed than the advisors or not does not determine per se any qualitative difference in the results.

First, we compute the decision-maker and the advisors' beliefs as a function of p. By Bayes rule, we can use the average signals precision (2/3) as a deterministic signal precision. Denote by $o(s)$ the number of 0's in a profile of truthfully revealed signals s. Then we have:

$$\Pr(\omega = 0|\sigma = 0) = \frac{2p}{p+1} = \Pr(\omega = 0|s_i = 0);$$

$$\Pr(\omega = 0|\sigma = 1) = \frac{p}{2-p} = \Pr(\omega = 0|s_i = 1);$$

$$\Pr(\omega = 0|\sigma = 0, s) = \frac{(\frac{2}{3})^{o(s)+1}(\frac{1}{3})^{3-o(s)}p}{(\frac{2}{3})^{o(s)+1}(\frac{1}{3})^{3-o(s)}p + (\frac{1}{3})^{o(s)+1}(\frac{2}{3})^{3-o(s)}(1-p)} = \begin{cases} \frac{16p}{1+15p} \text{ if } o(s) = 3 \\ \frac{4p}{1+3p} \text{ if } o(s) = 2 \\ p \quad \text{if } o(s) = 1 \\ \frac{p}{4-3p} \text{ if } o(s) = 0 \end{cases}$$

$$\Pr(\omega = 0|\sigma = 1, s) = \begin{cases} \frac{4p}{1+3p} \text{ if } o(s) = 3 \\ p \quad \text{if } o(s) = 2 \\ \frac{p}{4-3p} \text{ if } o(s) = 1 \\ \frac{p}{16-15p} \text{ if } o(s) = 0 \end{cases}$$

$$\Pr(\omega = 0|m^1) = \frac{(2\Pr(m^1|\sigma = 0) + \Pr(m^1|\sigma = 1))p}{(2\Pr(m^1|\sigma = 0) + \Pr(m^1|\sigma = 1))p + (2\Pr(m^1|\sigma = 1) + \Pr(m^1|\sigma = 0))(1-p)}.$$

As p changes, we have the following situations.

- $p \geq \frac{4}{5}$. Then $\Pr(\omega = 0|\sigma = 0, s) \geq \frac{1}{2}$ for $o(s) = 0$. This case contradicts A1.and thus it is not analyzed.
- $\frac{2}{3} < p < \frac{4}{5}$. Then $\Pr(\omega = 0|\sigma = 1) = \Pr(\omega = 0|s_i = 1) > \frac{1}{2}$. This is Case 1;

moreover the advisors herd in case of pooling on asking. Signal-type 0 changes her mind only if all the advisors suggest 1. Signal-type 1, instead, follows the majority of the advisors.

- $\frac{1}{2} < p \leq \frac{2}{3}$. Then $\Pr(\omega = 0|\sigma = 1) = \Pr(\omega = 0|s_i = 1) \leq \frac{1}{2}$. This is Case 2; moreover the advisors report truthfully in case of pooling on asking. The reactions of the decision-maker to the advices are the same as in the previous case.

- $p = \frac{1}{2}$. We are still in Case 2, but also signal-type 1 now changes her mind only if all the advisors suggest 0. The analysis of this case is left to the reader

For no value of p we fall in Case 3, for which it is necessary (but not sufficient) that the advisors' signals have worse average precision than the decision-maker's one.

So, we call $\frac{2}{3} < p < \frac{4}{5}$ Case 1 and $\frac{1}{2} < p \leq \frac{2}{3}$ Case 2.

Both signal-types of the decision-maker react to the advisors' suggestions in the same way in the two cases. Moreover, signal-type 0 always decides 0 after not asking. Thus, we can compute all values of instrumental utility and reputation in the same way for both cases, except for signal-type 1 when she does not ask.

