Reputation and Information Aggregation*

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Abstract

We analyze how reputation concerns of a partially informed decision-maker affect her ability to extract information from reputation-concerned advisors. Too high decision-maker's reputation concerns destroy her incentives to seek advice. However, when such concerns are low, she is tempted to solicit advice regardless of her private information, which can undermine advisors' truth-telling incentives. The optimal strength of the decision-maker's reputation concerns maximizes advice-seeking while preserving advisors' truth-telling. Prior uncertainty about the state of nature calls for a more reputation-concerned decision-maker. Higher expected competence of the advisors may worsen information aggregation, unless the decision-maker's reputation concerns are properly adjusted.

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1 Introduction

According to the case study by Huy et al. (2016),[1] one of the reasons for the downfall of Nokia was the failure of top managers to aggregate information from middle managers in face of the iPhone challenge. Although the middle managers received signals suggesting that a radical change in the strategy was needed, they did not communicate those signals to the top managers, despite being routinely asked for advice. Apparently, one of the reasons for this behavior was the failure of the top managers to credibly convey their concerns about the situation to the middle managers. As a result, middle managers succumbed to the generalized optimism about Nokia's current strategy and hid warning signals.

We analyze how incentive problems of a decision-maker can undermine the incentives of advisors to provide the former with truthful information. In our story, the decision-maker's incentive problems arise due to her either excessive or *insufficient* reputation concerns, which can provoke insufficient or *excessive advice-seeking* respectively. We focus on the latter problem.

Our setup is rather general and fits a variety of real-life settings. For instance, the decision-maker can be a CEO, a politician, a head of a university department, and the advisors can be her colleagues, subordinates, designated advisors, or any kind of experts in the domain of the decision-maker's responsibilities. For a variety of reasons, the advising process typically occurs in the form of direct communication between advisors and the decision-maker (rather though voting or secret polls, for example) — this is the mode of

[1] See also Vuori and Huy (2016).

advice provision we study in our work.

The problem of insufficient advice-seeking is well-known in the literature. Several works document that people can be reluctant to ask for advice or help from other people, even when such advice/help can improve the quality of their decisions (e.g., Lee (2002), Brooks et al. (2015)). One frequently cited reason for such behavior in the management and psychology literature is the fear to appear incompetent, inferior, or dependent (e.g., DePaulo and Fisher (1980), Lee (1997), Lee (2002), Brooks et al. (2015)). Levy (2004) provides a model in which a decision-maker excessively ignores/neglects the opportunity to ask for advice in order to be perceived competent.

Overall, the existing studies suggest that too high reputation concerns of a decision-maker may be detrimental to her ability to collect information from potential advisors. We argue that *low* reputation concerns generate the opposite problem – excessive advice-seeking, which is also detrimental to information aggregation. Consequently, some intermediate level of reputation concerns is generally optimal. The key feature of our story, which distinguishes it from the previous literature, is that the decision-maker's advice-seeking behavior affects advisors' information provision incentives. Without reputation concerns the decision-maker will *always* ask for advice. As we explain below, this adversely affects the advisors' incentives. The positive role for reputation concerns then is to ensure that the decision-maker asks for advice more often when it is needed more, that is, when her available information leaves high uncertainty about the state of the world. This behavior improves the advisors' incentives and, therefore, results in better aggregation of information.

In our model, a decision-maker faces a binary decision problem. The optimal action depends on the binary state of nature, which is unknown. At the beginning of the game, the decision-maker receives an informative binary signal about the state. The quality of the signal depends on the decision-maker's type, either good or bad, which is unknown to everybody including the decision-maker himself. In addition, she can solicit advice from other agents ("advisors"), each of whom (potentially) receives a binary signal about the state as well. The decision-maker cares about taking the right action but also has reputation concerns — she wants to appear competent, i.e., able to receive precise signals.

A key element of our story is that the advisors are more willing to provide the decision-maker with (truthful) information when they perceive the latter to be less confident about the state of nature. Such incentives may arise for various reasons. For example, if information is costly to acquire or transmit, an advisor will be less willing to do that if he thinks that the decision-maker has a strong belief about the right course of action. Another possible reason are the advisors' reputation concerns, which push advisors to bias their reports towards the prior belief, as shown in Ottaviani and Sørensen (2001, 2006a, 2006b). In the baseline model we adopt the latter way of modeling the advisors' incentives, but in the Supplemental Appendix, Section II.I, we show that our results can also be obtained in a costly information acquisition setting.

So, exactly like the decision-maker, we assume that each advisor receives a binary signal at no cost and can be of two precision types, high and low, unknown to everybody including the advisor himself. For simplicity, all advisors are ex-ante identical and only have reputation concerns. In this setup, similarly to Ottaviani and Sørensen (2001), an advisor reports truthfully in (the most informative) equilibrium if and only if his belief about the state *before* accounting for his own signal (i.e., based only on the prior and decision-maker's decision to ask for advice)[2] is sufficiently close to 1/2, so that different

[2]We assume that all advisors speak simultaneously. Sequential advice would not alter our results qualitatively, as we argue in the Supplemental Appendix, Section II.F.

signals result in different states appearing more likely for the advisor. Otherwise, no informative advice takes place ("babbling" or "herding" by the advisors).[3]

In this model, if the decision-maker cares *only* about the quality of decisions, she will *always* want to ask for advice. This means that, in equilibrium, no information can be inferred by the advisors from the decision-maker's behavior. Suppose the prior belief about the state is sufficiently far from 1/2 (as in the case of Nokia's self-confidence of having the best strategy in the mobile industry). Then the advisors will herd on the prior, and no informative advice will be provided. This is what we call the problem of "excessive advice-seeking": the decision-maker's "unrestrained" advice-seeking behavior destroys provision of advice.

Now suppose the decision-maker could *commit* to ask for advice only when she receives a signal that contradicts the prior. Such commitment could induce the advisors to report truthfully, provided that the combination of the prior and the decision-maker's signal results in a belief sufficiently close to 1/2. As a consequence, the decision-maker would manage to receive decision-relevant information precisely when it is most needed (when her signal confirms the prior, extra information is of much lower value).

We show that the decision-maker's reputation concerns can help to implement such commitment as a separating equilibrium. The key intuition can be explained through a kind of "single-crossing" argument. A decision-maker who received the signal that confirms the prior is confident about the correctness of the signal and has a strong reputational motive to show this. In contrast, a decision-maker with the signal that contradicts the prior has either a weaker reputational incentive or a disincentive to be perceived

[3] Ottaviani and Sørensen find binary or no information transmission in a very general setting with a continuum of signal realizations: there is informative communication only when the advisor's prior is sufficiently close to 1/2, in the form of a binary message that only says on which side of a threshold the advisor's signal lies — see, for instance, Ottaviani and Sørensen (2006a, p. 132). Our binary setup replicates these general insights for the different purpose of our model.

as having received the signal that confirms the prior. Coupled with a higher need for advisors' information of the latter signal-type, these incentives separate the two signal-types on asking or not asking for advice, provided that the weight of reputation in the decision-maker's utility function is sufficiently high.

We also show that, for a range of weights on reputation, there exists an equilibrium with even more information aggregation. In this equilibrium, the decision-maker always asks for advice when her signal contradicts the prior and *mixes* between asking and not asking when her signal confirms the prior, and the advisors report truthfully when asked. We call this equilibrium partially separating. Then, the optimal weight on reputation is the one that maximizes the frequency of asking for advice in the partially separating equilibrium, without damaging the advisors' truthtelling incentives. A further rise in the reputation concerns destroys this equilibrium and results in excessive advice-avoidance.

Next, we study the interaction between the prior uncertainty about the state of the world and the decision-maker's reputation concerns. We show that greater uncertainty leads to a higher optimal weight on reputation. The intuition is that higher prior uncertainty increases the decision-maker's incentives to ask for advice even when her signal confirms the prior. A higher weight on reputation is then needed to restrain this temptation.

There may be various ways of adjusting reputation concerns in an organization. One way is to pick managers with certain characteristics (for instance, younger managers are likely to have stronger reputation concerns). Another way is to calibrate practices of rewarding and punishing managers: increasing explicit rewards for high performance or raising the likelihood of dismissal for underperformance is equivalent to lowering the weight of reputation. Then, our findings imply that, as uncertainty about the right strategy for an organization kicks in, one should relieve the anxiety of the manager for the correct decision by making explicit rewards and/or the probability of dismissal less sensitive to performance.

Going back to Nokia, Huy et al. (2016) argue that Nokia's top managers were not technological experts (in contrast to Apple's Steve Jobs) and routinely relied on information provided by middle managers (that is, they "always asked for advice", in our terminology). In addition, the top managers were constantly under strong pressure from investors to deliver short-term results. Our model suggests that greater top managers' concerns for being perceived as technological experts and lower external pressure could have induced advice-seeking behavior that alleviated the middle managers' incentive problems.

We also study the impact of the prior competence of the decision-maker and the advisors on information aggregation. Higher prior competence of either party allows to aggregate more information, provided that the organization can adjust the decision-maker's reputation concerns accordingly. If the advisors are more confident about their own information, they reveal it truthfully also when the decision-maker asks for advice more frequently (with a signal confirming the prior). If the decision-maker receives signals of higher quality, she can avoid asking for advice less frequently (with a signal confirming the prior) and still transmit to the advisors sufficient uncertainty about the state for them to be willing to report their signal truthfully. This result provides an additional rationale for the decision-maker's reputation concerns: A decision-maker who is perceived smarter by her subordinates will be more able to steer the organization along a truthful revelation path.

Yet, if the organization does not adjust the decision-maker's incentives properly, these opportunities may not be exploited, and higher prior competence of the advisors can undermine information aggregation and worsen the quality of decisions. For low reputation concerns, higher quality of the advisors may provoke excessive advice-seeking. Instead, when reputation concerns are high, it can cause excessive advice-avoidance. The latter effect arises because higher-quality advice is more likely to be followed by the decision-maker independently of her private information, with the result that a correct decision will not be ascribed to her ability.

Related literature

There are a number of papers arguing that reputation concerns can be detrimental for efficiency, because they distort behavior of agents (e.g., Scharfstein and Stein (1990), Trueman (1994), Prendergast and Stole (1996), Effinger and Polborn (2001), Morris (2001), Levy (2004), Prat (2005), Ottaviani and Sørensen (2001, 2006a, 2006b), Ely and Välimäki (2003)).[4] In these papers, like in our work, reputation concerns are "career concerns for expertise" which arise due to the future gains from being perceived smart (except for Morris (2001) and Ely and Välimäki (2003), in which the agent have concerns for being perceived as having certain preferences).

Of these papers, Levy (2004) and Ottaviani and Sørensen (2001, 2006a, 2006b) most closely relate to our work. Ottaviani and Sørensen consider aggregation of information from agents possessing private signals about the state of nature. Due to their reputation concerns, agents have incentives to misreport their signals, which may result in herd behavior in reporting. Levy (2004) presents a model in which a decision-maker, who knows her ability, cares both about the outcome of her action and the public perception

[4] A few papers provide a positive view of reputation concerns. Suurmond et al. (2004) present a model in which reputation concerns of an agent incentivize him to acquire more information. Klein and Mylovanov (2017) show that reputation concerns may provide incentives for truthful reporting in a model of long-term dynamic interaction between the agent and the principal. Also, in Morris (2001), reputation concerns of an advisor may actually make the reporting behavior of a misaligned advisor less biased.

of her ability. Levy shows that the decision-maker excessively contradicts prior public information or may abstain from asking for valuable advice in order to raise her perceived competence.

