MICROECONOMICS

SELF-STUDY PROBLEMS

WITH

ANSWERS/HINTS/SOLUTIONS

Content

Consumer's Choice: Exercises

1. Consider a person that spends all his fixed income M on two goods. Currently he spends one third of the income on good 2. If the price of good one rises by 50% and consumer's income by one third, what is the change in the consumer's welfare?

2. During a war, food (good 1) and clothing (good 2) are rationed. In addition to a money price, p_i (i=1,2), a certain number of ration coupons q_i must be paid to obtain good i. Each consumer has an allocation of ration coupons Q (coupons are infinitely divisible) which may be used to purchase either good, and also has a fixed income M.
(a) Illustrate the budget set. Explain carefully.
(b) Suppose the money income of a consumer is raised and he buys more food and less clothing. It follows that clothing is an inferior good. True or false?
(c) Suppose the consumer's preferences can be described by $\sqrt{x_1}+\sqrt{x_2}$. Assume P_1=3, P_2=1, M=120 and Q=90. The purchase of each unit of good 1 requires one coupon and the purchase of good 2 - two. Find the optimal consumption bundle. Illustrate graphically.

(d) Assume that coupons can be traded and their price is 1 per coupon. Redraw the budget line, compute the optimum consumption bundle. Will the consumer buy or sell coupons? How many?

(e) At what price (of coupons) will the consumer not trade in coupons? What happens at higher and lower prices?

3. Bob uses his monthly income (M) to pay for water services and all other goods ("other goods" represents a composite commodity of all other goods. The price of the water services is p per m^3 (assume that water meters are already installed), and the price of composite commodity is 1. Bob's preferences are represented by differentiable utility function. The local water company cannot cover its cost and considers two options to solve the problem. It could raise the price by 10%. In this case Bob's utility level is reduced from u^0 to u^1. Alternatively, the water company may keep per unit price constant but in addition introduce fixed per month charge that results for Bob exactly in the same utility loss. Which scheme brings more revenue to the water company? Which scheme results in greater water conservation? Provide graphical and analytical solution.

4. Explain, why an increase in the basic wage rate per hour offered to a worker may decrease the number of hours she wishes to work while an overtime premium offered to the same worker may increase the number of hours she wishes to work?

5. At the backward bending part of individual labour supply curve leisure is a Giffen good. True or false? Explain.

Consumer’s Choice: Answers and Hints

1. Not worse off as initial bundle is just affordable (proof). Better off under smooth IC**.** Example (graphical or analytical)

2. (a) Hint: illustrate on the same graph money constraint and coupons constraint and look at the intersection of the two sets.
(b) False. Counterexample with normal good (graphical or analytical)

(d) Hint: combine both constraints by calculating the ‘full price’ for each good

(e) At p=2, sell coupons at p>2 and buy at p<2**)**

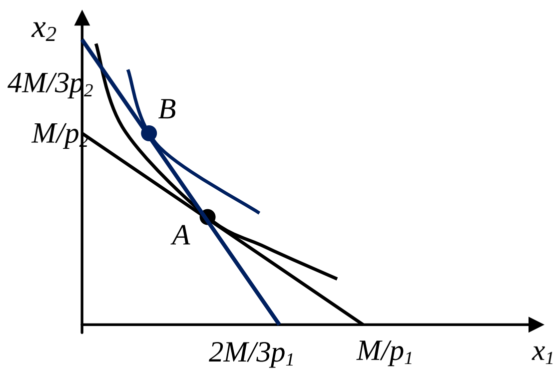
3. Water conservation is higher under the first scheme but the second scheme brings more revenue
Hint: look at substitution effect
Hint for graphical analysis:
$TR_1 = p^1 x(p^1, M) = M - y(p^1, M)$ and $TR_2 = F + p^0 x(p^0, M - F) = M - y(p^0, M - F)$.

4. Look at SE/IE

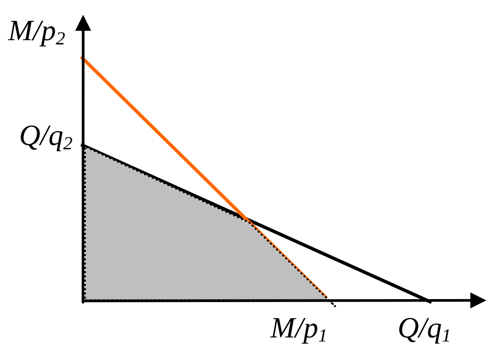
5. False.

Consumer's Choice: Solutions

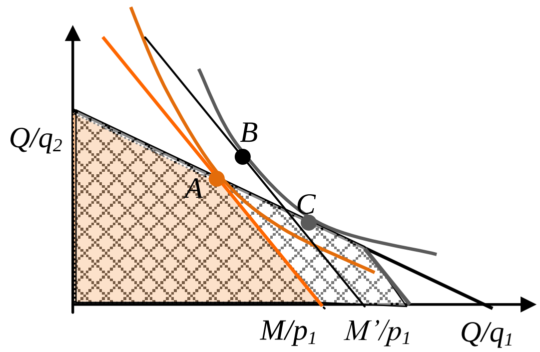
1. Not worse off. Initial bundle is just affordable (proof)
Initial bundle: $x_1 = M/3p_1$ and $x_2 = 2M/3p_2$.
Initial bundle is just affordable under new prices and income
$M/3 + 1{,}5 * 2M/3 = 4M/3 = M^{new}$
It means that consumer is not worse off (as he can choose initial bundle).
But as relative price of good 1 is different he can substitute away from good 1 that is relatively more expensive. Thus in case of smooth IC he is better off.
Example (graphical or analytical).



2. (a) Graph with comments.
Now in addition to money constraint $p_1x_1 + p_2x_2 \le M$ we have coupons constraint $q_1x_1 + q_2x_2 \le Q$ and these constraints should be satisfied simultaneously.



(b) False. Counterexample with normal good (graphical or analytical)



Comments: good 1 is normal as its consumption increases [movement from A to B] when income goes up and there are no additional constraints (like coupons).

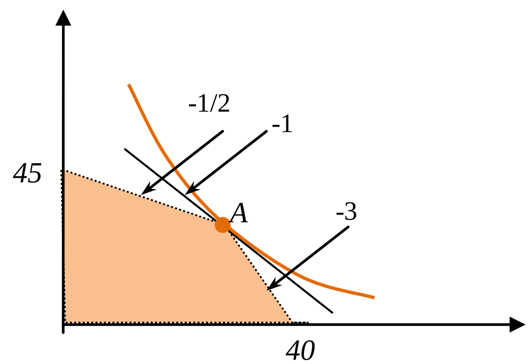
(c) Let's find the kink of the BC: $\begin{cases} 3x_1 + x_2 = 120 \\ x_1 + 2x_2 = 90 \end{cases}$, then $x_1 = 30, x_2 = 30$.

$$MRS_{12}(30{,}30) = \sqrt{\frac{x_2}{x_1}} = 1$$

Absolute value of the slope of money constraint line is 3, while for coupons constraint the absolute value of the slope is 1/2. Thus the slope of IC at the kink is in between these two slopes, which means that utility is maximized at this point.

Optimal consumption bundle (A): $x_1 = 30, x_2 = 30$

Graph



(d) Denote the quantity of traded coupons by z ($z > 0 \Rightarrow$ coupons are purchased, $z < 0 \Rightarrow$ coupons are sold)

New coupons constraint: $x_1 + 2x_2 \leq 90 + z$

New money constraint: $3x_1 + x_2 \leq M - z$

At the optimal point both constraints are binding. If coupons constraint is not binding while money is binding, then consumer can reduce z a bit (so that this constraint doesn't bind) then his income $M-z$ will increase and he can use it to increase consumption of any of the good. This improves his welfare. It proves that initial bundle does not maximize utility.

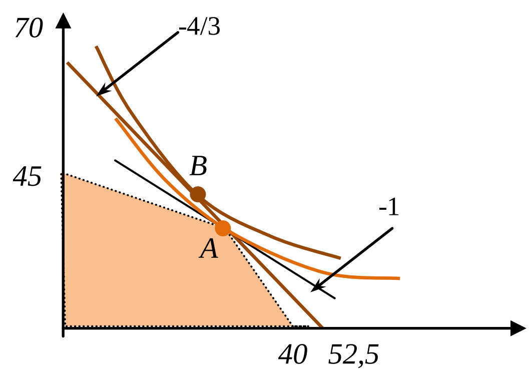
Use the same approach to justify that money constraint is binding.

From coupons constraint we get $z = x_1 + 2x_2 - 90$ and plug it into the money constraint that results in the following budget line: $4x_1 + 3x_2 = 210$. As coupons can be traded, the full price of good 1 is 3+1=4, the full price of good 2 is 1+2=3 and the total income is 90+120=210.

$MRS_{12} = \sqrt{\frac{x_2}{x_1}} = \frac{4}{3}$, $3x_2 = \frac{16x_1}{3}$, $4x_1 + \frac{16}{3}x_1 = 210$, Optimal consumption bundle (B) $x_1 = \frac{210\times 3}{28} = 22{,}5$ and $x_2 = \frac{16x_1}{9} = 40$.

To the left from A coupons constraint is binding, thus he will purchase coupons.

He needs 22,5+2*40=102,5 coupons. Thus he will buy 102.5-90=12.5 coupons.



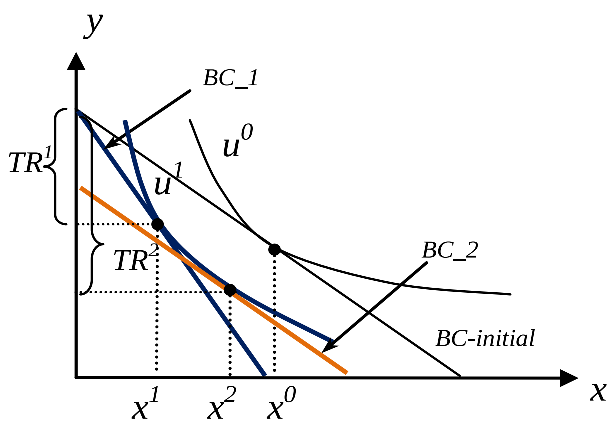
Graph

(e) If he doesn't trade then he stays at the point, where both constraints are binding (point A). This point is optimal iff IC is tangent to the budget line.

The slope of the budget line is given by the price ratio that reflects the full price of every good: $MRS_{12}(30{,}30) = 1 = \frac{3+p}{1+2p}$, $3+p = 1+2p$, $p=2$. Thus at $p=2$ he will stay with his endowment of coupons.

If $p > 2$ then $MRS_{12}(30{,}30) = 1 > \frac{3+p}{1+2p}$, i.e. agent is willing to increase consumption of good 1. As we know from (c) at any point to the right from the kink money constraint is binding while coupons constraint is not. Thus agent will sell coupons at any p>2. At p<2 he will buy coupons.

3. Graph.
Let x stays for water consumption and y-for AOG. Revenue of water company is given by the sum of revenue from sales (price multiplied by quantity) and fixed charge. As his income is the same, then water expenditures equals M-y. Graphically we compare $TR_1 = p^1 x(p^1, M) = M - y(p^1, M)$ and $TR_2 = F + p^0 x(p^0, M - F) = M - y(p^0, M - F)$. Thus from graph we get $TR_1 < TR_2$ and $x^0 - x^1 > x^0 - x^2$. Thus the second scheme brings more revenue but the first scheme provides greater water conservation.



