

## **Philosophical and Methodological Problems of the Principle of Least Action**

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The primary goal of this thesis is to find the common ontological foundations of the variational or extremal principles of the natural sciences. For this, I use the ontological probabilistic interpretation of the principle of least action. A research strategy is based on: a classification of philosophical and scientific problems of all extremal principles; examination the question of how the extremal principles are connected with philosophical concepts “possible” and “actual”; discussion of place of the principle of least action in a system of other laws of motion.

The theoretical backgrounds of the thesis include the classical works on the variational principles; old ontological concept of a transition from a potential to actual mode of existence; philosophical hypothesis of internal activity of any objects; views of quantum behavior as the transition from a potential state to actual one; contemporary interpretations of quantum mechanics including decoherence; relationship between Feynman’s path integral method and the principle of least action; connection quantum and classical processes through optical-mechanical analogy; concept that probabilistic approach is more fundamental than deterministic.

I begin the thesis with a historical review of the development of the extremal principles in optics, classical mechanics, field theory, relativity, cosmology, thermodynamics, information theory, and biology. Then I summarize the common properties of these principles. I propose the hypothesis that the unique role of the principle of least action is based on its probabilistic interpretation. I suggest to combine a realistic interpretation of virtual paths in the path integral model and realistic interpretation of possible trajectories in the principle of least action. So we can consider them as two particular mechanisms of a general ontological process, under which any existence changes from its possible mode to actual mode. My arguments are based on the replacement of the classical representation of the motion of objects along a single trajectory by the simultaneous motions along an infinite set of the possible trajectories or world-lines.

I use Feynman’s idea that at the quantum scale, coherent virtual trajectories in superposition interfere with each other so that the actual trajectory of the system arises. Quantum amplitudes of the virtual trajectories aggregate to the probability of single actual trajectory, which possesses a maximum probability. Due to decoherence, the actual systems in a mixed state interact with each other similarly to the classical ones. It follows that most of the variational or extremal principles can be reduced to the principle of maximal probability. According to it, the maximum probability corresponds to one of the main characteristics of the actual systems. The characteristic is always stationary, taking a minimum or maximum value. In different extremal principles, this characteristic may be the action, optical length, difference between kinetic and potential energy, constraint, proper time, curvature of space-time, thermodynamic potentials (for instance, entropy), and others.

From this ontological hypothesis, I formulate some consequences. Possible or virtual motions in the calculus of variations and Feynman integral can be considered not merely heuristic and useful mathematical tool for writing the laws of motion, but also as a description of real processes taking place in our universe, but only in the possible mode of existence. Each variation of the system corresponds to one of its virtual trajectories under given boundary conditions.

It is known that the principle of least action of classical mechanics can be derived from the laws of quantum mechanics as an approximation at the large scale. At the same time, the principle of least action is the approximation of general relativity for low speeds and weak gravity. I assume that the equations of general relativity could be also considered as the approximation of the laws of quantum mechanics. When the scale and mass of the systems increase, extra dimensions are collapsed and flattened, reducing into the stable, smooth four-dimensional space-time. Under the influence of massive systems, every virtual trajectory becomes curved and consequently the actual trajectory—the result of interference of the virtual trajectories—also becomes curved.

The differential equations of physical motion theories are equivalent the variational principles, which can be derived from Feynman's path integral. Thus, mechanical and geometrical descriptions of motion can be derived from the quantum behavior of systems. Due to this reason, the calculus of variations is so widely distributed in linear physics, non-linear thermodynamics, biology, the theory of information, and theory of management. To demonstrate this idea, I reformulate twenty extremal principles from different fields of science under the general ontological scheme—as the special cases of the principle of maximal probability, which is based on the interference-decoherence cycle of possible trajectories.

I examine whether the probabilistic interpretation of the principle of least action can explain why the deterministic behavior of classical and relativistic systems can be considered as the approximation to the probabilistic behavior of quantum systems. At the possible mode, every system tends to use the maximal number of its possibilities. To achieve this, the systems move simultaneously along all trajectories that are possible under the given conditions. The systems do not know in advance, which of their trajectories will be the actual one. They do not choose their particular actual trajectory. Each system tries or examines every virtual trajectory to include them in system's coherent superposition. Due to interference, the maximal number of virtual trajectories unites with each other in the actual trajectory. Moving along the actual trajectory, the system can take the maximal number of its potential states. A habitual view of causality that mutual effect between actual systems within the framework of classical space-time can be used only for solving practical problems. So I conclude that the principle of least action is of great significance in the philosophical explanation and unification of the laws of nature.

In conclusion, I discussed thirteen potential philosophical and physical objections to my hypothesis and answered them.