In light of these works, when both the advisors and the decision-maker have reputation concerns, one may expect that the problem of insufficient information transmission is exacerbated. We show that the opposite can occur. We have a reputation-concerned decision-maker who decides whether to ask for advice or not, like in Levy (2004), and reputation-concerned advisors who are tempted to herd on the public belief in their reporting behavior, like in the papers by Ottaviani and Sørensen. The crucial distinction of our paper is the strategic interaction between reputation-concerned agents.[5] In our model, the strategy of the decision-maker (to ask for advice or not depending on her signal) impacts on the advisors' behavior. Absent such influence, the problem of excessive advice-seeking would not exist, and the results would be similar to the ones in Levy (2004), i.e., the decision-maker's reputation concerns could only harm.

Our paper is also related to works on communication with two-sided private information, especially those in which the decision-maker can (attempt to) reveal her private information before the expert talks. Chen (2009) considers a Crawford and Sobel (1982) type of framework, but assumes that the decision-maker also has private information about the state. She provides conditions under which the decision-maker fails to reveal her signal to the expert in equilibrium and discusses when such revelation (full or partial) is possible. In a subsequent paper, Chen and Gordon (2014) argue that full revelation of the decision-maker's information is possible only if her signal is sufficiently informative.

[5] Levy (2004) has an extension in which she considers a strategic advisor, who has both instrumental and reputational payoff. However, in contrast to our model, the decision-maker does not exercise any influence on the advisor's truthtelling incentives. Instead, it is the advisor who tries to affect the decision-maker's actions by distorting the information he transmits. Thus, strategic interactions in Levy (2004) are orthogonal to those in our paper.

However, these papers do not discuss whether the decision-maker would benefit or lose from the ex-ante perspective by hiding her information.

Chen (2012) considers the effects of public information in a Crawford-Sobel framework. The paper shows that, depending on the magnitude of the bias and the precision of the public signal, the receiver may be either better or worse off when the sender is asked to report after the public signal arrives (rather than before). Since in Chen (2012) the public signal always arrives prior to the decision-maker choosing her action, her setting is equivalent to a setup in which the receiver has private information and can choose *ex-ante* whether to *commit* to reveal or to conceal it before the sender's communication.

A more relevant paper to ours is de Bettignies and Zabojnik (2019). There is a manager and a worker. The manager decides whether to reveal or to conceal her signal about the optimal action for an organization. This signal is hard information but the manager does not always receive it, thus she can pretend she does not have it even when she actually does. The worker can then exert effort to search for additional information and improve the accuracy of the action. In equilibrium, the manager excessively conceals information, as she is tempted to boost the worker's effort by pretending being uninformed. Then, commitment to information revelation is needed to improve the worker's incentives when no information is revealed. In our paper, commitment not to ask for advice would play a similar role: improving the advisors' incentives when they are asked for advice. Note however that we obtain this type of behavior as an equilibrium result without commitment, thanks to the reputation concerns of the decision-maker.

Apart from the difference in the incentive problem of advisors, our work differs from the mentioned studies on communication with two-sided private information in one important aspect: In these papers the decision-maker has the only goal of extracting information from the sender. In our model, instead, the decision-maker's incentives are shaped by the trade-off between the desire to receive information and the desire to appear competent.

The rest of the paper is organized as follows. In Section 2 we set up the model. Section 3 carries out the equilibrium analysis. In Section 4 we examine the effects of the prior uncertainty about the state as well as the impact of advisors' and the decision-maker's expected competence. Section 5 concludes the paper. The Appendix contains the proofs for Section 3. The Supplemental Appendix (for online publication) contains the proofs for Section 4 and shows the robustness of our results to alternative modeling assumptions.

2 The model

2.1 Players and information

There is a state of the world $\omega \in \{0,1\}$. A decision-maker has to take a decision $d \in \{0,1\}$. The instrumental utility for the decision-maker from the decision is 1 if the decision matches the state of the world and 0 otherwise. The decision-maker receives a private signal $\sigma \in \{0,1\}$ about the state. There are N advisors, each of whom has also received a private signal $s_i \in \{0,1\}$, $i = 1, ..., N$. Conditional on the state, all signals are independent.

The decision-maker can be of two types, $\theta \in \{G, B\}$, which influence the precision of her signal. Specifically, for any ω,

$$g := \Pr(\sigma = \omega | \theta = G) > b := \Pr(\sigma = \omega | \theta = B) \geq 1/2,$$

That is, the Good type of the decision-maker receives a more informative signal than the Bad type.

Analogously, each advisor $i = 1, ..., N$ can be of type $t_i \in \{H, L\}$, with the High type receiving a more informative signal than the Low type. Namely, for any ω:

$$h := \Pr(s_i = \omega | t_i = H) > l := \Pr(s_i = \omega | t_i = L) \geq 1/2.$$

The types of all agents are independent of each other and of the state of the world. No agent knows his/her own type and types of others. There are common priors about the state of the world, the type of the decision-maker, and the type of each advisor, namely:

$$p := \Pr(\omega = 0);\ q := \Pr(\theta = G);\ r := \Pr(t_i = H),\ i = 1, ..., N;\ p, q, r \in (0, 1).$$

Without loss of generality, we assume that $p \geq 1/2$.

We will call the decision-maker "signal-type 0" when she has received signal $\sigma = 0$ and "signal-type 1" otherwise (not to confuse the private information of the decision-maker with her unknown type θ.)

2.2 Sequence of the events and payoffs

The sequence of events is as follows:

1. Nature draws the state ω and the competences of all players.

2. All players receive their private signals.

3. The decision-maker decides whether to ask for advice or not. This is a binary choice $m \in \{m^0, m^1\}$, where m^0 and m^1 denote "not asking" and "asking" respectively. It is impossible to ask a subgroup of advisors: Either all advisors are invited to provide advice or none. If the decision-maker does not ask, the game proceeds to stage 5; if she

asks, the game proceeds to stage 4.

4. If asked, the advisors provide their advice publicly to the decision-maker. Specifically, all advisors *simultaneously* and *publicly* send binary *cheap-talk* messages $a_i \in \{0, 1\}$, $i = 1, ..., N$.

5. The decision-maker takes a decision $d \in \{0, 1\}$.

6. The state is revealed and players receive their payoffs.

The decision-maker cares about matching her action with the state (instrumental objective). However, she would also like to appear informed (reputation concerns). We model the decision-maker's reputational payoff as the posterior belief about her ability in the eyes of an "external observer", who observes the whole course of the game (m, d, and $a = (a_i)_{i=1}^N$ if $m = m^1$) and the realized state (ω). We will analyze equilibrium behavior, and thus, whenever possible, we will compute agents' beliefs via Bayesian updating given the priors and the equilibrium strategies. Anticipating this, we denote the observer's posterior belief at terminal node (m, a, d, ω) as $\Pr(G|m, a, d, \omega)$ (a to be omitted if $m = m^0$). For an off-path terminal node, $\Pr(G|m, a, d, \omega)$ is to be interpreted as the observer's subjective belief, which we will refine in Section 3.

The observer could be a decision-maker's boss (say, the board of directors) who cares about the decision-maker's quality, or could be the outside market. More generally, "external observer" should be viewed as a shortcut for some (internal or external) mechanism that rewards decision-makers for higher perceived skills (through a higher future wage or better job opportunities). Alternatively, the decision-maker could care about his reputation in the eyes of the advisors (who may be her colleagues or subordinates) either for a mere desire for esteem by colleagues or because better regarded decision-makers are able to receive better advice in the future.[6]

[6] In Section 4 we show that *a priori* more competent decision-makers, under the right reputational

Thus, the decision-maker's aggregate payoff is a convex combination of the instrumental and reputational objectives with weight $\rho \in [0, 1]$ attached to reputation:

$$u_D(m, a, d, \omega) = (1 - \rho)I(d, \omega) + \rho \Pr(G|m, a, d, \omega), \text{ where}$$

$$I(d, \omega) = \begin{cases} 1 \text{ if } d = \omega; \\ 0, \text{ if } d \neq \omega. \end{cases}$$

For simplicity, we assume that the advisors only have reputation concerns: Each advisor cares only about his reputation in the eyes of the decision-maker.[7] An advisor's payoff is thus

$$u_i(m, a, d, \omega) = \Pr(H|a_i, \omega), \; i = 1, ..., N,$$

provided that the decision-maker asked for advice.[8] If a_i is off-path, $\Pr(H|a_i, \omega)$ is to be interpreted as the decision-maker's subjective belief.

2.3 Restrictions to parameters and modeling assumptions

To avoid uninteresting cases and simplify exposition, we focus on an intermediate region of the parameters where, roughly speaking, signals are not "too strong" or "too weak" in expectation. In particular, we assume the following:

A1 If the decision-maker learns that all the advisors have received the same signal, she

incentives, are able to receive truthful advice more often.

[7] This implies that the decision-maker cares about the quality of advisors, which means that learning the advisors' types should be valuable for her ex-post. Incorporating this consideration in the decision-maker's utility would have no effect in our setup if the benefit from knowing an advisor's type is linear in the ex-post belief about the advisor. If it is convex, like in Prat (2005), it would increase the decision-maker's temptation to ask for advice, but our qualitative results would not change. Alternatively, we could assume that the advisors care about their reputation in the eyes of the external observer.

[8] If the decision-maker did not ask for advice, an advisor's payoff is simply the prior belief r, but this does not play any role in the model.

believes that the corresponding state is more likely, regardless of the own signal:

$$\Pr(\omega = \overline{s}|\sigma, s = (\overline{s}...\overline{s})) > 1/2 \quad \forall \overline{s} = 0, 1, \forall \sigma = 0, 1.$$

A2 Absent any information about the decision-maker's signal, an advisor who receives signal 1 still believes that state 0 is more likely; if an advisor learns that the decision-maker received signal 1, his own signal determines which state he considers more likely.

A1 means that both signal-types can change their mind after truthful advice, that is, advice is potentially useful for the decision-maker regardless of her own signal. For our analysis it is important that at least signal-type 1 can change her mind after advice. Assuming that advice is potentially useful also for signal-type 0 greatly simplifies the exposition.

The first part of A2 rules out the uninteresting situation where always asking results in truthful advice. Mathematically, it reads

$$\Pr(\omega = 0|s_i = 1) > 1/2,$$

and it is straightforward to show that it translates into

$$p > rh + (1 - r)l;$$

that is, the advisor's signal is, in expectation, weaker than the prior. The second part of A2 mathematically reads

$$\Pr(\omega = s_i | s_i, \sigma = 1) > 1/2, \quad s_i = 0, 1,$$

which translates into

$$rh + (1 - r)l > \Pr(\omega | \sigma = 1), \quad \omega = 0, 1.$$

It means that the advisor's signal is stronger than the posterior that takes into account $\sigma = 1$. We assume $\Pr(\omega = 1 | s_i = 1, \sigma = 1) > 1/2$ to rule out the uninteresting situation where an advisor always maintains the belief that state 0 is more likely, whatever he infers about the decision-maker's information. We assume $\Pr(\omega = 0 | s_i = 0, \sigma = 1) > 1/2$ only to streamline exposition: in the Supplemental Appendix, Section II.J, we show that our main findings are confirmed when this restriction is relaxed.

The model uses a number of assumptions. First, the decision-maker is only allowed to ask either all advisors or none. Second, if the advisors are not asked, they cannot report anything. Third, asking cannot be accompanied by any additional statements from the decision-maker. Fourth, both asking for advice and providing advice are public. Fifth, advice is simultaneous rather than sequential. Sixth, no player knows his/her own type. Finally, the advisors only care about their reputation in front of the decision-maker. In the Supplemental Appendix, Section II, we argue that relaxing these assumptions does not have a qualitative impact on our results.

