Independent versus Collective Expertise*

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Abstract

We consider the problem of a decision-maker who seeks for advice from several experts. The experts have reputation concerns which generate incentives to herd on the prior belief about the state of the world. We address the following question: Should the experts be allowed to exchange their information before providing advice ("collective expertise") or not ("independent expertise")? Allowing for such information exchange modifies the herding incentives in a non-trivial way. The effect is beneficial for the quality of advice when there is low prior uncertainty about the state and detrimental in the opposite case. We also show that independent expertise is more likely to be optimal when the decision-maker has a valuable enough "safe" option with a state-independent payoff. Finally, collective expertise is more likely to be optimal as the number of experts grows.

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1 Introduction

Decision-makers routinely rely on expert advice, and often there are multiple experts available. In this paper we address the following question: Should experts be given the opportunity to share their information before talking to the decision-maker? A peer review process in academic journals is a typical example where experts (referees) cannot talk to each other (we call it "independent expertise"), as they are simply not aware of each other's identity. At the other extreme, a CEO openly asking her colleagues for advice on the firm's strategy would naturally induce (some) information sharing between them before they deliver their advice at the next meeting (we call it "collective expertise").

In such and many other examples, experts care about their reputation for being considered smart. These reputation concerns are the key friction in our paper. As argued in a series of papers by Ottaviani and Sørensen (2001, 2006a, 2006b), reputation concerns can make advisors herd on the prior belief, and, consequently, lead to loss of information for the decision-maker.

We show that, due to aggregation of information *prior* to advice, collective expertise is better at predicting *which* state of the world is more likely. However, it fails to provide the decision-maker with the information on individual signals of experts, which is valuable when it is also important to know *how* likely the more likely state is.

We consider a model with two states of the world, 0 and 1, and three potential actions to choose from: 0, 1 and a *safe* action. Action i is optimal in state i. The safe action has a state-independent payoff which, in any state, is below the payoff from the optimal action in that state. It can be interpreted as the option to wait until the realization of uncertainty (which involves a cost of delay), costly investment in learning the state now, or implementing a safe project with low return. We assume that the state of nature is revealed at the end of the game regardless of the action chosen.

This payoff structure yields the following optimal decision rule: take the action corresponding to the more likely state if you are sufficiently confident about the state, otherwise take the safe action. Thus, for the decision-maker, it is important to learn not only which state is more likely, but also how likely it is, given the experts' information.

The experts receive informative *non-verifiable* signals about the state. The informativeness of an expert's signal depends on his/her ability, which is unknown to anyone, including him/herself. The objective of each expert is to maximize the decision-maker's posterior belief about his/her *absolute* ability (i.e., the experts do not care about their relative standing in the eyes of the decision-maker). In the baseline model we have two experts with the same expected ability.

We compare two communication schemes. Under "independent expertise", each experts sends a report to the decision-maker without knowing anything about the other expert's signal. Under "collective expertise", the experts share their signals before submitting a joint report. Regardless of the communication scheme, all reports (including reports between the experts) are non-contractible "cheap talk" messages.

The potential benefit of signal-sharing between the experts is alleviation of the herding-on-the-prior incentives. This effect materializes when each expert's signal is weaker than the prior (so that herd behavior results under independent expertise), but the combination of two signals contradicting the prior generates sufficiently strong evidence against the prior.

The potential cost of signal-sharing is that it aggravates herding incentives when the experts receive opposite signals. Two opposite signals of the same strength just leave the experts' beliefs at the prior, which implies that they will herd on the prior *regardless* of its strength. In fact, we show that a fully revealing equilibrium never exists under collective expertise, and the partially informative equilibrium that exists for the widest range of priors has the following structure: When both experts have received signals countering the prior, this fact is revealed; all other vectors of signals are pooled.

Therefore, the main conclusion of our model is that collective expertise is better than independent expertise when there is sufficiently low prior uncertainty about the state, whereas independent expertise dominates for a sufficiently high prior uncertainty.

We further show that collective expertise is more likely to be preferred when the value of the *safe* action is lower. The intuition is as follows. The advantage of independent expertise is that, conditional on truthful reporting, it provides the decision-maker with the most accurate information about the likelihood of each state, given the experts' information. However, when the safe action has a sufficiently low value, this accuracy is of no use, because the safe action is never taken anyway. In such a case, rough information on just what state is more likely becomes sufficient to take an optimal decision, and collective expertise achieves this for a wider range of priors.

Finally, we turn to the following question: What happens if we enlarge the number of experts? Under independent expertise, the incentives of each expert are unaffected by their number. Thus, when the prior is stronger than a single expert's signal, all experts herd and having more experts is of no use. When the prior is weaker than an expert's signal, the experts tell the truth and, thus, having more experts results in more information.

Collective expertise, in contrast, becomes more informative both for strong and weak priors. First, a higher quantity of signals is capable of generating stronger beliefs. This attenuates herding on the prior and, thus, allows partially informative communication for higher values of the prior. Second, any loss of information that arises due to partial pooling of vectors of signals is less important, because aggregation of signals gives a more precise information about the state as the number of experts grow.

To be specific, we first show that, in any equilibrium, the experts' messages partition the set of all signal profiles into *at most* two ordered subsets, according to the total number of zeros the experts receive. In particular, there is always an equilibrium in which the experts truthfully communicate one of two messages: "we received at least l zeros" and "we received at most l zeros" (with a possible randomization for l zeros). Now, if we increase the number of experts, these messages become more informative about the state. As an extreme example, consider an infinite number of experts. By learning each other's signals, the experts will simply learn the state (by the law of large numbers) and just report whether they received more zeros or more ones. Each of the two messages will perfectly inform the decision-maker about the state, regardless of the prior.