Proof. As bundles (x^1, y^1) and (x^2, y^2) provide the same utility, then the change in quantity demanded is due to Hicksian SE only. We know that own SE is nonpositive. As relative price goes up when we proceed from x^2 to x^1 and ICs are smooth (due to differentiability of utility function) then $\Delta x^{SE} = x^1 - x^2 < 0$. Thus $x^1 < x^2$, which means that water conservation is higher under the first scheme.
Due to nonsatiation with lower consumption of x we can have the same utility only with greater consumption of y: $u(x^1, y^1) = u(x^2, y^2)$ and $x^1 < x^2$ implies $y^1 > y^2$. Thus $TR_1 = M - y^1 < M - y^2 = TR_2$.

Choice under Uncertainty: Exercises

1. Dan has utility function $u(w)=\sqrt{w}$, where w is his wealth. All his initial wealth, equal to \$36, is deposited at bank M. With probability of 0.5 this bank can become bankrupt. Had this happened, Dan would get only \$4 guaranteed by the government. A risk neutral firm N proposes Dan to purchase his problem deposit (before the uncertainty is resolved) for \$X.

(a) Find all values of X that are mutually beneficial for Dan and firm N, provide graphical solution.
(b) Suppose that X=20. A corrupted manager from bank M possesses information about the bank's position and can say with certainty whether bankruptcy will take place. He offers Dan to sell this information. What is the maximum amount that Dan is willing to pay for this information? Provide algebraic solution and illustrate your solution on a diagram with contingent commodities.
(c) Suppose that Zara faces exactly the same problem as Dan but she is risk neutral. Find the maximum sum that Zara is willing to pay for the information offered by the corrupted manager described in (b) and compare with the maximum sum that Dan is willing to pay. Illustrate on the same graph.
(d) Compare the maximum prices found in (b) and (c). Would the result of this comparison be different if Dan had different preferences but the same type of risk attitude?

2. In P&R textbook you can find figure 5.6, that represents preferences of a risk averse agent in terms of expected return and standard deviation. In this exercise you are asked to analyze the implicit assumptions that stay behind this graph.
(a) Suppose that individual has quadratic Bernoulli utility function $u(x)=x-bx^2$, where $b>0$. Show that for this individual the expected utility from a distribution is determined by the mean and variance of the distribution and, in fact, by these moments alone. Draw the resulting indifference curves in the same axes as in figure 5.5. [Note: to guarantee non-satiation we assume that distribution cannot take values larger than $1/(2b)$.]
(b) Suppose that individual considered in (a) can invest in treasury bills or stocks or in some combination of the two. Treasury bills bring zero expected return and are assumed to be risk-free. Stocks bring positive expected return R_s and are assumed to be risky with standard deviation of $\sigma_s>0$. Suppose this investor has initial wealth equal to 1.

- **(i)** Derive the budget constraint in terms of mean and standard deviation of the portfolio and illustrate it graphically.
- **(ii)** Solve the resulting utility maximization problem to get demand for risky and risk-free assets (consider interior solution only). Illustrate graphically.

(c) Suppose that individual from (b) invests in both assets. How is the optimal portfolio affected by:

- **(i)** an increase in coefficient b?
- **(ii)** an increase in variance of risky asset?

2. A farmer can grow wheat or potatoes or both. If the summer is shiny then a unit of land yields a profit of \$7 if devoted to wheat, of \$3 if devoted to potatoes. Should the summer be rainy, a unit of land yields a profit of \$1 if devoted to wheat, of \$3 if devoted to potatoes. Good and bad weather is equally likely. Farmer is risk averse and has 40 units of land. Let the farmer's utility function be given by $u(x)=4\sqrt{x}$, where x stays for wealth.

(a) What is the optimal allocation of land between potatoes and wheat. Illustrate graphically and explain the result intuitively.

(b) Suppose that farmer is offered to rent his land out for one period for \$X. Find the smallest X at which the farmer would accept the offer. Denote this sum by $X_{\min}$ and illustrate graphically. Compare $X_{\min}$ with the one period expected profit from his business and explain the result.

(c) Now, suppose that the farmer can purchase an insurance against bad weather that pays \$1 per unit of insurance in case of bad weather at a price of \$0.5 per unit. Find the optimal allocation of land and the quantity of insurance purchased. Illustrate graphically and explain the result intuitively.

Choice under Uncertainty: Answers and Hints

1. (a) X from 16 to 20;

(b) Max_sum=7.2;

(c) Max_sum=8;

(d) The max sum will change but it will be less than 8

2. (a) $Eu(x) = Ex - b\left(\sigma^2 + (Ex)^2\right)$;

(b) (i) budget line: $\sigma_p = \frac{\sigma_S}{R_s} E_p$, where $0 \le E_p \le R_s$, E_p-portfolio expected return, σ_p - portfolio standard deviation;

(ii) $E_p = 0.5 \frac{(R_s)^2 / b}{(R_s)^2 + \sigma_s^2}$, which gives the optimal investment in risky asset $\alpha = 0.5 \frac{R_s / b}{(R_s)^2 + \sigma_s^2}$;

(c) investment in risky asset goes down in both cases (i) and (ii)

3. (a) 30 units (wheat) and 10 units (potatoes);

(b) $X_{\min} = 135 < \text{Exp\_profit} = 150$;

(c) 40 units (wheat) and 0 units (potatoes) +full insurance

Choice under Uncertainty: Solutions

1. (a) Dan doesn't reject iff his EU does not go down as a result of this sale: $\sqrt{x} \geq 0.5\sqrt{4} + 0.5\sqrt{36} = 4$, $x \geq 16$

Firm accepts iff its expected utility (equal to the expected profit due to risk neutrality) is not reduced as a result of this transaction: $0.5 \times 4 + 0.5 \times 36 - x = 20 - x \geq 0$

Mutually beneficial $20 \geq x \geq 16$

Graph should be provided

(b) With information Dan sells his deposit in case of bankruptcy and gets $20 - Q$ (Q - price of information) and keeps deposit otherwise (in this case his wealth is $36 - Q$). The resulting expected utility is $EU^{\inf} = 0.5\sqrt{20 - Q} + 0.5\sqrt{36 - Q}$. Without information he is better off by selling this deposit as price exceeds 16 and his utility is $\sqrt{x} = \sqrt{20}$. Thus he will purchase information iff $EU^{\inf} = 0.5\sqrt{20 - Q} + 0.5\sqrt{36 - Q} \geq \sqrt{20}$. The maximum price makes Dan indifferent:

$$\sqrt{36 - Q} = 2\sqrt{20} - \sqrt{20 - Q}\,.$$

Then $36 - Q = 4 \times 20 + 20 - Q - 4\sqrt{(20 - Q)20}$, which can be rewritten as $4\sqrt{(20 - Q)20} = 100 - 36 = 64$. Thus $\sqrt{(20 - Q)5} = 8$, which implies $(20 - Q)5 = 64$. Solving equation we get $Q = 20 - \frac{64}{5} = 7.2$

Graph should be provided

(c) Zara has the same utility function as bank N, so without information she is indifferent b/w selling deposit at X=20 or keeping it. $EU_B^{\inf} = 0.5(20 - Q) + 0.5(36 - Q) = 28 - Q = 20$, which gives $Q = 8$

Graph should be provided

(d) $Q_{Zara} = 8 > 7.2 = Q_{Dan}$

General case:

$0.5u(4) + 0.5u(36) < u(0.5 \times 4 + 0.5 \times 36) = u(20)$. Thus without information offer of the corrupted manager is still accepted as it gives the same EV: (36+4)/2=20 but with certainty

Maximum price of information should make this agent indifferent:

$$u(20) = EU^{\inf}(Q) = 0.5u(20 - Q) + 0.5u(36 - Q)$$

Due to risk-aversion $0.5u(20 - Q) + 0.5u(36 - Q) < u\left(\frac{20 - Q}{2} + \frac{36 - Q}{2}\right) = u(28 - Q)$

It implies that $u(20) = EU^{\inf}(Q) = 0.5u(20-Q) + 0.5u(36-Q) < u(28-Q)$

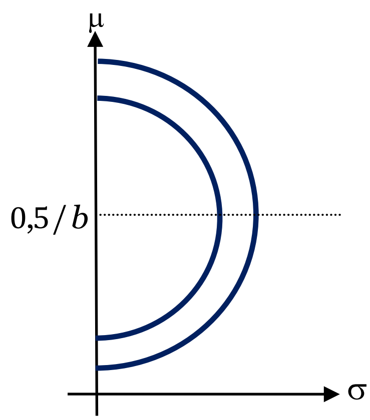
As $u(w)$ is increasing then $20 < 28 - Q$, $Q_{Dan} < 8 = Q_{Zara}$

4. (a) Calculate EU: $Eu(x) = Ex - bEx^2 = Ex - b(\sigma^2 + (Ex)^2)$

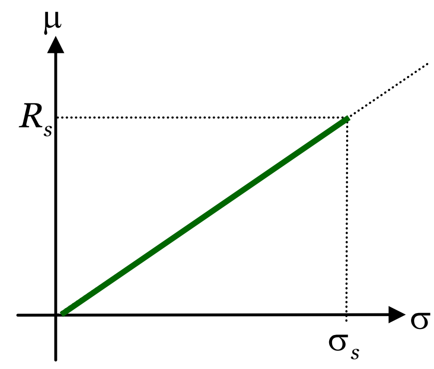
Define IC $Ex - b(\sigma^2 + (Ex)^2) = \bar{u}$ and re-arrange:

$\sigma^2 + (Ex)^2 - Ex/b + \bar{u}/b = \sigma^2 + (Ex - 0.5/b)^2 + \bar{u}/b - 0{,}25/b^2 = 0$

Thus ICs represent circles $\sigma^2 + (Ex - 0.5/b)^2 = 0{,}25/b^2 - \bar{u}/b$ with center at $(0, 0.5/b)$ and radius $\sqrt{0{,}25/b^2 - \bar{u}/b}$. Due to the assumption we operate only at upward sloping parts of ICs. Graph.



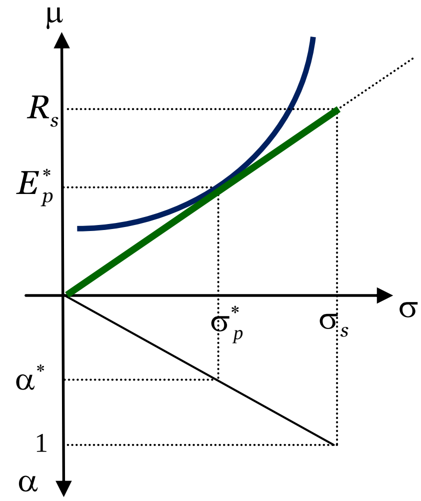
(b) (i) Budget line. Suppose he invests α in risky asset, then expected return of portfolio is $E_p = \alpha R_s$ and variance of portfolio is $\sigma_p^2 = \alpha^2 \sigma_s^2$, which implies $\sigma_p = \alpha\sigma_s$. From the first equation $\alpha = \dfrac{E_p}{R_s}$ and plugging in the second we get BL $\sigma_p = \dfrac{\sigma_s}{R_s} E_p$. As $0 \le \alpha \le 1$, we get additional restrictions: $0 \le E_p \le R_s$.