3 Equilibrium analysis

We are going to look for Perfect Bayesian Equilibria that satisfy the following restrictions to beliefs.

1. After decision d, the observer believes that the decision-maker considers $\omega = d$ more likely, if possible,[9] or received signal $\sigma = d$, otherwise.

2. After report a_i, the decision-maker believes that advisor i considers state $\omega = a_i$ more likely, if possible,[10] or received signal $s_i = a_i$, otherwise.

We will show that, given such beliefs, the decision-maker does have the incentive to take the decision that corresponds to the state she considers more likely, and the advisors do have the incentive to report the state that they consider more likely. We restrict our analysis to these beliefs for different reasons. First, we find them simple and natural. Second, they promote the locally optimal behavior of the decision-maker and of the advisors for the quality of the decisions. Indeed, we will argue that the induced advisors' behavior is the most informative among all possible equilibria. Third, as one would expect, these locally optimal behaviors are also part of the "second-best" under our reputational frictions, which we are going to characterize and achieve in equilibrium at an intermediate value of reputation concerns.

It is worth noting that, for a fixed level of reputation concerns, selecting alternative equilibria with a locally lower quality of advising and decision-making may improve the overall quality of the decisions via an increase of the equilibrium frequency of advice-seeking. The feasibility of these equilibria is questionable: the decision-maker will want

[9] That is, if at least one signal-type considers $\omega = d$ more likely, given the received advice (if any).

[10] That is, if at least one signal-type considers $\omega = a_i$ more likely, given the asking/not asking policy of the decision-maker.

to induce the most informative advising behavior by simply asking the advisors to "speak their mind";[11] in a related vein, the observer might have the incentive to induce the locally optimal decision policy. Be as it may, we will show at the end of the section that these other equilibria cannot achieve the second-best at any level of reputation concerns and generate a lower welfare than our separating equilibrium at low levels of reputation concerns.

3.1 The decision stage

Proceeding by backward induction, we start the equilibrium analysis from the final decision stage.

Take the viewpoint of a signal-type of the decision-maker who considers state $\overline{\omega}$ more likely. Given our assumption on the observer's beliefs, the decision-maker will be perceived as being signal type $\sigma = 1 - \overline{\omega}$ for sure if she takes decision $d = 1 - \overline{\omega}$. In terms of expected reputation, this is the worst possible scenario. In terms of expected instrumental utility, it is optimal for the decision-maker to take decision $d = \overline{\omega}$. These considerations give rise to the following result.

Lemma 1 *Consider an arbitrary history of events ψ prior to the decision stage (that is, ψ is either m^0 or (m^1, a)). If at ψ the decision-maker believes that one state is weakly more likely, she weakly prefers to take the corresponding decision.*

To break the tie when the decision-maker is indifferent between the two decisions,[12]

[11] That is, he could tell the advisors: "Tell me which state you consider more likely, and I will take your reports at face value." Then, given this promise, each advisor will indeed find it optimal to behave as asked, as we formally show in Lemma 2

[12] This happens in two scenarios: (1) when the decision-maker considers the two states equally likely, for then the observer will believe that both decisions are taken by the corresponding signal-type; (2) when (i) $\rho = 1$, (ii) $\sigma = \overline{\omega}$, (iii) the other signal-type considers $\overline{\omega}$ *strictly* more likely (so $\sigma = \overline{\omega}$ as well), and (iv) the observer gives probability 1 to the other signal-type after $d = \overline{\omega}$, for then the two decisions are reputationally equivalent.

we assume that she takes the decision that corresponds to her signal-type. Then, the optimal behavior of the decision-maker coincides with what the observer expects under our assumption. Thus, we can fix this behavior as an equilibrium, which is locally optimal for the quality of the decisions.

3.2 The advising stage

Each advisor cares only about his reputation. Thus, given our assumption on the decision-maker's beliefs, the incentives for the advisor parallel the reputational incentives for the decision-maker at the decision stage. Under A2, an advisor with signal 0 always considers state 0 strictly more likely, even after being asked for advice. An advisor with signal 1 considers state 1 weakly more likely when

$$\Pr(\omega = 1 | s_i = 1, m^1) \geq 1/2,$$

which translates into

$$\Pr(\omega = 0 | m^1) \leq rh + (1 - r)l. \tag{1}$$

Then, assuming that each advisor reports $a_i = s_i$ when indifferent, the optimal reporting strategy is the following (and coincides with what the decision-maker expects).

Lemma 2 *When (1) holds, each advisor reports his signal (i.e., he "reports truthfully"); when (1) is not satisfied, each advisor reports* 0 *regardless of his signal.*

The absence of informative communication when (1) is not satisfied does not depend on our assumption on the decision-maker's beliefs: For the binary message space, Ottaviani and Sørensen (2001) have argued that, when (1) is violated,[13] there is no informative

[13] Condition (1) is equivalent to condition $q \leq \rho^I$ from Ottaviani and Sørensen (2001), where q denotes the prior belief before advisors speak, and ρ^I is the average precision of an advisor's signal. To be precise,

equilibrium between the advisor and the decision-maker. Furthermore, it is easy to show that this conclusion generalizes to a setting with an arbitrary discrete message space. Specifically, when both signal-types of an advisor believe that $\omega = 0$ is more likely (i.e., (1) is violated), one can show the following. First, for any two hypothetical equilibrium messages a' and a'', such that $\Pr(a_i = a'|s_i = 0)/\Pr(a_i = a'|s_i = 1) > \Pr(a_i = a''|s_i = 0)/\Pr(a_i = a''|s_i = 1)$, signal-type 0 will strictly prefer to send a'. Second, if signal-type 0 never sends a'', signal-type 1 will strictly prefer not to send a''. This implies that any equilibrium message must be sent with the same probability under both signals, meaning that communication is uninformative.

Ottaviani and Sørensen (2001) have also shown that, when (1) is satisfied, there is also a mixed-strategy equilibrium in which the less confident signal-type of the advisor randomizes between reporting his true signal and pooling with the opposite signal-type. This behavior is obviously less informative. Thus, the behavior stated in Lemma 2 is locally optimal for the quality of decisions.

3.3 Second best

Before moving to the asking/not asking stage, we ask the following question: What is the highest frequency of asking that is compatible with the advisors' incentives to report truthfully? Whenever condition (1) is satisfied, it is satisfied also under a higher frequency of asking by signal-type 1, due to A2. Hence suppose that signal-type 1 always asks for advice. Then, we call μ the probability of asking by signal-type 0, and $\overline{\mu}$ the value of μ

in Ottaviani and Sørensen (2001) the condition is $1 - \rho^I \leq q \leq \rho^I$, because they do not restrict q to be greater than 1/2.

that satisfies (1) with equality:

$$\Pr(\omega = 0|m^1)|_{\mu=\overline{\mu}} = rh + (1-r)l.$$

Note that $\Pr(\omega = 0|m^1)|_{\mu=0} = \Pr(\omega = 0|\sigma = 1)$ and $\Pr(\omega = 0|m^1)|_{\mu=1} = p$. By A2, $\Pr(\omega = 0|\sigma = 1) < rh + (1-r)l < p$. Hence, $\overline{\mu}$ always exists. Condition (1) remains satisfied if $\mu < \overline{\mu}$, and is violated if $\mu > \overline{\mu}$. Thus, $\overline{\mu}$ is the *highest* value of μ that satisfies (1). Furthermore, because $\Pr(\omega = 0|m^1)$ is increasing in p and decreasing in μ, $\overline{\mu}$ is decreasing in p.

We will refer to the situation where signal-type 1 always asks and signal-type 0 asks with probability $\overline{\mu}$ as the "second best". This is the asking policy that maximizes information aggregation, hence the quality of decisions. By A1, $\mu < \overline{\mu}$ is suboptimal, in that it entails a strictly lower probability that signal-type 0 takes the correct decision.

Note that it is impossible to improve upon the second best by selecting different equilibrium behavior at the decision and/or advising stages. Since the behavior described by Lemmas 1 and 2 is locally optimal, a welfare improvement could only be achieved through more frequent asking for advice. However that would result in the violation of (1). Hence, we consider the equilibrium behavior that we selected at the decision and advising stages as a part of the second-best solution.

Note that the second-best has been defined with no reference to reputations. In any equilibrium of the game, the *ex-ante* expected reputation of any player is equal to the prior reputation. Thus, since the agents' payoffs are linear in reputation, the *ex-ante* welfare comparisons boil down to comparing the likelihoods of taking a correct decision.

3.4 The choice between asking and not asking and overall equilibrium behavior

Before presenting our main result, we formulate two key lemmas. The first one concerns the behavior of expected reputation for signal-type 0.

Lemma 3 *Suppose signal-type* 1 *always asks for advice. Then the following is true:*

i) The expected reputation of signal-type 0 *conditional on asking is strictly increasing in* $\mu \equiv \Pr(m^1|\sigma = 0)$.

ii) The expected reputation of signal-type 0 *is strictly higher for* $m = m^0$ *than for* $m = m^1$*, for any* $\mu < 1$.

Lemma 3 says that when signal-type 1 always asks, the expected reputation of signal-type 0 after asking is increasing in the probability that she asks (by part (i)) and is anyway higher after not asking (part (ii)).

Now we can formulate the "single crossing" result outlined in the Introduction.

Lemma 4 *Consider a strategy of the decision-maker such that:*

1. *given the asking/not asking behavior prescribed by this strategy, truthful reporting occurs after asking, i.e., (1) holds;*

2. *signal-type* 1 *always asks, and signal-type* 0 *does not always ask;*

3. *signal-type* 0 *weakly prefers to ask.*

Then signal-type 1 *strictly prefers to ask.*

3.4.1 Separating and partially separating equilibria

We want to establish the existence conditions for the following two equilibria:

- Separating: signal-type 0 never asks for advice, signal-type 1 always asks;
- Partially separating: signal-type 0 asks with probability $\mu \leq \overline{\mu}$, signal-type 1 always asks.

The separating equilibrium corresponds to a simple asking policy of the decision-maker and delivers a straightforward intuition for how reputation concerns help to avoid excessive advice-seeking. Moreover, if signal-type 0 seldom changes her mind after receiving advice, the separating equilibrium gets close to the second best, and it may do so up to higher levels of reputation concerns with respect to the partially separating equilibrium. However, only the partially separating equilibrium allows to reach the second best for the right level of reputation concerns, and improves upon the separating equilibrium also for lower levels.

The following theorem provides the main insight of the paper.

Theorem 1 *The following holds:*

1. *A separating equilibrium in which signal-type* 0 *never asks for advice and signal-type* 1 *always asks for advice exists if and only if* $\rho \in [\underline{\rho}, \overline{\rho}]$, *with* $\underline{\rho} \in (0,1)$ *and* $\overline{\rho} \in (\underline{\rho}, 1]$, *where* $\overline{\rho} = 1$ *if and only if* $\Pr(\omega = 1|\sigma = 1) \geq 1/2$*;*
2. *A partially separating equilibrium in which signal-type* 0 *is indifferent between asking and not asking for advice and signal-type* 1 *always asks exists if and only if* $\rho \in [\underline{\rho}, \widehat{\rho}]$, *where* $\widehat{\rho} \in (\underline{\rho}, 1)$*;*

3. *In both equilibria the advisors report their signals truthfully. In the partially separating equilibrium μ is strictly increasing in ρ, ranging from* 0 *at $\underline{\rho}$ to $\overline{\mu}$ at $\widehat{\rho}$.*

The intuition behind Theorem 1 is as follows. A decision-maker who received the signal confirming the prior (signal-type 0) has a strong reputational incentive to convey this news to the observer. At the same time, her need for extra information is low, because she is already quite confident about the state. In contrast, a decision-maker who received the signal contradicting the prior (signal-type 1) has either a weaker reputational incentive to be perceived as signal-type 0 (when the signal is weaker than the prior) or even a reputational incentive to reveal her true signal (when the signal is stronger than the prior). At the same time, such decision-maker cares more about information aggregation, because the signal contradicting the prior results in higher uncertainty compared to the signal confirming the prior.