The expected instrumental utility for signal-type 0 after not asking is $\Pr(\omega = 0|\sigma = 0) = \frac{2p}{p+1}$ and for signal-type 1 it is $\Pr(\omega = 0|\sigma = 1) = \frac{p}{2-p}$ in Case 1 and $\Pr(\omega = 1|\sigma = 1) = \frac{2-2p}{2-p}$ in Case 2. After asking, the expected instrumental utility for signal-type 0 is

$$\textstyle\sum_{s:o(s)\geq 1} \Pr(\omega = 0, s|\sigma = 0) + \Pr(\omega = 1, s = (1,1,1)|\sigma = 0) =$$
$$= \textstyle\sum_{s:o(s)\geq 1} \Pr(s|\omega = 0) \Pr(\omega = 0|\sigma = 0) + \Pr(s = (1,1,1)|\omega = 1) \Pr(\omega = 1|\sigma = 0) =$$
$$= (1 - \frac{1}{3^3})\frac{2p}{p+1} + \frac{2^3}{3^3}(1 - \frac{2p}{p+1}) = \frac{2}{3}\frac{2p}{p+1} + \frac{8}{27} = \frac{44p+8}{27p+27};$$

and for signal-type 1 it is

$$\textstyle\sum_{s:o(s)\geq 2} \Pr(\omega = 0, s|\sigma = 1) + \sum_{s:o(s)<2} \Pr(\omega = 1, s|\sigma = 1) =$$
$$= \textstyle\sum_{s:o(s)\geq 2} \Pr(s|\omega = 0) \Pr(\omega = 0|\sigma = 1) + \sum_{s:o(s)<2} \Pr(s|\omega = 1) \Pr(\omega = 1|\sigma = 1) =$$
$$= (\frac{2^3}{3^3} + 3 \cdot \frac{1}{3} \cdot \frac{2^2}{3^2})\frac{p}{2-p} + (\frac{2^3}{3^3} + 3 \cdot \frac{1}{3} \cdot \frac{2^2}{3^2})(1 - \frac{p}{2-p}) = (\frac{2^3}{3^3} + 3 \cdot \frac{1}{3} \cdot \frac{2^2}{3^2}) = \frac{20}{27}.$$

Suppose now that signal-type 1 always asks and signal-type 0 asks with probability μ. Then, after not asking, the advisors believe that the decision-maker has received signal

0 after decision 0 (by equilibrium strategy or A3) and signal 1 after decision 1 (by A3). Using the same notation as in the Appendix ($x := \Pr(G|\sigma = \omega)$, $y := \Pr(G|\sigma \neq \omega)$), the expected reputation for signal-type 0 after not asking is:

$$\begin{aligned}
&\Pr(\omega = 0|\sigma = 0)x + \Pr(\omega = 1|\sigma = 0)y = \\
&= \frac{2p}{p+1} \cdot \frac{7}{12} + (1 - \frac{2p}{p+1}) \cdot \frac{1}{3} = \frac{1}{4}\frac{2p}{p+1} + \frac{1}{3} = \frac{5p+2}{6p+6};
\end{aligned}$$

and for signal-type 1, in Case 1 (where she optimally decides 0), it is:

$$\begin{aligned}
&\Pr(\omega = 0|\sigma = 1)x + \Pr(\omega = 1|\sigma = 1)y = \\
&= \frac{p}{2-p} \cdot \frac{7}{12} + (1 - \frac{p}{2-p}) \cdot \frac{1}{3} = \frac{1}{4}\frac{p}{2-p} + \frac{1}{3} = \frac{8-p}{24-12p}.
\end{aligned}$$

and in Case 2 (where she optimally decides 1), it is:

$$\begin{aligned}
&\Pr(\omega = 0|\sigma = 1)y + \Pr(\omega = 1|\sigma = 1)x = \\
&= \frac{p}{2-p} \cdot \frac{1}{3} + (1 - \frac{p}{2-p}) \cdot \frac{7}{12} = \frac{7}{12} - \frac{1}{4}\frac{p}{2-p} = \frac{7-5p}{12-6p}.
\end{aligned}$$