(ii) utility maximization problem

$$\max_{0 \le E_p \le R_s} E_p - b(\sigma_p^2 + E_p^2) \quad s.t. \quad \sigma_p = \frac{\sigma_s}{R_s} E_p$$

Plugging into objective function, we get $\max\limits_{0 \le E_p \le R_s} E_p - b\left(\frac{\sigma_s}{R_s} E_p\right)^2 - bE_p^2$.



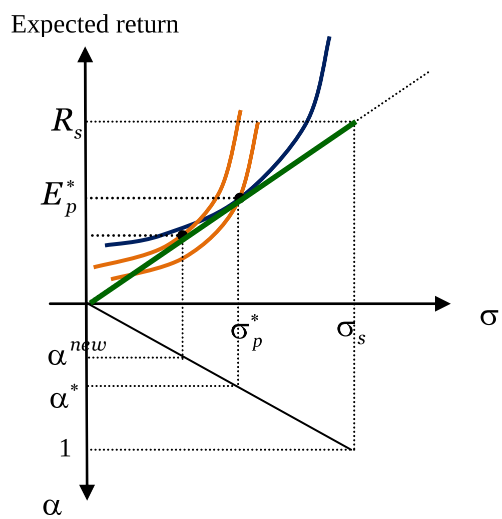
Function is strictly concave, so FOC is both necessary and sufficient

$$1 - 2b\left(\frac{\sigma_s}{R_s}\right)^2 E_p - 2bE_p = 0,\ 1 = 2bE_p\left(1 + \left(\frac{\sigma_s}{R_s}\right)^2\right),$$

$$E_p = 0.5\frac{(R_s)^2 / b}{(R_s)^2 + \sigma_s^2},\ \alpha = \frac{E_p}{R_s} = 0.5\frac{R_s / b}{(R_s)^2 + \sigma_s^2}.$$

Graph with IC tangent to BL [the lower part of the graph is not required but comments concerning α should be provided if this part is absent].

(c) (i) $\alpha = 0.5\frac{R_s / b}{(R_s)^2 + \sigma_s^2}$ is decreasing in b. It means that with higher b we get higher coefficient for variance in utility function, which indicates higher risk aversion that results in the reduction in investment in risky asset.



Graphically ICs become steeper as

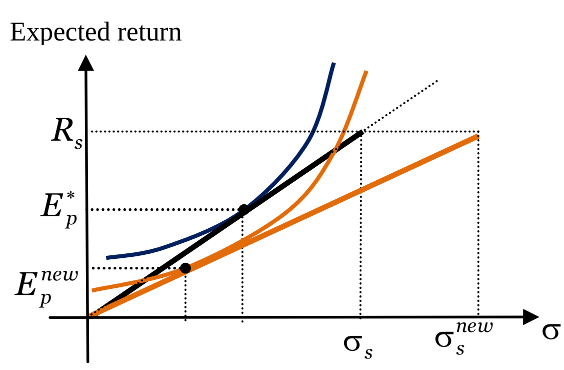
$$\frac{dEx}{d\sigma} = \frac{\sigma}{0.5/b - Ex} \text{ is increasing in } b,$$

$$\alpha = 0.5\frac{R_s / b}{(R_s)^2 + \sigma_s^2} \text{ is decreasing in } \sigma_s^2,$$

which is also reasonable as risk averse agent is willing to reduce his investment in risky asset when its riskiness went up while expected gain stays the same.

(ii) Budget line becomes steeper and IC is tangent to the new budget line somewhere to the left.

As a result new optimal portfolio has lower expected return. As expected returns of the two assets stay the same, this implies that agent invests less in risky asset.



2. (a) Optimal allocation of land is derived from EU maximization:

$$\max_{0\le y\le 40}\left(2\sqrt{7y+3(40-y)}+2\sqrt{y+3(40-y)}\right),$$

Function is strictly concave, so FOC is both necessary and sufficient

$$\frac{4}{\sqrt{120+4y}}-\frac{2}{\sqrt{120-2y}}=0,$$

$$2\sqrt{60-y}=\sqrt{60+2y};$$

$$4(60-y)=60+2y;\ y=60\times 3/6=30$$

Graph should be provided

Intuition: potato serves as risk-free asset while wheat is risky. Exp.profit from wheat exceeds the profit from potatoes (7+1)/2=4>3. Agent is risk averse (u-concave) and is willing to accept some risk as game is favourable.

(b) $X_{\min}$ makes this farmer indifferent between renting out and continuing his business.

Expected utility from business: $EU^{optimal}=2\left(\sqrt{210+30}+\sqrt{30+3\times 10}\right)=6\sqrt{60}=12\sqrt{15}$, $4\sqrt{X_{\min}}=12\sqrt{15}$, $X_{\min}=9\times 15=135$.

Graph should be provided

$E\pi=4\times 30+3\times 10=150>X_{\min}$.

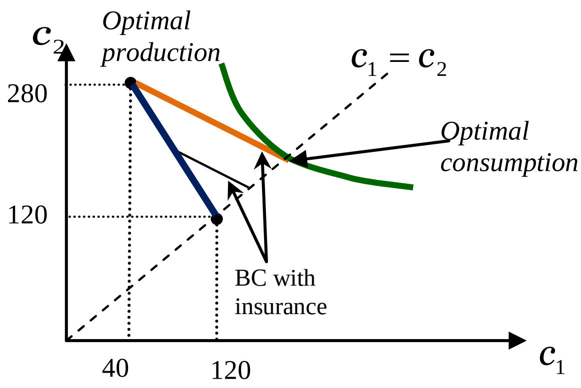
Explanation. As agent is risk averse he prefers expected value of the risky prospect (150) to the risky prospect, i.e $u(EV=150)>EU=12\sqrt{15}=u(X_{\min})$. Thus $X_{\min}<EV=150$.

(c) Derivation of optimal allocation with insurance

Suppose that y units of land are allocated to wheat and $(40-y)$- to potatoes. Then we have $(120+4y)$ in case of shiny and $(120-2y)$ in case of rainy summer. This point is risky if $y>0$.

Note that insurance is offered at actuarially fair terms (price equals the probability of loss), then risk averse agent will purchase full insurance. Thus if $y>0$, then he purchases $z=6y$ units of insurance. Finally consumption with insurance is $120+4y-0.5\times 6y=120+1y$ in each state of the world, which exceeds consumption under $y=0$. Thus it is profitable to have $y>0$. Now EU maximization problem is $\max_{0\le y\le 40}\sqrt{120+y}=\sqrt{120+40}$, i.e. $y=40$. It could be also derived algebraically without using the claim about full insurance but then intuitive explanation for the full insurance should be proved afterwards.

Graph.


c_2
Optimal
production
$c_1 = c_2$
280
Optimal
consumption
120
BC with
insurance
40
120
c_1

Theory of firm and Perfect competition: Exercises

1. Production function of the firm is given by $Q = 4\sqrt{L} + \sqrt{K}$. The wage of labor is $8 per hour and the rental price of capital is $2 per hour.

(a) Derive short run AC and AVC. Explain the shapes of AVC and AC.

(b) Derive long run total costs and find long run average cost. Are there economies of scale? Explain the result.

(c) Illustrate on the same graph LRAC and several SRAC curves.

2. A constant-cost industry consists of a large number of firms, each of which has a cost function of the form: $C(q) = \begin{cases} 9q^2 + 16, & q > 0 \\ 0, & q = 0 \end{cases}$.

(a) Find the long run supply curve of an individual firm;

(b) Find the long run equilibrium price and the number of operating firms, if the market demand is given by $Q^d(p) = 150 - 4p$.

3*[1]. Consider a perfectly competitive industry that produces good X. All firms in this industry have identical technologies with cost function $c(q)$, where $c'(0)=0$, $c'(q)>0$, $c''(q)>0$ for $q > 0$. Unfortunately a fraction α of the output produced by each firm is defective and cannot be sold. Moreover firm experiences some utilization cost for unsold output and the corresponding cost function is given by $l(z)$, where z is the volume of utilized output, $l'(0)=0$, $l'(z)>0$ and $l''(z)>0$ for $z > 0$. Both production and utilization cost are zero if output is zero.

(a) Suppose that improvement in management brings a reduction of α while cost function stays the same. What is the impact on individual supply of each firm? Provide both graphical and analytical solution (Note: change in α is not necessarily small).

(b) Suppose that there are N firms in the industry. Denote by $p(\alpha)$ the equilibrium price in this market. Find the impact of reduction in α on this short-run equilibrium price. Explain the result. (No need in graphical solution)

4. Consider perfectly competitive constant cost industry with identical firms. Suppose that, a per-unit sales subsidy is replaced by a lump sum subsidy that every active firm gets. The lump-sum subsidy leave the equilibrium price the same as it was under the per-unit subsidy scheme. Compare the total government expenditures under per unit and lump-sum subsidy schemes: (i) graphically assuming U-shaped AC and (ii) analytically (i.e. for any type of AC consistent with the conditions of the problem).

[1] Difficult problems are indicated by *

5. Consider the rental housing market with linear demand and supply curves. Suppose that house owners are required to pay rental tax equal to share α ($0 < \alpha < 1$) of rental price paid by tenants. All these tax revenue goes to the local government budget.

(a) Local government decided to introduce rent control policy. According to this regulation the rental price for tenants cannot exceed $p^{control}$. Assume that $p^{control}$ corresponds to the market-clearing price in absence of rental tax. Illustrate the initial equilibrium and new equilibrium at the same graph in terms of price paid by tenants. Compare initial (with tax but before rent control) and new (with tax and rent control) values of CS, PS, local government surplus (GS) and TS by filling in the following table. Comment on the welfare impact of the policy.

	Initial	New	Change
CS			
PS			
GS			
TS			

(b) What was the implicit assumption about who would get the apartment under price control? How the value of loss would change if this assumption is altered (illustrate by graph the maximum possible additional loss).

Theory of firm and Perfect competition: Answers and Hints

1. (a) $AVC = wL(Q,\bar{K})/Q = \begin{cases} 0.5\left(Q-\sqrt{\bar{K}}\right)^2, & Q > \sqrt{\bar{K}} \\ 0, & Q \le \sqrt{\bar{K}} \end{cases}$.

$$ATC^{SR} = AVC + AFC = \begin{cases} 0.5\left(Q-\sqrt{\bar{K}}\right)^2/Q + 2\bar{K}/Q, & Q > \sqrt{\bar{K}} \\ 2\bar{K}/Q, & Q \le \sqrt{\bar{K}} \end{cases},$$

(b) $TC(Q) = 0.4Q^2$.

$AC = 0.4Q$.

Diseconomy of scale, explanation via returns to scale

2.

(a) $q^s(p) = \begin{cases} p/18, & p \ge 24 \\ 0, & p < 24 \end{cases}$

(b) $N = 40$, $p = \dfrac{150}{40/18+4} = \dfrac{75\times 9}{10+2\times 9} = \dfrac{675}{28} = 24\dfrac{3}{28}$

3. (a) Reduction in α (under given price) results in an increase in quantity supplied by any firm

Hint: Use FOC and prove from the contrary

Use graph with MC

(b) Short-run equilibrium price falls as α goes down

Prove from the contrary

4. Lump sum subsidy is more expensive for the government

Hint: Use revealed choice

5. (a) Assuming that the graph is produced in terms of producer price this policy results in proportional upward shift of supply curve.