Thus, the rationale for separation (full or partial) of the two signal-types arises. However, the weight of reputation should be sufficiently high for such separation to emerge. When ρ is below $\underline{\rho}$, the instrumental incentive to receive additional information dominates and signal-type 0 prefers to deviate to asking for advice.

At $\rho = \underline{\rho}$, the incentive compatibility of signal-type 0 binds. Hence, by Lemma 4, in the separating equilibrium, signal-type 1 strictly prefers to ask for advice at $\underline{\rho}$ as well as for some $\rho > \underline{\rho}$, by continuity. However, when signal-type 1 believes that $\omega = 0$ is more likely, she has a reputational incentive to mimic signal-type 0. As we increase ρ, this incentive grows and finally prevails once ρ passes $\overline{\rho}$. Consequently, for $\rho > \overline{\rho}$, full separation cannot be supported anymore.

Consider now the partially separating equilibrium. If signal-type 1 always asks for advice, then, by Lemma 3, part (ii), the expected reputation of signal-type 0 from asking is lower than from not asking for any probability of asking, μ. However, by part (i) of the lemma, it grows with μ, thus making asking more attractive to her. Since asking generates a higher instrumental payoff, then, provided that ρ is neither too low nor too high, there will be μ that makes signal-type 0 indifferent between asking and not asking (given that the advisors report truthfully).

Since an increase in ρ makes not asking more attractive, μ has to go up with ρ in equilibrium, in order to preserve the indifference. Eventually ρ becomes so high that μ hits $\overline{\mu}$ – the maximum μ compatible with truthtelling by the advisors. The corresponding value of ρ is denoted $\widehat{\rho}$. A further increase in ρ, while keeping μ at $\overline{\mu}$, will make signal-type 0 deviate to not asking.

We have just seen that, our separating and partially separating equilibria are destroyed when ρ fall below $\underline{\rho}$, due to the "excessive" temptation of signal-type 0 to ask for advice. A stronger result is true:

Proposition 1 *Under our equilibrium selection at the advising and decision stages, there exist no equilibria with information aggregation for* $\rho < \underline{\rho}$.

The intuition behind this proposition is rather simple. Potential (informative) equilibria that we have not considered are those in which signal-type 1 does not always ask for advice. Such behavior of signal-type 1 makes the reputational loss from asking smaller for signal-type 0, which amplifies her incentive to ask. Then, for $\rho < \underline{\rho}$, signal-type 0 will always prefer to ask, no matter her frequency of asking. But when signal-type 0 always asks, (1) cannot be satisfied (a straightforward consequence of A2). As a result, any hypothetical informative equilibrium is destroyed.

3.4.2 The overall picture of equilibria and welfare

The analysis of the previous subsection suggests that both too high and too low reputation concerns worsen information aggregation. We now drop the equilibrium restrictions introduced in the beginning of Section 3 and argue that this result holds in general.

To begin with, it is clearly impossible to achieve the second best for $\rho \neq \widehat{\rho}$ *irrespective* of the equilibrium choices at the decision and advice stages. As the previous section shows, the second best cannot be attained for $\rho \neq \widehat{\rho}$ under the locally optimal decision-making and advising behavior, while deviations from local optimality lead to a strictly lower welfare compared to the second best, as follows from subsection 3.3.

Moreover, for small enough ρ, *any* equilibrium necessarily results in a lower welfare compared to the separating equilibrium of the previous subsection arising at $\underline{\rho}$. Let us elaborate on that.

As we have shown, there are no informative equilibria with locally optimal behavior below $\underline{\rho}$. For an equilibrium with locally suboptimal behavior not to be inferior to our separating one, it must be that signal-type 0 sometimes asks, but not always, so he must be indifferent between asking and not asking. As ρ goes to zero, for signal-type 0 to stay indifferent there must be at least one advisor i who reports some a_i with probability *bounded away* from zero both when $s_i = 0$ and when $s_i = 1$.[14] But then, there is a discrete instrumental loss for signal-type 1 compared to the truthful revelation case: sometimes she takes the wrong decision when advisor's i signal is pivotal. On the other hand, the instrumental gain from asking for signal-type 0 must go to zero as ρ goes to zero. Note that this argument does not depend on the message space of the advisors, which we

[14]If this probability converges to zero, by A1 the advice eventually becomes strictly useful for signal-type 0, which will destroy her indifference at small enough ρ.

Note also that signal-type 0's benefit from asking cannot be bounded via a deviation from local optimality at the decision stage, because taking the decision that corresponds to the state that is considered more likely eventually becomes dominant as ρ decreases.

assumed being binary earlier on because it sufficed for our existence results. Therefore, in the proof of the proposition below we do not restrict ourselves to considering a binary message space.

The following proposition summarizes our findings.

Proposition 2 *In equilibrium, the quality of decisions:*

- *is generically lower at $\rho \in [0, \underline{\underline{\rho}}]$ where $\underline{\underline{\rho}} \in \left(0, \underline{\rho}\right)$ than at $\rho \in \left[\underline{\rho}, \overline{\rho}\right]$ in the separating equilibrium;*
- *is below the second best for low values of reputation concerns ($\rho \in (0, \widehat{\rho})$) and for high values of reputation concerns ($\rho \in (\widehat{\rho}, 1]$);*
- *reaches the second best in the most informative equilibrium for an intermediate value of reputation concerns ($\rho = \widehat{\rho}$).*

Note that away from the second best, not only the quality of decisions is ex-ante inferior, but the amount of information available for/incorporated in the decisions is both weakly lower for signal-type 1 and strictly lower for signal-type 0.

Our overall analysis suggests the effect of the decision-maker's reputation concerns on information aggregation is generally non-monotonic. Both too high and too low reputation concerns are detrimental for information aggregation. Too low reputation concerns provoke excessive advice-seeking, which undermines the advisors' reporting incentives. Too high reputation concerns result in excessive advice avoidance.[15]

[15] If the first part of A2 is violated, always asking by both signal-types results in truthful reporting by the advisors. Such "first-best" equilibrium can be achieved for all values of ρ from 0 to $\widehat{\rho}$, where $\widehat{\rho}$ is determined by the indifference of signal-type 0 between asking and not asking. Thus, too low reputation concerns do not harm in this case, while too high reputation concerns are still detrimental.

4 Comparative statics

Now we ask: What is the impact of the priors (about state of nature, competence of the advisors, competence of the decision-maker) on the optimal level of reputation concerns and on the ultimate quality of the decisions?

We start from the role of the prior uncertainty about the state. As it decreases (p goes up), signal-type 0 becomes more confident about the state and less tempted to ask for advice. Therefore, for a given probability of asking, she will be indifferent between asking and not asking for lower levels of reputation concerns, that is, the thresholds $\underline{\rho}$ and $\widehat{\rho}$ decrease. In addition, the second-best probability of asking, $\overline{\mu}$, has to decline in order to preserve the advisors' uncertainty at the level consistent with truthful reporting. This makes not asking for $\mu = \overline{\mu}$ even more tempting for signal-type 0, as his expected reputation after asking declines (due to Lemma 3, part (i)). So, $\widehat{\rho}$ decreases further. Hence, the following proposition is true:

Proposition 3 *The values of $\underline{\rho}$ and $\widehat{\rho}$ rise with an increase in the prior uncertainty about the state of nature.*

Proof. See the Supplemental Appendix for a formal proof. ■

Thus, higher prior uncertainty about the state calls for a more reputation-concerned decision-maker.

It seems obvious that more competent advisors or decision-maker improve the quality of decisions. This is certainly the case when asking and reporting behavior of the parties is fixed. However, the competence of the advisors and/or the decision-maker do affect both asking and reporting, and, as we argue below, these changes in behavior can be detrimental to information aggregation.

If the organization is able to adjust the relative reputation concerns of the decision-maker, the effect can only be positive: the second-best frequency of asking increases in the prior competence of decision-maker and advisors. For the competence of the advisors, the argument is very simple: More confident advisors believe that the state that corresponds to their signal is more likely for a wider range of prior beliefs about the state (i.e., condition (1) is relaxed). For the competence of the decision-maker, the mechanism is a bit more subtle. When signal-type 0 asks less than signal-type 1, asking is informative about the decision-maker's signal. Consequently, the greater her signal precision is, the higher is the informational content of asking. This implies that, for a fixed μ, asking has a higher effect on the advisors' belief about the state. Hence, a "less informative" asking strategy, i.e., higher μ, becomes enough to ensure truthful reporting by the advisors.

Proposition 4 *The second-best probability of asking by signal-type* 0*,* $\overline{\mu}$*, is increasing in the decision-maker's or the advisors' prior competence.*

Proof. See the Supplemental Appendix for a formal proof. ■

Yet, for fixed values of ρ, higher prior quality of the advisors can surprisingly harm information aggregation. For low levels of reputation concerns, it can induce excessive advice-seeking (in other words, $\underline{\rho}$ increases), which can destroy the incentive of the advisors to report truthfully (cf. Proposition 1).

For higher values of reputation concerns, higher prior competence of the advisors can provoke excessive advice avoidance. Higher quality of advice induces the decision-maker to follow it more often independently from her private information. This reduces the opportunity for signal-type 0 to reveal herself through the decision *after* asking, thereby lowering her chances to take credit for a correct decision. As a result, signal-type 0 may prefer to abstain from asking.

The above arguments lead to the following proposition.

Proposition 5 *For given reputation concerns, greater prior competence of advisors can* ***worsen*** *information aggregation and the quality of decisions.*

Proof. See the Supplemental Appendix for a formal proof. ■

5 Conclusion

In this paper we have studied how reputation concerns of a decision-maker affect her ability to extract decision-relevant information from potential advisors. Too high reputation concerns provoke excessive advice-avoidance due to the decision-maker's desire to appear well informed. Too low reputation concerns result in excessive advice-seeking, which destroys advisors' incentives to provide truthful information. In general, some intermediate reputation concerns are optimal, as they create a credible commitment (in equilibrium) to abstain from asking for advice too frequently and, at the same time, do not trigger too much advice-avoidance.

A rise in the prior uncertainty about the state of nature increases the temptation to ask for advice, which may disrupt aggregation of information due to excessive advise-seeking. Hence, greater prior uncertainty calls for higher optimal reputation concerns, in order to restrain this incentive.

We have also shown that an increase in the prior competence of the advisors or the decision-maker improve information aggregation, provided that the reputation concerns of the decision-maker are properly adjusted. However, absent such an adjustment, higher quality of the advisors can worsen information aggregation and the quality of decisions, as it may provoke excessive advice-seeking (when the decision-maker's reputation concerns are not strong enough) or excessive advice-avoidance (when the reputation concerns are sufficiently high).