After asking, the expected reputation of the two signal-types is different since they decide differently if $o(s) = 1$. Using v and w as defined in the Appendix, for signal-type 0 it is:

$$\begin{aligned}
&\Pr(\omega{=}1, o(s) \neq 1|0)v + \Pr(\omega{=}0, o(s) \neq 1|0)w + \Pr(\omega{=}1, o(s){=}1|0)y + \Pr(\omega{=}0, o(s){=}1|0)x = \\
&= (1 - 3\frac{1}{3}\frac{2^2}{3^2})(1 - \frac{2p}{p+1})v + (1 - 3\frac{2}{3}\frac{1}{3^2})\frac{2p}{p+1}w + (3\frac{1}{3}\frac{2^2}{3^2})(1 - \frac{2p}{p+1})y + (3\frac{2}{3}\frac{1}{3^2})\frac{2p}{p+1}x = \\
&= \frac{5}{9}(1 - \frac{2p}{p+1})\frac{7+2\mu}{12+6\mu} + \frac{7}{9}\frac{2p}{p+1}\frac{7\mu+2}{12\mu+6} + \frac{4}{9}(1 - \frac{2p}{p+1})\frac{1}{3} + \frac{2}{9}\frac{2p}{p+1}\frac{7}{12} = \\
&= \frac{35+10\mu}{108+54\mu} + \frac{2p}{p+1}(\frac{49\mu+14}{108\mu+54} - \frac{35+10\mu}{108+54\mu} - \frac{1}{54}) + \frac{4}{27};
\end{aligned}$$

and for signal-type 1:

$$\begin{aligned}
&\Pr(\omega{=}1, o(s) \neq 1|1)v + \Pr(\omega{=}0, o(s) \neq 1|1)w + \Pr(\omega{=}1, o(s){=}1|1)x + \Pr(\omega{=}0, o(s){=}1|1)y = \\
&= (1 - 3\frac{1}{3}\frac{2^2}{3^2})(1 - \frac{p}{2-p})v + (1 - 3\frac{2}{3}\frac{1}{3^2})\frac{p}{2-p}w + (3\frac{1}{3}\frac{2^2}{3^2})(1 - \frac{p}{2-p})x + (3\frac{2}{3}\frac{1}{3^2})\frac{p}{2-p}y = \\
&= \frac{5}{9}(1 - \frac{p}{2-p})\frac{7+2\mu}{12+6\mu} + \frac{7}{9}\frac{p}{2-p}\frac{7\mu+2}{12\mu+6} + \frac{4}{9}(1 - \frac{p}{2-p})\frac{7}{12} + \frac{2}{9}\frac{p}{2-p}\frac{1}{3} = \\
&= \frac{35+10\mu}{108+54\mu} + \frac{p}{2-p}(\frac{49\mu+14}{108\mu+54} - \frac{35+10\mu}{108+54\mu} - \frac{5}{27}) + \frac{7}{27}.
\end{aligned}$$

To look for $\underline{\rho}$ and $\overline{\rho}$ we need to compute the values of reputation for $\mu = 0$. Then, for signal-type 0 the expected reputation after asking is:

$$\frac{35}{108} + \frac{2p}{p+1}\left(\frac{28}{108} - \frac{35}{108} - \frac{2}{108}\right) + \frac{16}{108} = \frac{51}{108} - \frac{9}{108}\frac{2p}{p+1} = \frac{33p+51}{108p+108}$$

For signal-type 1 it is:

$$\frac{35}{108} - \frac{27}{108}\frac{p}{2-p} + \frac{28}{108} = \frac{126-90p}{108(2-p)} = \frac{7-5p}{12-6p}.$$

The difference in expected payoff between asking and not asking for signal-type 0 is zero for $\underline{\rho}$ such that:

$$(1-\underline{\rho})\left(\frac{44p+8}{27p+27} - \frac{2p}{p+1}\right) + \underline{\rho}\left(\frac{33p+51}{108p+108} - \frac{5p+2}{6p+6}\right) = 0$$
$$(1-\underline{\rho})(32-40p) + \underline{\rho}(15-65p) = 0$$
$$\underline{\rho} = \frac{32-40p}{17+25p}.$$

As expected, for $p \uparrow 4/5$, $\underline{\rho} \downarrow 0$: signal-type 0 has no strict incentive to follow three 1 suggestions, so there is no gain from asking. For $p \downarrow 2/3$, $\underline{\rho} \uparrow \frac{16}{101}$. So, in Case 1 $\underline{\rho} \in (0, \frac{16}{101})$. Recall that in Case 1 pooling on asking does not trigger truthful reporting. Thus, there is no information aggregation up to $\underline{\rho}$, i.e. there is no information aggregation for too low reputation concerns. As uncertainty increases, i.e. as p decreases, $\underline{\rho}$ increases. That is, higher reputation concerns are needed to obtain some degree of information aggregation. For $p \downarrow 1/2$, $\underline{\rho} \uparrow \frac{24}{59}$, so in Case 2, $\underline{\rho} \in [\frac{16}{101}, \frac{24}{59})$. However in Case 2, pooling on asking triggers truthful reporting and can be implemented from $\rho = 0$.