TS is reduced as a result of rent control policy

(b) Hint: think who gets the apartments that are in excess demand

Under alternative distribution of the apartments the society loss would be even higher

Theory of firm and Perfect competition: Solutions

1. (a) Derivation of AVC

Factors employment in SR can be found from the following cost minimisation problem:

$$\min_{L\geq 0} \quad wL + r\overline{K}$$
$$s.t. \quad F(L,\overline{K}) \geq Q$$

As $Q = 4\sqrt{L} + \sqrt{K}$, we get $L = \left(Q - \sqrt{\overline{K}}\right)^2 / 16$ if $Q > \sqrt{\overline{K}}$ and $L = 0$ otherwise.

$$AVC = wL(Q,\overline{K})/Q = \begin{cases} 0.5\left(Q - \sqrt{\overline{K}}\right)^2, & Q > \sqrt{\overline{K}} \\ 0, & Q \leq \sqrt{\overline{K}} \end{cases}.$$

Derivation of ATC

$$ATC^{SR} = AVC + AFC = \begin{cases} 0.5\left(Q - \sqrt{\overline{K}}\right)^2 / Q + 2\overline{K}/Q, & Q > \sqrt{\overline{K}} \\ 2\overline{K}/Q, & Q \leq \sqrt{\overline{K}} \end{cases},$$

AVC is increasing in Q for large Q:

$AVC'_Q = 0.5\dfrac{2Q(Q - \sqrt{\overline{K}}) - \left(Q - \sqrt{\overline{K}}\right)^2}{Q^2} = 0.5\dfrac{\left(Q - \sqrt{\overline{K}}\right)\left(Q + \sqrt{\overline{K}}\right)^2}{Q^2} = 0.5\dfrac{Q^2 - \overline{K}}{Q^2} > 0$ if $Q > \sqrt{\overline{K}}$ and constant for small Q (when only fixed factor is used).

AVC is constant when output is produced by capital only and AVC increases when L is used due to diminishing marginal product of labor and $F(L=0) = 4\sqrt{K} \geq 0$.

AFC is always diminishing due to fixed level of capital. ATC in SR consists of two parts: constant and then increasing AVC and diminishing AFC. As a result it diminishes initially due to AFC and then starts to increase (due to AVC) as with large output levels the contribution of AFC becomes negligible.

(b) Derivation of AC

Cost minimization problem. $\begin{aligned} &\min_{L,K\geq 0} \quad wL + rK \\ &s.t. \quad F(L,K) = Q \end{aligned}$.

The necessary condition for interior solution is an equality of the slope of isoquant (MRTS) and the slope of isocost (ratio of factors' prices). $MRTS_{LK} = 4\sqrt{\dfrac{K}{L}} = \dfrac{w}{r} = 4$ - and we get $\dfrac{K}{L} = 1$.

Note that corner solution is impossible as with K=0 MRTS=0<w/r=4 and firm will be able to substitute L by K. Similarly with L=0 MRTS approaches infinity, while w/r=4<infinity and firm will substitute K by L.

$Q = 4\sqrt{L} + \sqrt{K}$. As $L = K$ plugging into production function, we get $Q = 4\sqrt{L} + \sqrt{L} = 5\sqrt{L}$ or $L(Q) = 0.04Q^2$. Thus we get the following cost function: $TC(Q) = 8L(Q) + 2K(Q) = 10L(Q) = 0.4Q^2$.

Then $AC = TC(Q)/Q = 0.4Q$.

Diseconomy of scale, explanation via returns to scale

Thus AC in LR is increasing, which implies that there is a diseconomy of scale. This is due to decreasing returns to scale: $Q(\lambda L, \lambda K) = 4\sqrt{\lambda L} + \sqrt{\lambda K} = \sqrt{\lambda}(4\sqrt{L} + \sqrt{K}) < \lambda\left(4\sqrt{L} + \sqrt{K}\right) = \lambda Q(L, K)$ for any $\lambda > 1$

(c) Graph should be provided

2. (a) Derivation of individual supply curve

$p = MC(q) = 18q$ if $q > 0$

$p \geq AC(q) = 9q + 16/q$, $18q \geq 9q + 16/q$, $q \geq 4/3$, $p \geq 18q \geq 24$

$$q^s(p) = \begin{cases} p/18, & p \geq 24 \\ 0, & p < 24 \end{cases}$$

(b) Derivation of LR equilibrium

$Q^s(p) = Nq = Np/18$

$Q^D(p) = 150 - 4p = Np/18$, $p(N/18 + 4) = 150$, $p = \dfrac{150}{N/18 + 4} \geq 24$

$150 \geq 4N/3 + 96$, $54 \times 3/4 \geq N$, $54 \times 3/4 \geq N$,

$N = 40$, $p = \dfrac{150}{40/18 + 4} = \dfrac{75 \times 9}{10 + 2 \times 9} = \dfrac{675}{28} = 24\dfrac{3}{28}$

3. (a) Profit maximization problem $(1-\alpha)pq - c(q) - l(\alpha q) \to \max\limits_{q \geq 0}$

Function is strictly concave (second derivative is negative due to the assumptions), thus FOC is both necessary and sufficient.

$(1-\alpha)p - c'(q) - \alpha l'(\alpha q) \leq 0$ and $(1-\alpha)p - c'(q) - \alpha l'(\alpha q) = 0$ if $q > 0$.

Note that $q > 0$ for any $p > 0$: Otherwise $(1-\alpha)p - c'(0) - \alpha l'(0) = (1-\alpha)p > 0$ which violates FOC.

$(1-\alpha)p = c'(q) + \alpha l'(\alpha q)$

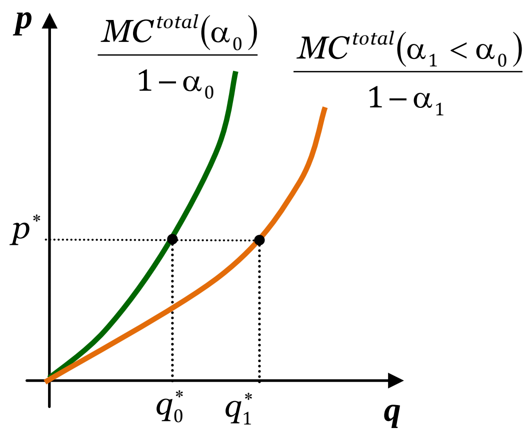
Algebraic analysis of response to reduction in α

Let us prove that reduction in α for given price results in an increase in quantity supplied by any firm. The LHS that represent marginal revenues goes up and so should do the RHS. If $\Delta q \le 0$ then marginal production cost would fall or stay the same (as MC is increasing) and the second term (marginal utilization cost) definitely falls as: $\Delta(\alpha q) < 0$ and MUC is increasing then $\Delta l'(\alpha q) < 0$ and with smaller α we have $\Delta(\alpha l'(\alpha q)) < 0$. Thus RHS goes down while LHS up and we get a contradiction. It means that $\Delta q > 0$.

Graphical analysis with comments

Supply is given by nondiminishing part of total MC per efficient unit (i.e. sold units) that lie above AC. Here as cost function is convex and goes from the origin than at any point MC>AC and MC is increasing. Thus total MC per efficient unit represents supply curve. With reduced α under the same output we utilize less and due to increasing marginal utilization cost assumption we get lower value of MUC. Thus total MC falls at every q. Moreover, as we sell more, the level TMC per efficient unit goes down which strengthens the effect of reduction of MUC.

As TMC per efficient unit shift downward, it means that firm is willing to produce the same output under lower price (i.e. supply curve shifts down or to the right).



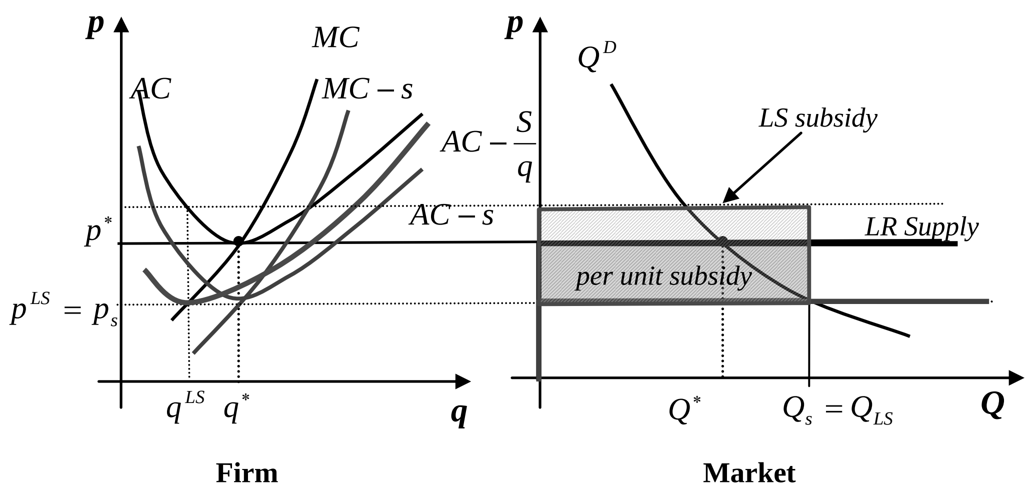
(b) In equilibrium $Q^d(p(\alpha)) = N(1-\alpha)q(p(\alpha),\alpha)$. Let us prove that $p(\alpha_0) > p(\alpha_1)$ if $\alpha_0 > \alpha_1$.

Suppose that $p(\alpha_0) \le p(\alpha_1)$, then $Q^d(p(\alpha_0)) \ge Q^d(p(\alpha_1))$ as demand is diminishing.

From part (a) we know that $(1-\alpha_1)q(p(\alpha_0),\alpha_1) > (1-\alpha_0)q(p(\alpha_0),\alpha_0)$. As supply is increasing in price $q(p(\alpha_1),\alpha_1) \ge q(p(\alpha_0),\alpha_1) > q(p(\alpha_0),\alpha_0)$ if $p(\alpha_0) \le p(\alpha_1)$. Thus market supply goes up $N(1-\alpha_1)q(p(\alpha_1),\alpha_1) > N(1-\alpha_0)q(p(\alpha_0),\alpha_0)$ while demand goes down, so market is not in equilibrium. It proves that $p(\alpha_0) > p(\alpha_1)$.

Intuition is straightforward: as each firm is willing to produce more, market supply goes up and results in excess supply that drives the price downward.

4. Graphical analysis



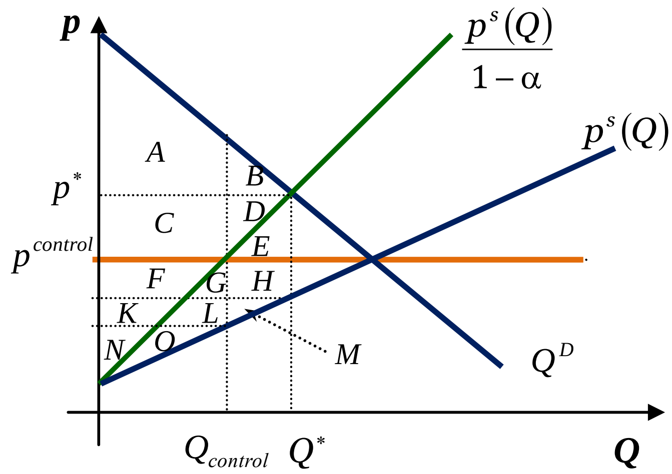
Analytical approach

$p_{LR} = AC(q^*) - s = AC(q_{LS}) - \frac{S}{q_{LS}}$, $AC'(q^*) = 0 = AC'(q_{LS}) + \frac{S}{(q_{LS})^2}$, which implies $AC'(q_{LS}) = -\frac{S}{(q_{LS})^2} < 0$, i.e. AC is not minimized at $q = q_{LS}$ but AC is minimized at $q = q^*$. Thus $AC(q^*) < AC(q_{LS})$ and $\frac{S}{q_{LS}} - s = AC(q_{LS}) - AC(q^*) > 0$. Cost of subsidy comparison: $S \times n_{LS} = S\frac{Q_{LS}}{q_{LS}} = S\frac{Q_s}{q_{LS}} > sQ_S$.