A legitimate question is how an organization can adjust the relative weight of reputation concerns in the decision-maker's utility function. One factor that can affect reputation concerns is the age of the decision-maker: Other things being equal, younger managers should have stronger career concerns. Alternatively, an organization could adjust practices of rewarding and punishing managers: Higher explicit rewards for good performance or higher likelihood of dismissal for underperformance is equivalent to a lower weight of reputation. In particular, our findings imply that, as uncertainty about the right strategy for an organization kicks in, one should relieve the anxiety of the manager on the correct decision by making explicit rewards and/or the probability of dismissal less sensitive to performance.

Appendix

Proofs for Sections 3.1 and 3.2

Proof of Lemma 1. Consider an arbitrary history of events ψ prior to the decision stage (that is, ψ is either m^0 or (m^1, a)). Fix a signal-type $\overline{\sigma}$, and without loss of generality suppose that she considers state 0 weakly more likely, that is $\Pr(\omega = 0|\overline{\sigma}, \psi) \geq 1/2$. Then, given our restriction to the observer's believes, she is perceived as signal-type 1 when she takes $d = 1$. Thus, in this case, her expected reputation is

$$\Pr(\omega = 0|\overline{\sigma}, \psi) \cdot \Pr(G|\sigma \neq \omega) + \Pr(\omega = 1|\overline{\sigma}, \psi) \cdot \Pr(G|\sigma = \omega).$$

When she takes decision $d = 0$, her expected reputation is

$$\Pr(\omega = 0|\overline{\sigma}, \psi) \cdot \Pr(G|\omega = 0, \psi, d = 0) + \Pr(\omega = 1|\overline{\sigma}, \psi) \cdot \Pr(G|\omega = 1, \psi, d = 0)$$
$$= \Pr(\omega = 0|\overline{\sigma}, \psi) \cdot \begin{bmatrix} \Pr(G|\sigma = \omega) \Pr(\sigma = 0|\omega = 0, \psi, d = 0) \\ + \Pr(G|\sigma \neq \omega) \Pr(\sigma = 1|\omega = 0, \psi, d = 0) \end{bmatrix}$$
$$+ \Pr(\omega = 1|\overline{\sigma}, \psi) \cdot \begin{bmatrix} \Pr(G|\sigma \neq \omega) \Pr(\sigma = 0|\omega = 1, \psi, d = 0) \\ + \Pr(G|\sigma = \omega) \Pr(\sigma = 1|\omega = 1, \psi, d = 0) \end{bmatrix}.$$

Since $\Pr(\omega = 0|\overline{\sigma}, \psi) \geq \Pr(\omega = 1|\overline{\sigma}, \psi)$ and $\Pr(G|\sigma = \omega) > \Pr(G|\sigma \neq \omega)$, for the second expected reputation not to be smaller than the first it is enough that $\Pr(\sigma = 0|\omega = 0, \psi, d = 0) \geq \Pr(\sigma = 0|\omega = 1, \psi, d = 0)$. If $\Pr(\sigma = 0|\psi, d = 0) = 0$, both probabilities are 0. Otherwise, for each $\omega = 0, 1$, we have

$$\begin{aligned} \Pr(\sigma &= 0|\omega, \psi, d = 0) = \frac{\Pr(\omega|\sigma = 0, \psi) \Pr(\sigma = 0|\psi, d = 0)}{num. + \Pr(\omega|\sigma = 1, \psi) \Pr(\sigma = 1|\psi, d = 0)} \\ &= \frac{1}{1 + \frac{\Pr(\omega|\sigma = 1, \psi) \Pr(\sigma = 1|\psi, d = 0)}{\Pr(\omega|\sigma = 0, \psi) \Pr(\sigma = 0|\psi, d = 0)}}, \end{aligned}$$

which is higher for $\omega = 0$ than for $\omega = 1$, or equal if $\Pr(\sigma = 1|\psi, d = 0) = 0$.

Obviously, instrumental utility only reinforces the no-deviation incentives. ■

Proof of Lemma 2. The statement of the lemma is equivalent to the statement that each advisor weakly prefers to report the state he considers weakly more likely. Then, the argument of the proof of the previous lemma applies, with s_i instead of σ, H instead of G, a_i instead of d, and ψ being the history of events prior to the advice stage, i.e., m^1.

■

Proofs for Section 3.4

Preliminaries

Throughout, we assume that the decision-maker takes the decision equal to the state that she considers strictly more likely (and follows her own signal if she considers both states equally likely), and that the advisors report their signals truthfully.

Vectors of advisors' signals

For any profile of advisors' truthfully reported signals s, let $o(s)$ denote the number of 0's in s. The decision after s is 1 if and only if $o(s) < j$ for some $j \leq n$ when $\sigma = 0$ and $o(s) < j'$ for some $j' \geq j$ when $\sigma = 1$. By A1, $j > 0$ and $j' \leq n$. Denote by S the set of all possible s. Let $\overline{S}$ be the set of s such that $j \leq o(s) < j'$ and $\widehat{S}$ its complement. In other words, $\overline{S}$ is the subset of S where, for any $s \in \overline{S}$, both signal-types take the decision equal to their own signal. In contrast, for any $s \in \widehat{S}$, both signal-types take the same decision, suggested by s. While $\overline{S}$ is empty when $j' = j$, $\widehat{S}$ is never empty.

For a profile s to belong to $\widehat{S}$, it must contain either enough 0's to let signal-type 1 believe that state 0 is more likely, or sufficiently many 1's (definitely more than $n/2$) to make signal-type 0 believe that state 1 is more likely. However, since $\omega = 0$ is weakly more likely a priori, the minimum number of 1's needed to "change the mind" of signal-type 0 is weakly higher than the minimum number of 0's needed to "change the mind" of signal-type 1. Therefore, the likelihood that s falls into $\widehat{S}$ should be weakly higher when $\omega = 0$.

To formalize this argument, consider first all profiles $s \in \widehat{S}$ such that $o(s) \leq n/2$. It must be that either $\Pr(\omega = 1|\sigma = 0, s) > 1/2$ (s contains so many 1's that signal-type 0 considers $\omega = 1$ more likely) or $\Pr(\omega = 0|\sigma = 1, s) > 1/2$ (despite $o(s) \leq n/2$, s contains enough 0's to let signal-type 1 still believe that $\omega = 0$ is more likely). Then, the profile $s' = \overrightarrow{1} - s$ with $o(s') = n - o(s)$ also belongs to $\widehat{S}$, because: (1) if $\Pr(\omega = 1|\sigma = 0, s) > 1/2$, then $\Pr(\omega = 0|\sigma = 1, s') > 1/2$ as well (s' contains as many 0's as s contains 1's, and $p \geq 1/2$), (2) if $\Pr(\omega = 0|\sigma = 1, s) > 1/2$, then $\Pr(\omega = 0|\sigma = 1, s') > 1/2$ (s' contains more 0's than s does).

Since all advisors are identical and, for every i, $\Pr(s_i = \omega|\omega)$ does not depend on ω, $\Pr(s|\omega = 1) = \Pr(s'|\omega = 0)$ and $\Pr(s|\omega = 0) = \Pr(s'|\omega = 1)$.

If there are any remaining profiles $s'' \in \widehat{S}$, they must have $o(s'') > n/2$, implying $\Pr(s''|\omega = 0) \geq \Pr(s''|\omega = 1)$. Thus, we conclude that

$$\Pr(\widehat{S}|\omega = 0) \geq \Pr(\widehat{S}|\omega = 1). \tag{2}$$

This formula will be used in the proof of Lemma 4.

Decision-maker's reputation at terminal nodes on-path

Fix a terminal history ξ on-path. Let

$$\gamma := \Pr(\sigma = \omega) = qg + (1 - q)b > 1/2.$$

Suppose first that, after observing ξ, the observer concludes that the decision-maker has definitely received a specific signal σ: $\Pr(\sigma|\xi) = 1$. Then, when state ω is observed, the reputation depends only on whether $\sigma = \omega$ or $\sigma \neq \omega$, i.e., one of these two values of reputation is realized:

$$\begin{aligned}\Pr(G|\xi,\omega) = \Pr(G|\sigma=\omega) = \frac{\Pr(\sigma=\omega|G)\Pr(G)}{\Pr(\sigma=\omega)} = \frac{gq}{\gamma} =: x;\\ \Pr(G|\xi,\omega) = \Pr(G|\sigma\neq\omega) = \frac{\Pr(\sigma\neq\omega|G)\Pr(G)}{\Pr(\sigma\neq\omega)} = \frac{(1-g)q}{1-\gamma} := y.\end{aligned} \tag{3}$$

It is straightforward to show that, since $1/2 \leq b < g$, we have $x > y$.

Suppose now that ξ does not necessarily reveal the signal-type perfectly. Specifically, suppose that either of these two cases is realized: (i) $\xi = (m^1, a, d)$ with $a = s \in \widehat{S}$ and d being the decision equal to the state that both signal-types consider strictly more likely, or (ii) $\xi = (m^0, d = 0)$ with $\Pr(m^0|\sigma) \neq 0$ for both σ and signal-type 1 considers state $\omega = 0$ strictly more likely. Then:

$$\Pr(\xi = (m^1, a, d)|\omega, \sigma) = \Pr(m^1|\sigma) \cdot \Pr(s|\omega) \cdot \Pr(d|\sigma, s, m^1) = \Pr(m^1|\sigma) \cdot \Pr(s|\omega),$$

$$\Pr(\xi = (m^0, d = 0)|\omega, \sigma) = \Pr(m^0|\sigma) \cdot \Pr(d = 0|\sigma, m^0) = \Pr(m^0|\sigma).$$

In the formulas above we have used the fact that m depends only on σ, $a = s$ and s depends only on ω, and d is deterministic given σ and s (when $m = m^1$) or just σ (when $m = m^0$).

So, the reputation of the decision-maker at ξ when state ω is observed is

$$\begin{aligned}\Pr(G|\xi,\omega) &= \Pr(G|\sigma=\omega)\Pr(\sigma=\omega|\xi) + \Pr(G|\sigma\neq\omega)\Pr(\sigma\neq\omega|\xi)\\ &= x\frac{\Pr(\xi|\sigma=\omega)\Pr(\sigma=\omega)}{numerator + \Pr(\xi|\sigma\neq\omega)\Pr(\sigma\neq\omega)} + y\frac{\Pr(\xi|\sigma\neq\omega)\Pr(\sigma\neq\omega)}{num. + \Pr(\xi|\sigma=\omega)\Pr(\sigma=\omega)}\\ &= x\frac{\Pr(m|\sigma=\omega)\cdot\gamma}{num. + \Pr(m|\sigma\neq\omega)\cdot(1-\gamma)} + y\frac{\Pr(m|\sigma\neq\omega)\cdot(1-\gamma)}{num. + \Pr(m|\sigma=\omega)\cdot(1-\gamma)} = \Pr(G|m,\omega).\end{aligned}$$

The formula is the same for cases (i) and (ii) because, after expressing $\Pr(\xi|\omega,\sigma)$ as

$\Pr(m|\sigma) \cdot \Pr(s|\omega)$ in case (i), $\Pr(s|\omega)$ cancels out.