The difference in expected payoff between asking and not asking for signal-type 1 in Case 1 is zero for $\overline{\rho}$ such that:

$$(1-\overline{\rho})\left(\frac{20}{27} - \frac{p}{2-p}\right) + \overline{\rho}\left(\frac{7-5p}{12-6p} - \frac{8-p}{24-12p}\right) = 0$$
$$(1-\overline{\rho})\left(\frac{160-188p}{108(2-p)}\right) + \overline{\rho}\left(\frac{54-81p}{108(2-p)}\right) = 0$$
$$\overline{\rho} = \frac{160-188p}{106-107p}.$$

As expected, for $p \downarrow 2/3$, $\overline{\rho} \uparrow 1$. Note indeed that in Case 2, as suggested by Proposition

1, $\overline{\rho} = 1$: The expected reputation after asking and not asking is the same, so no value of ρ makes signal-type 1 indifferent between asking and not asking. For $p = 4/5$, $\overline{\rho} = \frac{24}{51}$. Thus, also $\overline{\rho}$ increases as p decreases.

To compute $\widehat{\rho}$, we need to compute the highest value of μ such that the advisors report truthfully. It solves:

$$\Pr(\omega = 0|m^1) = \frac{2\overline{\mu}p + p}{\overline{\mu}p + 2 + \overline{\mu} - p} = \frac{2}{3}.$$
$$\overline{\mu} = \frac{4 - 5p}{4p - 2}.$$

For $p \uparrow 4/5$, $\overline{\mu} \downarrow 0$, so the "second best" partially separating equilibrium approaches the separating equilibrium with weak incentive for signal-type 0. As anticipated, for $p = 2/3$, $\overline{\mu} = 1$ so for every $p > 2/3$ there is no pooling-on-asking equilibrium with truthful reporting. Now we look for the value of ρ such that signal-type 0 is indifferent between asking and not asking for $p = 2/3$ and $\mu = 1$: this is the upper bound for $\widehat{\rho}$ in Case 1. Substituting $\mu = 1$ in the reputation after asking, the difference in expected payoff for signal-type 0 between asking and not asking is zero for $\widehat{\rho}$ such that:

$$(1 - \widehat{\rho})(\frac{44p + 8}{27p + 27} - \frac{2p}{p + 1}) + \widehat{\rho}(\frac{45}{162} + \frac{2p}{p + 1}(\frac{15}{162}) + \frac{4}{27} - \frac{5p + 2}{6p + 6}) = 0$$
$$(1 - \widehat{\rho})(48 - 60p) + \widehat{\rho}(15 - 36p) = 0$$
$$\widehat{\rho} = \frac{48 - 60p}{33 - 24p}.$$

For $p \downarrow 2/3$, $\widehat{\rho} \uparrow \frac{8}{17}$. So, in Case 1, $\widehat{\rho} \in (0, \frac{8}{17})$. Note that for every p, as expected, $\widehat{\rho} > \underline{\rho}$.

In case 2, pooling on asking triggers truthful reporting. So, we are interested in $\widehat{\rho}$ as the maximum weight on reputation such that the pooling-on-asking equilibrium exists under A3. For $p \downarrow 1/2$, $\widehat{\rho} \uparrow \frac{6}{7}$. Thus, in Case 2, the pooling-on-asking equilibrium exists up to $\widehat{\rho} \in \left[\frac{8}{17}, \frac{6}{7}\right)$.

Also $\widehat{\rho}$ increases as p decreases. That is, more uncertainty requires (in Case 1) or allows (in Case 2) higher reputation concerns to achieve the best feasible level of information aggregation.

References

[1] Grossman, S.J., and M. Perry, 1986, "Perfect sequential equilibria," *Journal of Economic Theory*, 39(1), 97–119.