5. (a) Graph

Comments on after-tax supply curve.

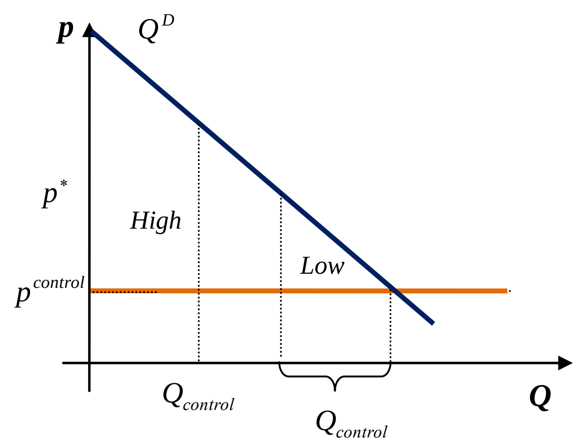
Initial inverse supply is drown in terms of producer price that reflects the marginal cost of house owners. Houseowners are willing to supply the same amount if they get the same net of tax price, i.e. under $p_t(1-\alpha) = p_s$. It means that tenants price should be $p_t = \frac{p_s}{1-\alpha}$. Thus inverse supply curve shifts upward proportionally.



	Initial	New	Change
CS	A+B	A+C	C-B
PS	K+L+M+N+O	N+O	-(K+L+M)
GS	C+D+E+F+G+H	F+G+K+L	K+L-(C+D+E+H)
TS			-(B+D+E+H+M)

Welfare impact: TS is reduced that results in additional DWL of (B+D+E+H+M). The reason is that imposed price ceiling results in rent decrease that is unfavourable for house owners and they are willing to supply less. Thus output that was less then efficient initially due to tax now is reduced even more due to price restriction.

(b) Implicit assumption: efficient rationing, i.e. those who most desire apartment (have higher valuation measured by CS) are the ones who get it). If this is not the case the loss of TS would be even higher and it would be the highest if we serve only consumers with lowest valuation. In this case TS is reduced by the difference in their CS, i.e. by High-Low.



Monopoly/Monopsony and Price Discrimination: Exercises

1. Question is based on Figure 11.5 ‘Third degree price discrimination’ (p.407 P&R). This figure represents graphical derivation of equilibrium under third-degree price discrimination when monopolist has increasing marginal cost. Read the following comment to Figure.11.5: “the total quantity produced, $Q_T = Q_1 + Q_2$, is found by summing the marginal revenue curves MR_1 and MR_2 horizontally, which yields the dashed curve MR_T, and finding its intersection with the marginal cost curve”.

As we know the first order conditions suggest that marginal revenue for each group separately, not the total marginal revenue should be equal to marginal cost. Is there a contradiction? What equation do you solve by constructing this MR_T? Explain your answers.

3. In town N only one firm offers job to engineers. The inverse labour supply curve by female engineers is given by $L^F\left(w^F\right) = \max\left(0, w^F - 4\right)$ and inverse labour supply curve of male engineers is $L^M\left(w^M\right) = w^M$, where w^k stays for wage rate of group k ($k = M, F$). Assume that labour is the only variable factor in the short run and the short run production function of the firm is $F(L) = \max\left(0,\ 13L - 0.5L^2\right)$. The final product is sold at perfectly competitive market and the price is \$4 per unit.

(a) Find the profit maximizing wage rates assuming that the firm can set different wage rates to male and female engineers. Illustrate by graph.

(b) Now assume that price discrimination is not allowed any more. Find equilibrium. Illustrate graphically on the same graph with different colour.

(c) Compare social welfare in (a) and (b) by calculating the difference in the total surplus? Provide intuitive explanation (explain carefully).

(d) Suggest taxes\subsidies which can be used to induce the discriminating monopsony from part (a) to choose efficient employment. Demonstrate that proposed solutions would result in efficient outcome.

3. A price discriminating monopolist allocates its output between domestic market and foreign market. The monopolist’s marginal cost schedule is rising and marginal revenue curves are declining on each market and current domestic price is below the foreign price. Suppose that the government decided to tax export by introducing a per-unit export duty equal to α. Assume that sales at both markets are positive both before and after the introduction of export duty.
(a) Analyze graphically the impact of this policy on domestic sales assuming that all curves are linear (you are expected to provide some comments to the graphs).
(b) Derive the analytical solution for general (non-linear) case for the impact of the policy on domestic sales. [Differentiability of MR and MC is not assumed].

Monopoly/Monopsony and Price Discrimination: Answers/Hints

1. No contradiction

2.

(a) $L_F = 4, L_M = 6, w_F = 8, w_M = 6$

(b) $L = 10$ and $w = 7$, $L_F = 3, L_M = 7$

(c) $TS^{(b)} - TS^{(a)} = 1$

(d) $s_F^* = 32/9$, $s_M^* = 68/9$

3.

(a) Domestic sales will increase

(b) Prove from the contrary

Monopoly/Monopsony and Price Discrimination: Solutions

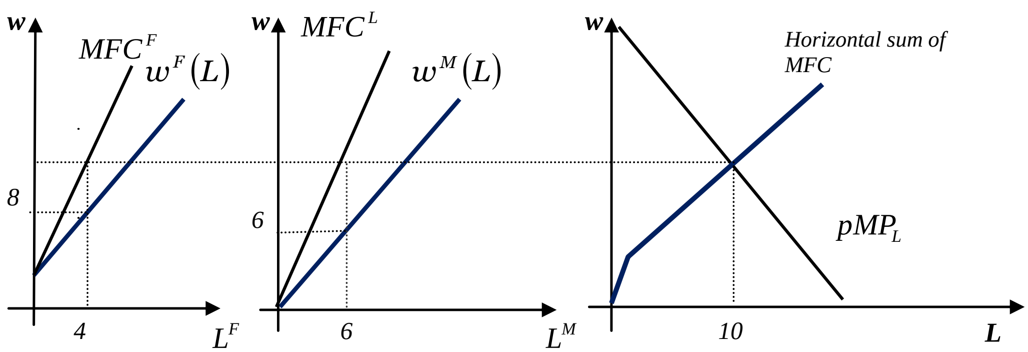
1. No contradiction as we do not sum up MR, we sum up sales for given value of MR. We do this to equalize MR at the two markets, i.e. we solve graphically the following equation $MR_1(q_1) = MR_2(q_2)$. We take some particular value of MR and look at volume of sales at each market that generates this value of MR, then we sum up this volumes and represent this total volume at the graph with MC (as MC also depends on total output). Thus each point along combined MR curve represent the total volume of sales that can be allocated across the two markets in such a way that MR are the same and equal to the given value.

2. (a) We can equivalently rewrite the problem in terms of employment rather than wage rate setting.

$$\max_{L_M, L_F \geq 0} \left(4\left(13L - 0.5(L_M + L_F)^2\right) - L_M \times L_M - L_F \times (L_F + 4)\right).$$

FOCs: $52 - 4(L_M + L_F) = 2L_M$, $52 - 4(L_M + L_F) = 4 + 2L_F$

Solution. $L_F = 4, L_M = 6, w_F = 8, w_M = 6$.



(b) Aggregate supply: $L^S(w) = \begin{cases} w, & w < 4 \\ 2w - 4, & w \geq 4 \end{cases}$, Inverse market supply

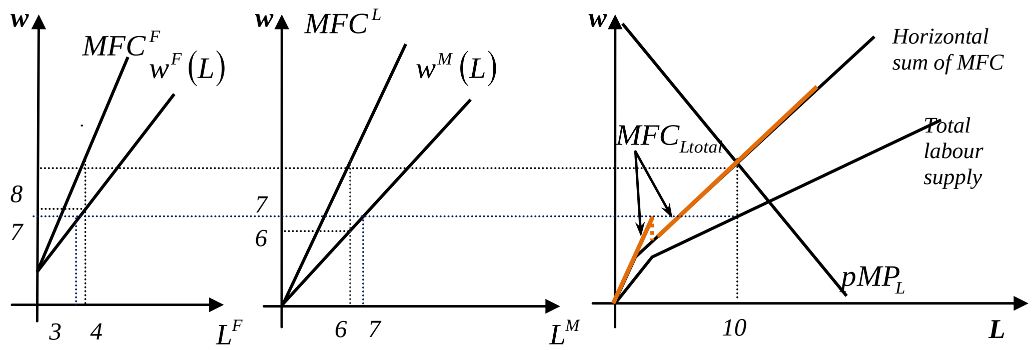
$$w^S(L) = \begin{cases} L, & L < 4 \\ (L+4)/2, & L \geq 4 \end{cases}.$$

Profit maximization problem $\max_{L \geq 0}\left(4\left(13L - 0.5L^2\right) - w^s(L) \times L\right)$. $MFC(L) = \begin{cases} 2L, & L < 4 \\ (L+2), & L \geq 4 \end{cases}$

FOC: $\begin{cases} 52 - 4L = 2L, & L < 4 \\ 52 - 4L = L + 2, & L \geq 4 \end{cases}$

Solution. $L = 10$ and $w = 7$, $L_F = 3, L_M = 7$

Graph



(c) Comparison of social welfare:

$$TS^{(a)} = 4\int_0^{10}(13-L)dL - \int_0^6 L_M dL_M - \int_0^4 (L_F+4)dL_F \ ,$$

$$TS^{(b)} = 4\int_0^{10}(13-L)dL - \int_0^7 L_M dL_M - \int_0^3 (L_F+4)dL_F \ , \ TS^{(b)} - TS^{(a)} = 1$$

Conclusion: if discrimination is prohibited, total employment stays the same but with uniform wage employment is efficiently allocated between the groups that increases TS.

Under market segmentation price discrimination any given level of employment is inefficiently allocated b/w the workers as $w^m \neq w^f$. If we keep the same total employment but reallocate employment between the groups then output (and TB) would stay the same. If we increase male employment by small unit and give compensation equal to $(w^m + w^f)/2 = 7$ then they would be better off as they were willing to work at lower wage. On the other hand we would reduce female employment by the same amount. The opportunity cost of this unit was $w^f = 8$. But we would take only $(w^m + w^f)/2 = 7$, which is less so this person is also better off. Thus we get a Pareto improvement that demonstrates inefficiency of initial allocation of total employment between the two markets. Thus $TS^{(a)} = TS^{segm} < TS^M = TS^{(b)}$.

(d) As employment is below efficient level we should subsidize employment that would reduce MFC. As after-subsidy MFC is lower, the monopsonist has an incentive to increase employment. To attain desired increase in employment for each group we need two different subsidies.