Let $\nu = \Pr(m|\sigma = 0)/\Pr(m|\sigma = 1)$. From the formula above, we get:

$$\begin{aligned}
\Pr(G|m, \omega = 1) &= \frac{gq + \nu(1-g)q}{\gamma + \nu(1-\gamma)} =: v(\nu); \\
\Pr(G|m, \omega = 0) &= \frac{\nu gq + (1-g)q}{\nu\gamma + 1 - \gamma} =: w(\nu).
\end{aligned} \tag{4}$$

It is easy to observe that:

$$x = v(0) > w(0) = y;$$

$$x > v(\nu) > w(\nu) > y \qquad \text{for } \nu \in (0, 1);$$

$$x > v(1) = w(1) > y;$$

$$x > w(\nu) > v(\nu) > y \qquad \text{for } \nu > 1.$$

Moreover, for any $\nu > 0$,

$$v(\nu) + w(\nu) > x + y,$$

because

$$v(\nu) = x \cdot \Pr(\sigma = 1|m, \omega = 1) + y \cdot \Pr(\sigma = 0|m, \omega = 1),$$

$$w(\nu) = x \cdot \Pr(\sigma = 0|m, \omega = 0) + y \cdot \Pr(\sigma = 1|m, \omega = 0),$$

$x > y$, and

$$\Pr(\sigma = 1|m, \omega = 1) + \Pr(\sigma = 0|m, \omega = 0) > \Pr(\sigma = 0|m, \omega = 1) + \Pr(\sigma = 1|m, \omega = 0).$$

Proof of Lemma 3. From Bayes rule, we get:

$$\Pr(\omega = 0|\sigma = 0) = \frac{p\gamma}{p\gamma + (1-p)(1-\gamma)}.$$

For $\Pr(m|\sigma = 1) = 1$ and $m = m^1$, $\nu \equiv \Pr(m|\sigma = 0)/\Pr(m|\sigma = 1) = \Pr(m^1|\sigma = 0) \equiv \mu$. Let

$$C(\mu) := \Pr(\omega = 0|\sigma = 0) \cdot w(\mu) + \Pr(\omega = 1|\sigma = 0) \cdot v(\mu).$$

For each profile of advisors' truthfully reported signals s, let

$$A(s) := \Pr(s|\omega = 0)\Pr(\omega = 0|\sigma = 0)w + \Pr(s|\omega = 1)\Pr(\omega = 1|\sigma = 0)v;$$

$$B(s) := \Pr(s|\omega = 0)\Pr(\omega = 0|\sigma = 0)x + \Pr(s|\omega = 1)\Pr(\omega = 1|\sigma = 0)y.$$

First, we prove Part (i). The expected reputation of signal-type 0 after asking is:

$$\sum_{s\in\widehat{S}} A(s) + \sum_{s\in\overline{S}} B(s).$$

Since $\sum_{s\in\overline{S}} B(s)$ does not depend on μ, we can focus on $\sum_{s\in\widehat{S}} A(s)$. As shown in Preliminaries, $\widehat{S}$ can be partitioned into pairs s, s' with $o(s') = n - o(s)$ and unpaired vectors s'' with $o(s'') \geq n/2$. Thus, $\sum_{\widehat{s}\in\widehat{S}} A(\widehat{s})$ is increasing in μ when both $A(s) + A(s')$ for any such pair s, s' and $A(s'')$ for any such s'' are increasing in μ. This is what we show next.

Since $\Pr(s_i = \omega|\omega)$ depends neither on ω, nor on i, we have $\Pr(s'|\omega = 1) = \Pr(s|\omega = 0)$ and $\Pr(s|\omega = 1) = \Pr(s'|\omega = 0)$. Thus,

$$A(s) + A(s') = [\Pr(s|\omega = 0) + \Pr(s'|\omega = 0)] \cdot C(\mu). \tag{5}$$

Now we show that $C(\mu)$ is increasing in μ. Fix $\mu_0 < \mu_1$. For brevity, let $\overline{p} := 1-p$, $\overline{\gamma} := 1-\gamma$, $\overline{g} = 1-g$. We have

$$C(\mu_0) < C(\mu_1) \Leftrightarrow$$

$$\Pr(\omega=0|\sigma=0)\cdot w(\mu_0) + \Pr(\omega=1|\sigma=0)\cdot v(\mu_0) \tag{6}$$

$$< \Pr(\omega=0|\sigma=0)\cdot w(\mu_1) + \Pr(\omega=1|\sigma=0)\cdot v(\mu_1) \Leftrightarrow$$

$$\frac{p\gamma}{p\gamma+(1-p)(1-\gamma)}\left(\frac{\mu_0 gq+(1-g)q}{\mu_0\gamma+1-\gamma} - \frac{\mu_1 gq+(1-g)q}{\mu_1\gamma+1-\gamma}\right)$$

$$< \frac{(1-p)(1-\gamma)}{p\gamma+(1-p)(1-\gamma)}\left(\frac{gq+\mu_1(1-g)q}{\gamma+\mu_1(1-\gamma)} - \frac{gq+\mu_0(1-g)q}{\gamma+\mu_0(1-\gamma)}\right) \Leftrightarrow$$

$$p\gamma\frac{\mu_0 g\mu_1\gamma+\mu_0 g\overline{\gamma}+\overline{g}\mu_1\gamma+\overline{g}\overline{\gamma}-\mu_1 g\mu_0\gamma-\mu_1 g\overline{\gamma}-\overline{g}\mu_0\gamma-\overline{g}\overline{\gamma}}{\mu_0\mu_1\gamma^2+\mu_0\gamma\overline{\gamma}+\mu_1\gamma\overline{\gamma}+\overline{\gamma}^2}$$

$$< \overline{p\gamma}\frac{g\gamma+g\mu_0\overline{\gamma}+\mu_1\overline{g}\gamma+\mu_1\overline{g}\mu_0\overline{\gamma}-g\gamma-g\mu_1\overline{\gamma}-\mu_0\overline{g}\gamma-\mu_0\overline{g}\mu_1\overline{\gamma}}{\gamma^2+\mu_1\gamma\overline{\gamma}+\mu_0\gamma\overline{\gamma}+\mu_0\mu_1\overline{\gamma}^2} \Leftrightarrow$$

$$p\gamma\frac{(\mu_1-\mu_0)(\overline{g}\gamma-g\overline{\gamma})}{\mu_0\mu_1\gamma^2+\mu_0\gamma\overline{\gamma}+\mu_1\gamma\overline{\gamma}+\overline{\gamma}^2} < \overline{p\gamma}\frac{(\mu_1-\mu_0)(\overline{g}\gamma-g\overline{\gamma})}{\gamma^2+\mu_1\gamma\overline{\gamma}+\mu_0\gamma\overline{\gamma}+\mu_0\mu_1\overline{\gamma}^2} \Leftrightarrow$$

$$\frac{p\gamma}{\overline{p\gamma}} > \frac{\mu_0\mu_1\gamma^2+\overline{\gamma}^2+\mu_0\gamma\overline{\gamma}+\mu_1\gamma\overline{\gamma}}{\mu_0\mu_1\overline{\gamma}^2+\gamma^2+\mu_1\gamma\overline{\gamma}+\mu_0\gamma\overline{\gamma}}, \tag{7}$$

where the last implication uses $\mu_0 < \mu_1$ and $\overline{g}\gamma - g\overline{\gamma} = \gamma - g < 0$. The last inequality is always true for $\mu_0, \mu_1 \leq 1$ because then, by $\gamma^2 > \overline{\gamma}^2$, the RHS is smaller than 1, whereas the LHS is always bigger than 1.

Finally, whenever $C(\mu) = \Pr(\omega=0|\sigma=0)\cdot w(\mu) + \Pr(\omega=1|\sigma=0)\cdot v(\mu)$ increases with μ,

$$A(s'') = \Pr(s''|\omega=0)\Pr(\omega=0|\sigma=0)\cdot w(\mu) + \Pr(s''|\omega=1)\Pr(\omega=1|\sigma=0)\cdot v(\mu)$$

$$= \Pr(s''|\omega=1)C(\mu) + \Pr(\omega=0|\sigma=0)\cdot[\Pr(s''|\omega=0) - \Pr(s''|\omega=1)]\cdot w(\mu)$$

does too, because $w(\mu)$ increases with μ, and $\Pr(s''|\omega=0) \geq \Pr(s''|\omega=1)$ (recall that

$o(s'') \geq n/2)$.

For Part (ii), write the expected reputation of signal-type 0 after not asking as $\sum_{s \in \overline{S} \cup \widehat{S}} B(s)$. Given Part (i), the expected reputation of signal-type 0 after asking when signal-type 1 always asks is maximal for $\mu = 1$. So, the difference in expected reputation between not asking and asking for signal-type 0 is bounded below by

$$\sum_{s \in \overline{S} \cup \widehat{S}} B(s) - \left(\sum_{s \in \overline{S}} B(s) + \sum_{s \in \widehat{S}} A(s)|_{\mu=1}\right) = \sum_{s \in \widehat{S}} (B(s) - A(s)|_{\mu=1}).$$

Similarly to the proof of Part (i), we can use the fact that $\widehat{S}$ can be partitioned into pairs s, s' with $o(s') = n - o(s)$ and unpaired s'' with $o(s'') \geq n/2$. Then it is enough to show that

$$B(s) + B(s') > (A(s) + A(s'))|_{\mu=1};$$

$$B(s'') > A(s'')|_{\mu=1}.$$

for any such pair s, s' and any such s'' respectively.

By equation (5),

$$(A(s) + A(s'))|_{\mu=1} = [\Pr(s|\omega = 0) + \Pr(s'|\omega = 0)] \cdot C(1),$$

and, analogously to the derivation of (5), we can derive

$$B(s) + B(s') = [\Pr(s|\omega = 0) + \Pr(s'|\omega = 0)] \cdot [\Pr(\omega = 0|\sigma = 0) \cdot x + \Pr(\omega = 1|\sigma = 0) \cdot y].$$

Abstracting from the fact that μ is a probability, note that

$$[\Pr(\omega = 0|\sigma = 0) \cdot x + \Pr(\omega = 1|\sigma = 0) \cdot y] = \lim_{\mu \to \infty} C(\mu),$$

and thus $B(s) + B(s') > (A(s) + A(s'))|_{\mu=1}$ is equivalent to $\lim_{\mu \to \infty} C(\mu) > C(1)$. Using inequality (7) with $\mu_0 = 1$ and $\mu_1 = \infty$, we get

$$\frac{p\gamma}{\overline{p}\overline{\gamma}} > \frac{\gamma^2 + \gamma\overline{\gamma}}{\overline{\gamma}^2 + \gamma\overline{\gamma}} \Leftrightarrow \frac{p}{\overline{p}} > \frac{\gamma + \overline{\gamma}}{\overline{\gamma} + \gamma} = 1,$$

which is always true because $p > 1/2$ by the first part of A2.

Thus,

$$[\Pr(\omega = 0|\sigma = 0) \cdot x + \Pr(\omega = 1|\sigma = 0) \cdot y] > C(1) = q,$$

and together with $\Pr(s''|\omega = 0) \geq \Pr(s''|\omega = 1)$, and $x > q > y$,

$$B(s'') = \Pr(s''|\omega = 0) \Pr(\omega = 0|\sigma = 0) \cdot x + \Pr(s''|\omega = 1) \Pr(\omega = 1|\sigma = 0) \cdot y$$
$$> \Pr(s''|\omega = 0) \Pr(\omega = 0|\sigma = 0) \cdot q + \Pr(s''|\omega = 1) \Pr(\omega = 1|\sigma = 0) \cdot q = A(s'')|_{\mu=1}.$$

■

Proof of Lemma 4. Recall first that in Preliminaries we defined j and j' as the critical numbers of 0's in s such that the decision after s is 1 if and only if $o(s) < j$ when $\sigma = 0$ and $o(s) < j'$ when $\sigma = 1$.