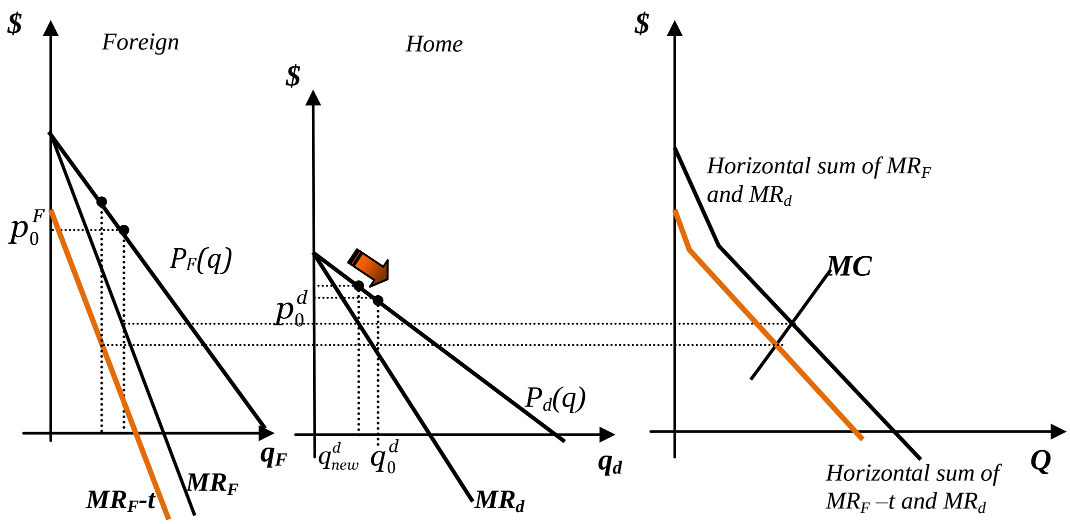
Efficient employment is given by

$$w_i(L_i^*) = MRP(L_i^* + L_j^*) \ . \ (*)$$

$L_M^* = 4(13 - L_M^* - L_F^*)$, $L_F^* + 4 = 4(13 - L_M^* - L_F^*)$, $L_F^* + 4 = L_M^*$, $L_M^* = 4(17 - 2L_M^*)$, $L_M^* = 68/9$, $L_F^* = 32/9$

Monopsonist would operate at point, where $MFC_i(L_i) - s_i = MRP(L_i + L_j)$. Thus we should set $s_i^* = MFC_i(L_i^*) - w_i(L_i^*)$. $s_F^* = 2L_F^* + 4 - L_F^* - 4 = L_F^* = 32/9$, $s_M^* = 2L_M^* - L_M^* = L_M^* = 68/9$

Note: the resulting allocation would be efficient as under given subsidies efficiency conditions (*) are satisfied.



By summing MR curves horizontally we equalize the values of MR across the two markets (monopolist will sell at both only if MR is the same). Then we find the optimal production by intersecting equalized MR with MC.

Per unit tax reduces MR from export. As a result it affects the locus of points where MRs are equalized and this new (grey) line intersects increasing MC at lower value of MC. Domestic MR curve is unaffected and we need lower value of MR which happens at increased domestic sales.

(b) Domestic sales will go up.

Proof. In equilibrium with export duty we have $\begin{cases} MR_F(q_F) - t = MC(q_F + q_d) \\ MR_d(q_d) = MC(q_F + q_d) \end{cases}$.

Note that initially $t = 0$ and then it increases by $\Delta t > 0$.

Let us prove that $\Delta q_d > 0$. Suppose that this is not the case and $\Delta q_d \le 0$, then $\Delta MR_d \ge 0$ as MR is diminishing. In equilibrium $\Delta MC = \Delta MR_d \ge 0$. (**)

As MC is increasing, then $\Delta Q = \Delta q_d + \Delta q_F \ge 0$. Thus $\Delta q_F = \Delta Q - \Delta q_d \ge 0$. MR_F is diminishing then $\Delta MR_F - t < 0$. This implies $\Delta MC = \Delta MR_F - t < 0$ which contradicts to (**). Contradiction proves that $\Delta q_F > 0$.

Strategic Interactions: Exercises

1. [P&R] Ch.12, #11 ‘Two firms compete by choosing prices. Their demand functions are $Q_i = 20 - P_i + P_j$. MC=0.

(a) Suppose the two firms set their prices at the same time. Find NE, What price will each firm charge, how much will it sell, and what will its profit be?
(b) Suppose firm 1 sets its price first and then firm 2 sets its price. What price will each firm charge, how much will it sell, and what will its profit be?
(c) Suppose you are one of these firms and that there are three ways you could play the game: (i) both firms set price at the same time; (ii) you set price first, (iii) your competitor sets price first. If you could choose among these options, which would you prefer? Why?’

2. Suppose firm A has two different technologies of producing output. Technology 1 requires four units of labor and one unit of raw material to produce each unit of output. Technology 2 requires two units of labor and one unit of raw material per unit of output but before technology 2 can be used firm has to incur setup costs equal to 70 and there are no setup costs for technology 1. Suppose that wage rate is 3 and the price of raw materials is 4. The inverse demand function is $P(Q) = 34 - Q$.

(a) If firm A is the only firm operating in this industry, which technology should it use. Find corresponding output, price and profit.

(b) Suppose that firm A is afraid that company B is going to enter the market. Firm B’s cost function is given by $TC^B(q) = 10q$ and it has to purchase a license that cost 90 had it decided to enter the industry. If B enters the market then the firms will compete by choosing quantities simultaneously. Which technology would you advise firm A to choose (if this choice is made before firm B decides, whether to enter or not)? Represent the game tree, find equilibrium and explain the result.

(c) Find efficient allocation. Calculate and compare the value of DWL in (a) and (b). What can you conclude about the role of potential competition? Explain the result.

(d) Explain intuitively the relevance of sunk costs to the ability of the incumbent (firm A) in part (b) to make a commitment to use different technology. Demonstrate, how the game tree and the resulting outcome would differ when the set-up cost associated with the second technology are not sunk.

3*. The inverse demand function for good X is given by $p(Q) = A - Q$, where Q is industry output. Suppose that there are two firms in the industry but firm 1 has a patent on the production technology that allows to produce good X with constant per unit cost c, where $c < A$. Firm 1 charges firm 2 a license fee of t per unit of output produced by firm 2.

(a) Suppose that for given value of t the two firms compete the two firms compete by choosing outputs simultaneously. Find Nash equilibrium, assuming that t is small enough so that both produce positive output levels.

(b) Now consider a dynamic game. Firm 1 starts the game by choosing the license fee t, then firm 2 observes the value of the license fee and both firms simultaneously chose their output levels. Find perfect Nash equilibrium.

(c) Compare profit of the first firm in case (b) with the profit in the case where firm one is the pure monopolist. Could this result be generalized for any downward sloping demand curve?

Strategic Interactions: Answers/Hints

1. Solution is available in [P&R] Ch.12, #11

2. (a) Technology 1, profit=81;

(b) Technology 2. Hint: produce a game tree and solve via backward induction

(c) Efficient allocation: use technology 2 and produce $Q = 24$,

$DWL_a = 96{,}5$

$DWL_b = 72 < DWL_a$

TS goes up, P goes down, potential competition might limit the market power

(d) Hint: threat of MC reduction is not credible if the costs are not sunk

3. (a) $q_1 = \dfrac{A-c+t}{3}$ and $q_2 = \dfrac{A-c-2t}{3}$;

(b) $t = \dfrac{A-c}{2}$, $q_1 = \dfrac{A-c+t}{3} = \dfrac{A-c}{2}$ and $q_2 = 0$

(c) $\pi_1^{(b)} = \pi^{monopoly}$. This result could be generalized.

Strategic Interactions: Solutions

1. Solution is available in [P&R] Ch.12, #11

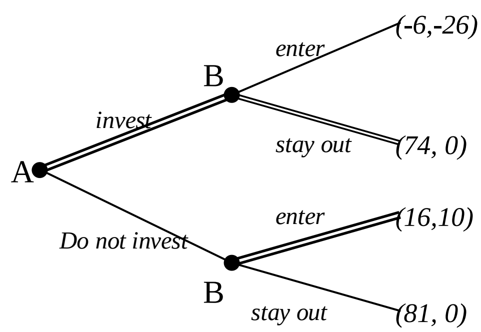
2. (a) Both technologies are CRS, which implies that cost function is linear in output and MC=AC=constant. $MC_1 = r + 4w = 4 + 12 = 16$ and $MC_2 = r + 2w = 4 + 6 = 10$.

If A believes that it will continue to be the only producer, then it should chose the technology that brings the maximum possible profit. Let us find the maximum profit college can get if technology 1 is used. The optimal output is determined from equality of MR and MC: $MR = 34 - 2Q = MC_1 = 16$, $Q = 9$, $p = 25$ and $\pi = (25 - 16)9 = 81$.

In case of technology 2 $MR = 34 - 2Q = MC_2 = 10$, $Q = 12$, $p = 22$, $\pi = (22 - 10)12 - 70 = 144 - 70 = 74$. Thus firm A should use technology 1.

(b) Technology 2 results in symmetric competition. $MR_i = 34 - q_j - 2q_i = MC_i = 10$. As equilibrium is symmetric, $q_i = q_j = 8$, $p = 34 - 16 = 18$ and $\pi_A = (18 - 10)8 - 70 = -6$, while $\pi_B = (18 - 10)8 - 90 = -26$.

Technology 1 results in asymmetric competition. $MR_A = 34 - q_B - 2q_A = MC_A = 16$ and $MR_B = 34 - q_A - 2q_B = MC_B = 10$. Summing up, we get $68 - 3Q = 26$ or $Q = 42/3 = 14$. Finally, $q_A = 34 - 16 - Q = 4$, $q_B = 10$, $p = 34 - 14 = 20$, $\pi_A = (20 - 16)4 = 16$, $\pi_B = (20 - 10)10 - 90 = 10$.



Thus it is profitable to use technology 2. Game tree and solution via backward induction see above.

Explanation of the result. Under the threat of losing its monopoly power, firm A switches to technology with lower marginal cost as it makes the treat of aggressive behaviour under Cournot competition (i.e. large output) credible and results in successful entry deterrence.

(c) Efficient allocation.

As we deal with constant MC then in absence of set-up cost it would be cheaper to produce with second technology. But in case of set-up choice this is not necessarily the case and we should compare the two options. The third option with combination of the technologies is definitely dominated by the second technology which will result in lower cost.

Technology 1. $34 - Q = 16$, $Q = 18$, $TS(1) = (34-16) \times 18/2 = 162$

Technology 2. $34 - Q = 10$, $Q = 24$, $TS(2) = (34-10) \times 24/2 - 60 = 288 - 70 = 218$

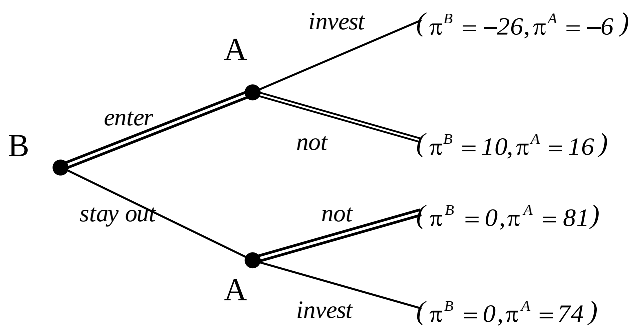
Thus it is efficient to use the second technology and $TS_{\max} = 218$

$$DWL_a = TS_{\max} - TS^a = 218 - (34 - 16 + 34 - 9 - 16) \times 9/2 = 218 - 27 \times 9/2 = 218 - 121.5 = 96.5$$
$$DWL_b = TS_{\max} - TS^b = 218 - [(34 - 10 + 34 - 12 - 10) \times 12/2 - 70] = 218 - [216 - 70] = 72$$
$$DWL_b = 72 < DWL_a$$

Thus the threat of potential competition may result in output expansion that is used as entry deterrence strategy. As a result output becomes closer to the efficient level which reduces DWL.

(d) Intuitive explanation

In the absence of sunk costs firm A could not credibly precommit to technology 2. Had firm B believe in technology 2 and decide to stay out, firm A prefers to produce with technology 1 as this allows to get profit of 81 instead of 74. Note that firm A has to pay for MC reduction anyway, but if this payment is reversible the game tree is different. Threat of MC reduction is no more credible and firm B finds optimal to enter the industry.



3. Cournot competition with different MC. Note: here license payment constitutes additional source of revenue for firm 1.

Profit maximization problem of firm 1: $\max\limits_{t \geq 0}\left((A - q_1 - q_2 - c)q_1 + tq_2\right)$

FOC: $A - 2q_1 - q_2 - c = 0$

Profit maximization problem of firm 2: $\max\limits_{t \geq 0}(A - q_1 - q_2 - c - t)q_2$

FOC: $A - q_1 - 2q_2 - c - t = 0$

Solution of the system: $q_1 = \dfrac{A - c + t}{3}$ and $q_2 = \dfrac{A - c - 2t}{3}$.

(b) Last part of the game is solved in part (a). Problem of firm 1 at the first stage of the game:

$$\max_{t\geq 0}\left(\left(A-\frac{2A-2c-t}{3}-c\right)\frac{A-c+t}{3}+t\frac{A-c-2t}{3}\right)=\max_{t\geq 0}\left(\left(\frac{A-c+t}{3}\right)^2+t\frac{A-c-2t}{3}\right)$$

FOC. $2\frac{A-c+t}{3}+A-c-4t=0$

Solution. $t=\frac{A-c}{2}$, $q_1=\frac{A-c+t}{3}=\frac{A-c}{2}$ and $q_2=0$. Thus the best option is to charge the fee that makes business of the second firm unprofitable so that firm 1 becomes a pure monopoly.

(c) $\pi_1^{(b)}=\pi^{monopoly}$. This result could be generalized.

By charging very high fee firm 1 can always make business of the second firm unprofitable and become the only producer in the industry, which implies that for any cost and demand functions $\pi_1^{(b)}\geq\pi^{monopoly}$.

On the other hand for any t

$$\pi_1(q_1(t),q_2(t))+\pi_2(q_1(t),q_2(t))=(q_1(t)+q_2(t))(P(q_1(t)+q_2(t))-c)\leq$$
$$\leq\max Q(P(Q)-c)=\pi^{monopoly},$$

which implies that $\pi_1(q_1(t),q_2(t))\leq\pi^{monopoly}$ for any t. If $t=t^b$ then $\pi_1(q_1(t^b),q_2(t^b))=\pi_1^{(b)}\leq\pi^{monopoly}$.

Thus we have proved that $\pi_1^{(b)}=\pi^{monopoly}$.

Externalities and Public Goods: Exercises

1. Two villagers (A and B) have to decide the number of cows **(** x_A and x_B) they will purchase. The cost of each cow is 6. The value to a villager of each cow depends on the total number of cows that will be purchased (which will then be grazed on the village green). The value per cow is $30 - X$, where X denotes the total number of cows purchased.

(a) Assuming that the two villagers make their respective purchases simultaneously, derive the number of cows purchased by each villager in equilibrium.

(b) Show that equilibrium found in (a) is inefficient. Explain the reason for market failure.

(c) Propose the way that would restore Pareto efficiency and show how this solution works.

2. Consider an urban pollution problem caused by car traffic, that emits 21 tones of pollution annually, and factories, that emit another 15 tones. The cost of reducing car pollution is $C_c(x_c) = 0.5(x_c)^2$ and the cost of reducing factory pollution is $C_f(x_f) = (x_f)^2$, where x_c and x_f are the reduction in tones in annual car and factory pollution, respectively. The benefits of reducing pollution are given by $B(x) = 0.25x^2 + 2x$, where x represents the total reduction in pollution.

(a) What is the socially efficient reduction in pollution?

(b) Discuss how tradable pollution permits could be used to achieve a socially efficient reduction found in (a) and specify the volume of permits (in tones of pollution) that should be issued. Suppose that each firm gets the same volume of permits and permits are freely traded. Find equilibrium at permits market and illustrate it graphically.

(c) Consider an alternative regulation that results in the same overall level of reduction as tradable permits but requires a uniform percentage reduction in emissions by all cars and factories. Compare the resulting abatement costs with the one from (b) and explain the result.

3. (a) With reference to a public good explain how the free-rider problem can be modeled as a prisoners' dilemma game.
(b) Consider two individuals (A and B) who are trying to decide how much to contribute to a public good. Their utility functions are $u^A(x_A, G) = x_A + 2\sqrt{Y}$ and $u^B(x_B, Y) = x_B + 4\sqrt{Y}$, where x_A, x_B are quantities of a private good (money) that they consume and Y is the amount of public good. Let their income levels be 100 each. Public good is produced by a perfectly competitive profit-maximising firm with cost function $C(Y) = Y^2$.

> **(i)** Suppose that individuals make their decisions about contributions to the public good independently and simultaneously. Find equilibrium.
>
> **(ii)** Comment whether it is possible to improve upon this allocation in terms of efficiency (do not calculate the efficient allocation).

4. Two companies, A and B, operate in the same gas field. Each company's costs depend on its own production level as well as production level of the other company: $C_A = 0.25(q_A + q_B)^2 + 0.5(q_A)^2$ and $C_B = 0.25(q_B + q_A)^2 + 0.5(q_B)^2$. Demand for gas is given by $Q(p) = 20 - p$.

(a) Suppose that the two firms are oligopolists that compete by choosing production levels simultaneously and independently. Find equilibrium.

(b) How would your answer to part (a) change if the two firms acted as price takers at gas market? Find equilibrium.

(c) Calculate and compare the values of deadweight loss in (a) and (b). Explain the result.

(d) Is it possible to eliminate the loss from inefficiency (if there is any) found in (c) for case (a) and /or case (b) via taxes/subsidies? Find the required taxes/subsidies or prove that it is impossible.

Externalities and Public Goods: Answers/Hints

1. (a) $x_a = x_b = 8$

(b) $X^{eff} = 12 < X^{eq} = 16$

(c) $t = 6$

2. (a) $x_f = 4, \quad x_c = 8$

(b) $p_q = 8$

(c) $TAC_{prop} = 49.5 > TAC_{permits} = 48$

3. (a) see the lecture

(b) (i) Equilibrium $p_Y = 2$, $Y = 1$

(ii) Yes, it is possible

4. (a) $q_i = q_j = 4$

(b) $Q = 10$

(c) Efficient outcome $Q = 40/5 = 8$, $q_i = q_j = 4$

$DWL^a = 0$

$DWL^b = 5$

(d) (a) is efficient

(b) it is possible via Pigouvian tax $t_i = 4$

Externalities and Public Goods: Solutions

1. (a) Net benefit maximization problem of villager i:

$\max(30 - x_i - x_j - 6)x_i$,

FOC: $24 - 2x_i - x_j = 0$,

Summing up: $48 - 3(x_i + x_j) = 0$. Thus $x_i + x_j = 48/3 = 16$ and $x_i = 24 - (x_i + x_j) = 8$

Nash equilibrium: $x_a = x_b = 8$

(b) Efficient outcome: $\max(24 - X)X$, $X^{eff} = 12 < X^{eq} = 16$

Reason: negative external effect due congestion

Each agent doesn't take into account decrease in marginal value of his neighbor cows due to increase in the number of cows: $MPB_i = 30 - 2x_i - x_j = 30 - 2X + x_j > MSB = 30 - 2X$

(c) Pigouvian tax: $t = MD = x = 6$ per cow

$\max(30 - x_a - x_b - 6 - t)x_a$

$24 - 2x_a - x_b - t = 0$, $x_a = x_b = 8 - t/3 = 6$

2. Consider an urban pollution problem caused by car traffic, that emits 21 tones of pollution annually, and factories, that emit another 15 tones. The cost of reducing car pollution is $C_c(x_c) = 0.5(x_c)^2$ and the cost of reducing factory pollution is $C_f(x_f) = (x_f)^2$, where x_c and x_f are the reduction in tones in annual car and factory pollution, respectively. The total benefits of reducing pollution are given by $B(x) = 0.25x^2 + 2x$, where x represents the total reduction in pollution.

(a) What is the socially efficient reduction in pollution?

Derivation of efficiency condition:

$MB = 0.5(x_c + x_f) + 2 = MC_f = 2x_f$

$MB = 0.5(x_c + x_f) + 2 = MC_c = x_c$

Solution of the system: $x_f = 4, \quad x_c = 8$.

(b) Verbal explanation should be provided

Assume that efficient number of permits is issued and allocated for free: $\overline{q}_c + \overline{q}_f = 15 + 21 - 4 - 8 = 24$

Firm C demand for permits we get from: $\max\limits_{q_c}\left(p_q(\overline{q}_c - q_c) - 0.5(21 - q_c)^2\right)$

FOC: $-p_q + (21 - q_c) = 0 \Rightarrow q_c = 21 - p_q$

Firm F demand for permits: $\max_{q_f}\left(p_q(\bar{q}_f - q_f) - (15 - q_f)^2\right)$

FOC: $-p_q + 2(15 - q_f) = 0 \Rightarrow q_f = 15 - 0.5p_q$

In equilibrium total quantity demanded equals to the quantity supplied:

$q_c + q_f = 21 - p_q + 15 - 0.5p_q = 24$

$p_q = 12/1.5 = 8$, $q_c = 13$, $q_f = 11$, $x_c = 21 - 13 = 8$, $x_f = 15 - 11 = 4$.

Graph should be provided

(c) Uniform rate of reduction in emission levels $12/36 = 1/3$ results in $x_c = 21/3 = 7$, $x_f = 15/3 = 5$

This outcome is different from (b) and it is inefficient as total costs are not minimized.

$TAC_{prop} = 0.5 \times 49 + 25 = 49.5 > TAC_{permits} = 0.5 \times 64 + 16 = 48$.

3. (a) see the lecture

(b) (i) $\max\left(100 - p_y y_i + 2i\sqrt{y_i + y_j}\right)$,

FOC: $-p_y + i/\sqrt{Y} \le 0$ and $-p_y + i/\sqrt{Y} \le 0$ if $y_i > 0$

Claim: only agent with low valuation of the public good free rides. Proof.

Assume that this is not the case and agent A contributes then $p_y = 1/\sqrt{Y}$ and $p_y \ge 2/\sqrt{Y}$, which is impossible.

Thus agent A free rides and only B contributes so that $p_y = 2/\sqrt{Y}$.

Thus $Y = (2/p_y)^2$

Profit maximization implies $p_Y = MC = 2Y$. Thus in equilibrium $p_Y/2 = (2/p_y)^2$

Equilibrium $p_Y = 2$, $Y = 1$.