Then, since signal-type 0 takes $d = 0$ after not asking, the difference in expected instrumental utility between asking and not asking for this signal-type is:

$$\Delta IU_0 := \sum_{s:o(s)<j} [\Pr(\omega = 1, s|\sigma = 0) - \Pr(\omega = 0, s|\sigma = 0)]$$

$$= \Pr(\omega = 1|\sigma = 0) \sum_{s:o(s)<j} \Pr(s|\omega = 1) - \Pr(\omega = 0|\sigma = 0) \sum_{s:o(s)<j} \Pr(s|\omega = 0).$$

Analogously, for signal-type 1, if $\Pr(\omega = 0|\sigma = 1) > 1/2$ it is:

$$\Delta IU_1 := \sum_{s:o(s)<j'} [\Pr(\omega = 1, s|\sigma = 1) - \Pr(\omega = 0, s|\sigma = 1)]$$

$$\geq \sum_{s:o(s)<j} [\Pr(\omega = 1, s|\sigma = 1) - \Pr(\omega = 0, s|\sigma = 1)]$$

$$= \Pr(\omega = 1|\sigma = 1) \sum_{s:o(s)<j} \Pr(s|\omega = 1) - \Pr(\omega = 0|\sigma = 1) \sum_{s:o(s)<j} \Pr(s|\omega = 0)$$

where the inequality holds because $j' \geq j$ and, for every s with $o(s) < j'$,

$$\Pr(\omega = 1, s|\sigma = 1) - \Pr(\omega = 0, s|\sigma = 1)$$

$$= [\Pr(\omega = 1|s, \sigma = 1) - \Pr(\omega = 0|s, \sigma = 1)] \cdot \Pr(s|\sigma = 1) \geq 0. \tag{8}$$

If $\Pr(\omega = 1|\sigma = 1) \geq 1/2$, it is:

$$\Delta IU_1' := \sum_{s:o(s)\geq j'} [\Pr(\omega = 0, s|\sigma = 1) - \Pr(\omega = 1, s|\sigma = 1)]$$

$$\geq \sum_{s:o(s)\geq n-j+1} [\Pr(\omega = 0, s|\sigma = 1) - \Pr(\omega = 1, s|\sigma = 1)]$$

$$= \Pr(\omega = 0|\sigma = 1) \sum_{s:o(s)>n-j} \Pr(s|\omega = 0) - \Pr(\omega = 1|\sigma = 1) \sum_{s:o(s)>n-j} \Pr(s|\omega = 1),$$

where the inequality holds because $j' \leq n - j + 1$ (due to $p \geq 1/2$) and, for every s with

$o(s) \geq j'$,

$$\Pr(\omega = 0, s|\sigma = 1) - \Pr(\omega = 1, s|\sigma = 1)$$

$$= [\Pr(\omega = 0|s, \sigma = 1) - \Pr(\omega = 1|s, \sigma = 1)] \cdot \Pr(s|\sigma = 1) > 0.$$

It follows immediately from $\Pr(\omega = 0|\sigma = 0) > \Pr(\omega = 0|\sigma = 1)$ that ΔIU_1 is bigger than ΔIU_0. Note furthermore that since $\Pr(s_i = \omega|\omega)$ depends neither on ω, nor on i, we have:

$$\sum_{s:o(s)<j} \Pr(s|\omega = 1) = \sum_{s:o(s)>n-j} \Pr(s|\omega = 0).$$

Then it follows immediately from $\Pr(\omega = 0|\sigma = 0) \geq \Pr(\omega = 1|\sigma = 1)$ that $\Delta IU_1'$ is weakly bigger than ΔIU_0.

When signal-type 1 always asks (Condition 2 of the lemma), the difference in expected reputation between asking and not asking for signal type 0 is:

$$\Delta R_0 := \sum_{s \in \widehat{S}} [\Pr(\omega = 0, s|\sigma = 0)(w - x) + \Pr(\omega = 1, s|\sigma = 0)(v - y)]$$

$$+ \sum_{s \in \overline{S}} [\Pr(\omega = 0, s|\sigma = 0)(x - x) + (\Pr(\omega = 1, s|\sigma = 0)(y - y)].$$

For signal-type 1, if $\Pr(\omega = 0|\sigma = 1) > 1/2$ it is:

$$\Delta R_1 := \sum_{s \in \widehat{S}} [\Pr(\omega = 0, s|\sigma = 1)(w - x) + \Pr(\omega = 1, s|\sigma = 1)(v - y)]$$

$$+ \sum_{s \in \overline{S}} [\Pr(\omega = 0, s|\sigma = 1)(y - x) + \Pr(\omega = 1, s|\sigma = 1)(x - y)],$$

and if $\Pr(\omega = 1|\sigma = 1) \geq 1/2$ it is:

$$\Delta R_1' := \sum_{s\in\widehat{S}}[\Pr(\omega = 0, s|\sigma = 1)(w - y) + \Pr(\omega = 1, s|\sigma = 1)(v - x)]$$
$$+ \sum_{s\in\overline{S}}[\Pr(\omega = 0, s|\sigma = 1)(y - y) + \Pr(\omega = 1, s|\sigma = 1)(x - x)].$$

The second terms of ΔR_0 and $\Delta R_1'$ are zero, whereas the second term of ΔR_1 is non negative because for every $s \in \overline{S}$, (8) holds and $x > y$. The first term of ΔR_1 is strictly bigger than the first term of ΔR_0 because $w - x < 0$, $v - y > 0$, $\widehat{S} \neq \varnothing$ (by A1), and

$$\Pr(\omega = 0, s|\sigma = 0) = \Pr(s|\omega = 0) \cdot \Pr(\omega = 0|\sigma = 0)$$
$$> \Pr(s|\omega = 0) \cdot \Pr(\omega = 0|\sigma = 1) = \Pr(\omega = 0, s|\sigma = 1).$$

So, if $\Pr(\omega = 0|\sigma = 1) > 1/2$, signal-type 1 strictly prefers to ask, given that signal-type 0 weakly prefers to ask (Condition 3 of the lemma).

If $\Pr(\omega = 1|\sigma = 1) \geq 1/2$, suppose by contraposition that signal-type 1 weakly prefers not to ask. Then, since by A1 $\Delta IU_1'$ is positive, $\Delta R_1'$ must be negative. Then, since (as we have shown in Preliminaries) $w - y > x - v$, it must be that

$$\Pr(\{\omega = 0\} \times \widehat{S}|\sigma = 1) < \Pr(\{\omega = 1\} \times \widehat{S}|\sigma = 1). \tag{9}$$

Rewrite the first term of $\Delta R_1'$ as:

$$(w - x) \sum_{s\in\widehat{S}} \Pr(\omega = 1, s|\sigma = 1) + (v - y) \sum_{s\in\widehat{S}} \Pr(\omega = 0, s|\sigma = 1)$$
$$+(v - w) \sum_{s\in\widehat{S}} \Pr(\omega = 1, s|\sigma = 1) + (w - v) \sum_{s\in\widehat{S}} \Pr(\omega = 0, s|\sigma = 1),$$

Due to Condition 2 of the Lemma, $\mu < 1$, implying $v > w$. Hence, together with inequality (9), we obtain that the second line is positive. The first line is weakly bigger than ΔR_0, because $w - x < 0$, $v - y > 0$, and, by $\Pr(\omega = 0|\sigma = 0) \geq \Pr(\omega = 1|\sigma = 1)$ and inequality (2) from Preliminaries,

$$\sum_{s \in \widehat{S}} \Pr(\omega = 1, s|\sigma = 1) = \Pr(\widehat{S}|\omega = 1)\Pr(\omega = 1|\sigma = 1)$$

$$\leq \Pr(\widehat{S}|\omega = 0)\Pr(\omega = 0|\sigma = 0) = \sum_{s \in \widehat{S}} \Pr(\omega = 0, s|\sigma = 0);$$

$$\sum_{s \in \widehat{S}} \Pr(\omega = 0, s|\sigma = 1) = \Pr(\widehat{S}|\omega = 0)\Pr(\omega = 0|\sigma = 1)$$

$$\geq \Pr(\widehat{S}|\omega = 1)\Pr(\omega = 1|\sigma = 0) = \sum_{s \in \widehat{S}} \Pr(\omega = 1, s|\sigma = 0).$$

So, $\Delta R'_1 > \Delta R_0$ and (as we have shown above) $\Delta IU'_1 \geq \Delta IU_0$. Thus signal-type 0 strictly prefers not to ask, contradicting Condition 3 of the Lemma. ■

Proof of Theorem 1. Take a candidate separating equilibrium in which signal-type 1 always asks and signal-type 0 never asks. It is easy to observe that the difference in expected reputation between asking and not asking is negative for signal-type 0 and, if $Pr(\omega = 1|\sigma = 1) < 1/2$, signal-type 1,[16] whereas it is zero for signal-type 1 when $Pr(\omega = 1|\sigma = 1) \geq 1/2$.[17] By truthful reporting after asking and A1, the difference in expected instrumental payoff between asking and not asking is positive for both signal-types.[18] Hence, the difference in expected payoff between asking and not asking is strictly decreasing in ρ for both signal-types. For $\rho = 0$, both signal-types strictly prefer to ask.

[16] The decision-maker prefers to be perceived as the signal-type that corresponds to the state that she considers more likely rather than as the opposite signal-type. For the formalization of this argument, see the proof of Lemma 1.

[17] If $Pr(\omega = 1|\sigma = 1) \geq 1/2$, after not asking signal-type 1 decides 1, so she is perceived as signal-type 1, just like after asking.

[18] This is because, by A1, advisors' information is decision-relevant with a positive probability. See the proof of Lemma 4 for the formal argument.

For $\rho = 1$, signal-type 0 strictly prefers not to ask and signal-type 1 strictly prefers not to ask when $Pr(\omega = 1|\sigma = 1) < 1/2$ and is indifferent when $Pr(\omega = 1|\sigma = 1) \geq 1/2$. Thus, each signal-type is indifferent in the candidate separating equilibrium only for one value of ρ. Let $\underline{\rho}$ be the value at which signal-type 0 is indifferent and let $\overline{\rho}$ be the value at which signal-type 1 is indifferent. By Lemma 4, at $\underline{\rho}$ signal-type 1 strictly prefers to ask. Thus $\overline{\rho} > \underline{\rho}$ ($\overline{\rho} = 1$ when $Pr(\omega = 1|\sigma = 1) \geq 1/2$) and at $\overline{\rho}$ signal-type 0 strictly prefers not to ask. Therefore, the separating equilibrium exists if and only if $\rho \in [\underline{\rho}, \overline{\rho}]$.

Consider now the partially separating equilibrium. For $\rho < \underline{\rho}$, no such equilibrium can exist: Since signal-type 0 strictly prefers to ask when $\mu = 0$, by Lemma 3 (part (i)) she strictly prefers to ask also when $\mu > 0$. For $\rho = 1$, by Lemma 3 (part (ii)) (and by continuity for the case $p = 1/2$), signal-type 0 weakly prefers not to ask for any value of μ. For a fixed μ, the expected payoff after asking or not asking is the convex combination of two constant terms (expected reputation and expected instrumental utility) with weights ρ and $(1 - \rho)$. Hence, the observations above about $\rho < \underline{\rho}$ and $\rho = 1$ imply that the difference in expected payoff between asking and not asking is strictly decreasing in ρ for signal-type 0. Therefore, for any μ, there must be a unique value of $\rho \in [\underline{\rho}, 1]$ such that signal-type 0 is indifferent between asking and not asking. For $\mu = 0$ such value of ρ is obviously $\underline{\rho}$. Furthermore, this value must be strictly increasing in μ. This is because, by Lemma 3 (part (i)), expected reputation of signal-type 0 after asking is increasing in μ. Hence, a higher μ requires a higher ρ to keep signal-type 0 indifferent. Let $\widehat{\rho}$ be such value for $\mu = \overline{\mu}$, i.e., the maximum value of μ compatible with truthful reporting by the advisors.