(ii) Yes, it is possible because $SMB = 1/\sqrt{Y} + 2/\sqrt{Y} = 3 > MC = 2Y = 2$

Intuitive explanation: More of public good should be produced as willingness to pay exceeds the MC. Reason for inefficiency: agents do not take into account positive external effect

4. (a) Profit maximisation problem. $\max_{q_i \ge 0}\left[(20 - q_i - q_j)q_i - 0.25(q_i + q_j)^2 - 0.5(q_A)^2\right]$

FOC $MR_i = 20 - 2q_i - q_j = 1.5q_i + 0.5q_j$, Summing up: $40-3Q=2Q$, $Q=8$, $q_i=q_j=4$

(b) Profit maximisation problem. $\max_{q_i\geq 0}\left[pq_i-0.25(q_i+q_j)^2-0.5(q_A)^2\right]$

FOC $p=1.5q_i+0.5q_j$,

Market equilibrium. $20-Q=p=0.5(q_i+q_j)+q_i$, $40-2Q=2Q$, $Q=10$,

(c) Efficient outcome can be derived from TS maximization.

FOC:

$$SMB=20-Q=MC=Q+q_i=Q+q_j,\ Q=40/5=8,\ q_i=q_j=4$$

$$TS^{\max}=\frac{20+(20-8)}{2}\times 8-0.5\times 8^2-0.5\times 2\times 4^2=128-32-16=80$$

$DWL^a=0$ as allocation is efficient

$$TS^b=\frac{20+(20-10)}{2}\times 10-0.5\times 10^2-5^2=150-75=75,\ DWL^b=80-75=5$$

Explanation

In (a) loss from underproduction due to market power is balanced by the gain from reduced negative external effects and as a result outcome is efficient.

In (b) huge loss from negative external effect due to overproduction as firms base their decision on private cost that are less than social due to negative external effect

(d) (a) results in efficient allocation

In (b) efficiency could be restored with the help of Pigouvian tax

$$t_i=MD_i\left(q_i^{eff},q_j^{eff}\right)=\frac{\partial c_j\left(q_i^{eff},q_j^{eff}\right)}{\partial q_i}=\left(q_i^{eff}+q_j^{eff}\right)/2=8/2=4$$

Asymmetric information: Exercises

1. Suppose that in the economy there 100 of workers: half of the workers are high productive (type H) with marginal revenue product of labour of 100 and half of the workers are low productive (type L) with marginal revenue product of labour of 50. Assume that reservation utility of high productive worker is 55, while for low productive worker it is 20. Let preferences of type t worker be given by $u_t = y - \alpha_t e$, where y-aggregate commodity with price equal 1, e-education, $\alpha_H = 1$ and $\alpha_L = 5/4$. Education does not affect productivity. Firms and workers are price takers. Firms are risk neutral.

(a) Suppose that each worker knows his/her type, but worker's type is not observable for the firms. Assume that education is not treated as a signal. Find the corresponding equilibrium. Is it efficient?

(b) Now, suppose that type is still unobservable for the firm but education is treated as a signal of productivity.

> **(i)** Find all levels of education that result in successful signaling. Compare resulting equilibria according to Pareto criteria and illustrate the best one.
>
> **(ii)** Compare the best equilibrium with signaling (one you look at in part (i)) with equilibrium from part (a). Which one is preferred from efficiency point of view? Explain, why high productive workers get education if they could be in a better position without signaling?
>
> **(iii)** Find all pooling equilibria and illustrate the Pareto superior one.

2. Consider insurance industry with 100 risk neutral insurance companies and risk averse individuals that differs only in probability of accident, which is 0.2 for individuals from group A and 0.4 for individuals from group B. Assume that there 50 individuals in each group. Initial wealth of each individual is 100 and in case of accident the loss constitutes 75% of initial wealth. Preferences are consistent with expected utility and described by the utility function $u(w) = \sqrt{w}$. Suppose that only full insurance is allowed.

(a) Suppose that information is symmetric. Derive demand for insurance, supply of insurance and find equilibrium. Illustrate graphically.

(b) How would your answers to part (a) change if information is asymmetric: insurance companies can't observe the probability of loss of each individual. Find equilibrium\equilibria.

(c) How would your answers to (a) and (b) change if there are 75 individuals in group A and 25 individuals in group B?

Asymmetric information: Answers/Hints

1. (a) Wage= 75, everybody is willing to work at this industry.

Equilibrium allocation is efficient.

(b) (i) e from 40 to 45

Pareto superior equilibrium corresponds to $e = 40$

(ii) Equilibrium from part (a) is preferred from efficiency point of view

(iii) $0 \le e \le 20$

Pareto superior pooling equilibrium corresponds to $e = 0$

2. (a) L-market: $\gamma_L = 0.2$ and every agent gets full insurance

H-market: $\gamma_H = 0.4$ and every agent gets full insurance

(b) Equilibrium is at $\gamma = 0.4$, low-risk consumers leave the market

(c)

(a) prices are the same, every consumer gets full insurance .

(b) two equilibria: $\gamma = 0.4$ and $\gamma = 0.25$

Asymmetric information: Solutions

1. (a) As education is not treated as a signal but workers get disutility from education, then e=0 for each worker.

In equilibrium wage rate is equal to the expected productivity of those workers that are willing to work at this wage rate. (At higher wage rate firm gets negative profit and labour demand is zero, at lower wage rate each worker brings positive profit on average and as a result demand goes to infinity).

Unconditional $MRP^{av}=150/2=75$, then u(75)=75>55>20 and everybody is willing to work at this industry.

Efficient allocation: $u^H(100,0)=100>55$ and $u^L(50,0)=50>20$. It is optimal for each worker to work at given industry. Thus equilibrium allocation is efficient.

(b) (i) Self-selection constraint: for signaling to be successful cost of education should exceed benefit for low productive: $u^L(50,0)\geq u^L(100,e)$ and cost of education should be less than benefit for high productive $u^H(50,0)\leq u^H(100,e)$.

$$\begin{cases}50\geq 100-1.25e\\ 50\leq 100-e\end{cases}\Rightarrow 40\leq e\leq 50.$$

Participation constraint: **i**n addition each agent's utility should be greater or equal the reservation one: $u^L(50,0)=50\geq 20$, $u^H(100,e)=100-e\geq 55 \Rightarrow e\leq 45$.

Conclusion $40\leq e\leq 45$.

Comparison of equilibria. Note, that in any equilibrium firms get zero profit (as each agent is paid his MRP), agent L has the same utility equal to 50 and utility of agent H is the highest, when education is at the lowest level (as education is costly but nonproductive).

(ii) In both cases firms get zero expected profit.

Utility of L-type worker is lower than in (a): $u_b^L=50<75$

Utility of H-type worker is also lower than in (a): $u_b^H=100-40<75$

Thus total surplus is lower than in (a), that is equilibrium from part (a) is preferred from efficiency point of view

High productive workers get education in this model because if they do not they will be treated as low productive and get utility of 50 instead of 100-50=60, that is the option of (a) is not affordable when firms treat education as a signal of productivity

(iii) Pooling equilibria should satisfy both participation and incentive compatibility constraints:

$$\begin{array}{l}75-e\geq 50\\ 75-5e/4\geq 50\\ 75-e\geq 55\\ 75-5e/4\geq 20\end{array},\ \begin{array}{l}75-5e/4\geq 50\\ 75-e\geq 55\end{array},\ 0\leq e\leq 20.$$

In all equilibria firms get the same (zero) expected profit but agents are better off with lower level of signaling. Thus Pareto superior pooling equilibrium corresponds to the lowest level $e = 20$.

Graph should be provided.

2. (a) With full insurance $U_t = \sqrt{100 - 75\gamma}$.

Without insurance $EU_t = p\sqrt{25} + (1 - p)\sqrt{100} = 5p + 10(1 - p) = 10 - 5p$

$U_L = \sqrt{100 - 75\gamma} \geq EU_L = 10 - 1 = 9$ iff $\gamma \leq (100 - 81)/75 = 19/75$

$U_H = \sqrt{100 - 75\gamma} \geq EU_H = 10 - 2 = 8$ iff $\gamma \leq (100 - 64)/75 = 36/75 = 12/25$

Demand at L market:

If $\gamma > 12/25$, $z_{dL} = 0$,

If $\gamma = 12/25$, $z_{dL} = [0, 75 \times 50]$,

If $\gamma < 12/25$, $z_{dL} = 75 \times 50$

Supply at L market:

If $\gamma > 0.2$, $z_{sL} \to \infty$,

If $\gamma = 0.2$, $z_{sL} - any$,

If $\gamma < 0.2$, $z_{sL} = 0$

Equilibrium at $\gamma_L = 0.2$.

Similarly $\gamma_H = 0.4$

(b) Under asymmetric we have one market instead of two

Market demand we get by summing up horizontally individual demands:

If $\gamma > 12/25$, $z_d = 0$,

If $\gamma = 12/25$, $z_d = [0, 75 \times 50]$,

If $19/75 < \gamma < 12/25$, $z_d = 75 \times 50$,

If $\gamma = 19/75$, $z_d = [75 \times 50, 75 \times 100]$,

If $\gamma < 19/75$, $z_d = 75 \times 100$

Supply:

Firms maximize expected total profit. They expect only high-risk agents if the price is above 19/75≈0.253 and both types if the price is below.

Since we have equal number of high and low-risk agents insurance companies are willing to sell to both starting from the price equal to average probability of accident, that is at any $\gamma \geq (0.4+0.2)/2 = 0.3$ but this price exceeds 0.253. It means that insurance companies can expect only high-risk agents at price of 0.3 which results in negative expected profit. Thus no firm is willing to sell until price increases up to 0.4.

If $\gamma > 0.4, \quad z_s = \infty$, and at $\gamma = 0.4, \quad z_s - any$

If the price is below: $\gamma < 0.4$ then $z_s = 0$.

Equilibrium is at $\gamma = 0.4$, where only high-risk agents get insurance and $z = 75 \times 50$.

(c) It has no implications for the prices in part (a) since these prices are determined by the horizontal supply curves. Thus in part (a) prices are the same and every consumer gets full insurance .

(b) Market demand we get by summing up horizontally individual demands:

If $\gamma > 12/25, \quad z_d = 0$,

If $\gamma = 12/25, \quad z_d = [0, 75 \times 75]$,

If $19/75 < \gamma < 12/25, \quad z_d = 75 \times 75$,

If $\gamma = 19/75, \quad z_d = [75 \times 75, 75 \times 100]$,

If $\gamma < 19/75, \quad z_d = 75 \times 100$

Insurance companies still expect only high-risk agents if the price is above 19/75≈0.253 and both types if the price is below.

Since 75% of the potential policyholders are low risk agents firms are willing to sell to both groups at lower price: $\gamma \geq (0.4 \times 0.25 + 0.2 \times 0.75) = 0.25$ and this price is less than 0.253.

Thus if the price is below 0.25 the quantity supplied is 0 but if $\gamma = 0.25$ then firms are willing to supply any quantity of insurance. If the price increases above 0.25 but stays below 19/75 then profit becomes unlimited $z_s = \infty$.

When price increases above 19/75 only high-risk agents stay at the market and firms stop selling at prices below 0.4.

At $\gamma = 0.4, \quad z_s - any$ and at $\gamma > 0.4, \quad z_s = \infty$.

Finally we have two equilibria: one with adverse selection at $\gamma = 0.4$, where only high-risk agents purchase insurance and another equilibrium at $\gamma = 0.25$, where both low-risk and high-risk agents get full insurance.