Thus, given that signal-type 1 always asks for advice, the range of ρ for which signal-type 0 is indifferent between asking and not asking for some μ is $[\underline{\rho}, \widehat{\rho}]$. By Lemma 4, whenever signal-type 0 is indifferent, signal-type 1 strictly prefers to ask if $\mu < 1$, and, by continuity, weakly prefers to ask if $\mu = 1$. Thus she will not deviate. Therefore, the partially separating equilibrium exists if and only if $\rho \in [\underline{\rho}, \widehat{\rho}]$. ■

Proof of Proposition 1. In order to prove the proposition, we need the following Lemma, which is a generalization of Lemma 3, part (i), from the main text, to allow signal-type 1 to ask for advice with probability less than 1.

Lemma 5 *The expected reputation of signal-type* 0 *conditional on a given* $m \in \{m^0, m^1\}$ *(i.e. conditional on not asking or asking) is strictly increasing in* $\nu := \Pr(m|\sigma = 0)/\Pr(m|\sigma = 1)$ *for* $m = m^1$ *and* $\nu \leq 1$*, and also for* $m = m^0$ *and any* ν *if* $p > qg + (1-q)b$*.*

Proof. For $m = m^1$ and $\nu \leq 1$, the proof is identical to the proof of Lemma 3, part (i), from the main Appendix, with μ substituted by ν, because the latter proof just relied on $\mu \leq 1$ but not on signal-type 1 always asking.

Let us consider $m = m^0$ and $p > qg + (1-q)b \equiv \gamma$. For $p > \gamma$, both signal-types take $d = 0$ after not asking. Therefore, expression $C(\nu) \equiv \Pr(\omega = 0|\sigma = 0) \cdot w(\nu) + \Pr(\omega = 1|\sigma = 0) \cdot v(\nu)$ represents the expected reputation of signal-type 0 after not asking. Thus, we need to show that $C(\nu)$ increases with ν for any ν.

Replace μ with ν in inequality (7) from the proof of Lemma 3. The inequality will then correspond to the condition $C(\nu_0) < C(\nu_1)$. In the proof of Lemma 3, we have shown that the condition holds for $\nu_0, \nu_1 < 1$. Furthermore, when $p > \gamma$, the condition is satisfied also for $\nu_1 > \nu_0 > 1$. This is because $p > \gamma > 1/2$ implies

$$\frac{p\gamma}{\overline{p}\overline{\gamma}} > \frac{\gamma^2}{\overline{\gamma}^2} = \frac{\nu_0\nu_1\gamma^2}{\nu_0\nu_1\overline{\gamma}^2} > \frac{\nu_0\nu_1\gamma^2 + \nu_0\gamma\overline{\gamma} + \nu_1\gamma\overline{\gamma} + \overline{\gamma}^2}{\nu_0\nu_1\overline{\gamma}^2 + \nu_0\gamma\overline{\gamma} + \nu_1\gamma\overline{\gamma} + \gamma^2}.$$

Thus, when $p > \gamma$, $C(\nu)$ increases with ν for all ν. ■

Now we are ready to prove Proposition 1. Fix $\rho < \underline{\rho}$. Consider a hypothetical asking policy that induces truthful reporting. Due to A2, a necessary condition for truthtelling by the advisors is that signal-type 0 asks less frequently than signal-type 1, that is, $\Pr(m^1|\sigma = 0)/\Pr(m^1|\sigma = 1) < 1$. Consider the case of perfect separation: $\Pr(m^0|\sigma = 0) = \Pr(m^1|\sigma = 1) = 1$. In this case, by Lemma 5, the expected reputation of signal-type 0 from asking is the lowest possible ($\nu = \Pr(m^1|\sigma = 0)/\Pr(m^1|\sigma = 1) = 0$), while her expected reputation from not asking is the highest possible if $p > qg + (1 - q)$ ($\nu = \Pr(m^0|\sigma = 0)/\Pr(m^0|\sigma = 1) = \infty$). If $p \leq qg + (1 - q)$, the expected reputation of signal-type 0 from not asking does not depend on the asking policy: $p \leq qg + (1 - q)$ means $\Pr(\omega = 1|\sigma = 1) \geq 1/2$, thus each signal type would be perfectly revealed through her decision after not asking. Hence, the expected reputational loss from asking for signal-type 0 is the highest possible under perfect separation. By Theorem 1, there is no separating equilibrium, because signal-type 0 would strictly prefer to ask. A fortiori, any other asking policy where signal-type 0 does not always ask cannot be sustained in equilibrium, because signal-type 0 would strictly prefers to ask. ■

Proof of Proposition 2. That the second best can be reached only at $\rho = \widehat{\rho}$ follows from the analysis of the partially separating equilibrium. We now formalize the argument provided in Section 3.4.2 for the comparison between the separating equilibrium and any equilibrium at low levels of reputation concerns.

By Proposition 1 there is no equilibrium with locally optimal behavior below $\underline{\rho}$. Then, the claim is automatically true if there does not exist a decreasing sequence of levels of reputation concerns $(\rho^k)_{k\in\mathbb{N}}$ that converges to 0 and a corresponding sequence of equilibria where signal-type 0 sometimes asks for advice and still (1) is satisfied; so suppose there is one. Thus, in these equilibria, signal-type 0 is indifferent between asking and not asking. Then, the instrumental gain from asking for signal-type 0 must converge to 0, since the weight ρ^k on reputation vanishes. By A1, signal-type 0 has a strictly positive instrumental benefit from asking if all advisors report truthfully signal 1. Fix $\varepsilon > 0$ such that the expected instrumental gain for signal-type 0 from deciding 1 instead of 0 when all the advisors receive signal 1 is higher than ε. For each $k > 0$, in the equilibrium with index k, assume without loss of generality that the message space of the advisors is discrete, and for each $i = 1, ..., n$, let

$$\begin{aligned} A^0_{i,k} \quad &: \quad = \left\{a_i : \Pr(a_i|s_i = 0, m^1) > \Pr(a_i|s_i = 1, m^1)\right\}; \\ A^1_{i,k} \quad &: \quad = \left\{a_i : \Pr(a_i|s_i = 1, m^1) \geq \Pr(a_i|s_i = 0, m^1)\right\}; \\ \gamma^k_i \quad &: \quad = \textstyle\sum_{a_i} \min_{s_i} \Pr(a_i|s_i, m^1) = \sum_{a_i \in A^0_{i,k}} \Pr(a_i|s_i = 1, m^1) + \sum_{a_i \in A^1_{i,k}} \Pr(a_i|s_i = 0, m^1) \end{aligned}$$

Each γ^k_i is a measure of "pooling" in the reporting strategy of advisor i — if i reports truthfully, γ^k_i is zero. Let $\gamma^k := \max_{i=1,...,n} \gamma^k_i$. So, γ^k is the γ^k_i of the "least informative" advisor i. Suppose that the sequence $(\gamma^k)_{k \in \mathbb{N}}$ has a subsequence that converges to 0. Then, for each $\delta > 0$, in every equilibrium of the subsequence with sufficiently high k, we have $\gamma^k < \delta$. For each advisor i, this has two implications. First, under $s_i = 1$, advisor i sends a report in $A^1_{i,k}$ with probability at least $1 - \delta$. Second, the reports in $A^1_{i,k}$ are sent under $s_i = 0$ with probability lower than δ. Then, for sufficiently small δ and sufficiently high k, when signal-type 0 of the decision-maker receives a report in $A^1_{i,k}$ from every advisor i it is strictly dominant to decide 1, and the expected instrumental gain from asking exceeds ε. This contradicts that the expected instrumental gain from asking converges to 0. So, there is no subsequence of $(\gamma^k)_{k \in \mathbb{N}}$ that converges to 0. Hence, there exist $\gamma > 0$ and $k' > 0$ such that, for each $k > k'$, $\gamma^k > \gamma$.

Now, suppose that there is $j \in \{1, ...n\}$ such that

$$\Pr(\omega = 0|\sigma = 1, s') < \frac{1}{2} < \Pr(\omega = 0|\sigma = 1, s)$$

for each s, s' with $o(s) = j$ and $o(s') = j - 1$ (j generically exists under A2). Let

$$\eta := \min \{\Pr(\omega = 0|\sigma = 1, s) - \Pr(\omega = 1|\sigma = 1, s), \Pr(\omega = 1|\sigma = 1, s') - \Pr(\omega = 0|\sigma = 1, s')\} > 0,$$

be the minimum between the expected instrumental gain from deciding 0 instead of 1 given s, and vice versa given s'. Fix $k > k'$ and let $\iota := \arg\max_i \gamma_i^k$. Fix s, s' with $o(s) = j$ and $o(s') = j - 1$ that differ only for the signal of advisor ι. Suppose that the other advisors report their signals truthfully and that the decision-maker takes the decision that corresponds to the state that she considers more likely; the argument will be true a fortiori if these assumptions are violated. Thus, when the true signal profile is s or s', the decision d of signal-type 1 will depend on the report a_ι of advisor ι. In particular, the decision will be 1 if and only if $\Pr(a_\iota|s_\iota = 1, m^1)/\Pr(a_\iota|s_\iota = 0, m^1) \geq t$ for some $t > 0$. Let

$$\begin{aligned}
\overline{\gamma}^0_{\iota,k} \quad &: \quad = \textstyle\sum_{a_i:\Pr(a_\iota|s_\iota=1,m^1)/\Pr(a_\iota|s_\iota=0,m^1)\geq t} \Pr(a_\iota|s_\iota = 0, m^1), \\
\overline{\gamma}^1_{\iota,k} \quad &: \quad = \textstyle\sum_{a_i:\Pr(a_\iota|s_\iota=1,m^1)/\Pr(a_\iota|s_\iota=0,m^1)<t} \Pr(a_\iota|s_\iota = 1, m^1);
\end{aligned}$$

$\overline{\gamma}^{s_\iota}_{\iota,k}$ is the probability that advisor ι induces signal-type 1 to take the suboptimal (in expected terms) decision given s_ι. Note that $\overline{\gamma}^0_{\iota,k} + \overline{\gamma}^1_{\iota,k} = \gamma^k_\iota \equiv \gamma^k$ if $t = 1$, and $\overline{\gamma}^0_{\iota,k} + \overline{\gamma}^1_{\iota,k} \geq \gamma^k$. Thus $\max\left\{\overline{\gamma}^0_{\iota,k}, \overline{\gamma}^1_{\iota,k}\right\} > \gamma/2$. But then, either conditional on s, or conditional on s', with probability at least $\gamma/2$ the decision will be suboptimal. This entails a discrete loss of ex-ante expected instrumental utility for signal-type 1 which is bounded below by

$$\eta \cdot \frac{\gamma}{2} \cdot \min\left\{\Pr(s,\omega=1),\Pr(s',\omega=0)\right\}.$$

This lower bound applies to every $k > k'$ because even if advisor ι changes, the probabilities of the new s and s' do not change, since they contain the same number of zeros. Coupled with the fact that the instrumental gain from asking for signal-type 0 converges to 0, this means that there is $\overline{k} > k'$, such that the ex-ante quality of decisions in equilibrium in the interval $\left[0,\overline{k}\right]$ is strictly lower than in the separating equilibrium. ■

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