Thus, if, for some reason we cannot condition the choice of the expertise scheme on the prior or/and the ex-ante quality of experts, the conclusion is that, as the number of experts grows, collective expertise becomes more likely to be optimal.

Our paper joins the literature that explores how information aggregation and decision-making can be improved in the presence of reputation concerns. Ottaviani and Sørensen (2001) examine the role of the order of speech in a public debate among reputation-concerned experts. Prat (2005) studies the effects of transparency of decisions on the actions of a reputation-concerned decision-maker (Levy (2007) addresses a similar question in a committee setting). Catonini and Stepanov (2019) show how the decision-maker can improve extraction of information from reputation-concerned experts by asking for advice only in certain circumstances.

This paper looks at how the adverse effects of reputation concerns can be alleviated by the optimal organization of expertise. In this sense, it is close to the work by Ottaviani and Sørensen (2001). The crucial distinction of our work from Ottaviani and Sørensen (2001) is that in our study, under collective expertise, the experts exchange their information *privately*, whereas in the latter paper they speak sequentially and *publicly*.[1] The impossibility of private information sharing changes the advisors' incentives fundamentally. The first speaker is forced to report *publicly* without knowing the information of subsequent speakers. A subsequent speaker's incentives are affected by earlier speakers' reports, but, differently from our paper, her reporting is constrained by the infeasibility to hide earlier speakers' reports. Put differently, in Ottaviani and Sørensen (2001), advisors can-

[1] Bag and Sharma (2019) also consider sequential advice. In their setup, the experts care about their reputation in eyes of the public, but not of the decision-maker. The latter observes the experts' deliberation and takes a publicly observable decision. The paper examines whether the experts' deliberation should be hidden from the public or not.

not coordinate their reporting behavior based on their joint information, whereas in our paper they can.

There are works on eliciting information from multiple advisors in a Crawford and Sobel (1982) type of setting (e.g., Gilligan and Krehbiel (1989), Krishna and Morgan (2001a,b). Battaglini (2002), Ambrus and Takahashi (2008), McGee and Yang (2013), Wolinsky (2002)). Due to a different nature of communication distortions, this literature is orthogonal to the "reputational cheap talk" literature. Moreover, most of this literature does not address the central question of our work: Should experts be allowed to talk to each other before reporting to the decision-maker?[2]

The only exception, to our knowledge, is Wolinsky (2002).[3] Wolinsky considers the problem of a decision-maker who wants to aggregate decision-relevant information that is disseminated among a number of experts. The decision is binary, and so each expert's piece of information (0 or 1). The experts care about the decision, and both for the experts and for the decision-maker the preferred decision depends on the sum of the experts' pieces of information. However, the experts are biased: For some values of this sum, their preferred decision is 0, while the decision-maker's is 1. Because of this, if the decision-maker asks each individual expert to reveal his piece of information, the expert will focus on the case when his advice is pivotal and will pretend that his information is 0 also when it is 1 (1 is verifiable but 0 is not). If instead subgroups of experts share their information before providing advice, informative equilibria arise: A subgroup of experts with many 1's will suggest to the decision-maker to take decision 1, because the increased weight of their advice on the final decision makes it pivotal also in situations where the experts prefer decision 1.

All in all, the information structure, the nature of distortions in communi-

[2]Although some of these models compare sequential and simultaneous communication, see Hori (2009), Li (2010), Li, Rantakari, and Yang (2016).

[3]Rather than studying ex-post information-sharing, Elliott, Golub, and Kirilenko (2014) consider sharing *technologies* for generating recommendations to the decision-maker in a setup where two experts have different attitude to type I versus type II errors. The authors show that allowing for such sharing can harm the decision-maker, because the resulting expansion in the sets of technologies available to each expert may make the experts switch to suboptimal choices of recommendation-generating procedures from the decision-maker's perspective.

cation, and, most importantly, the channel through which information sharing among experts improves the informativeness of communication all differ with respect to our work. In our model, the beliefs about the state are the key determinant of the effect of reputation concerns on the experts' reporting behavior, and information sharing acts through changing these beliefs. In contrast, there are no reputation concerns in Wolinsky's paper, and merging experts in teams acts through changing the "pivotality" of experts: It helps them to coordinate on disclosing a critical mass of information that is sufficiently influential to be willingly (but coarsely) transmitted to the decision-maker.

Finally, there are works on deliberation in committees (see Austen-Smith and Feddersen (2009) for a survey). These papers however do not examine whether committee members *should* be allowed to share their information before voting or not.[4] Instead, they are focused on distortions (in both information sharing and voting outcomes) created by divergence of preferences, reputation concerns and strategic voting considerations, and how such distortions can be alleviated through the design of optimal voting rules (Coughlan (2000), Austen-Smith and Feddersen (2005, 2006), Visser and Swank (2007), Gerardi and Yariv (2007)) deliberation rules (Van Weelden (2008)) and transparency regulations (e.g., Meade and Stasavage (2008), Swank and Visser (2013), Fehrler and Hughes (2018))

The rest of the paper is organized as follows. Section 2 sets up a model with two homogeneous experts. Sections 3 and 4 analyze independent and collective expertise, respectively, in this setup. Section 5 deals with the welfare analysis. Section 6 studies the case of more than two experts. Section 7 concludes. The Appendix contains the proofs omitted from the main text. We also provide an online appendix in which we show that introducing heterogeneity between the experts does not change the insights of the model, and argue that our solution under collective expertise is robust to the communication protocol.

[4]An exception is Ali and Bohren (2019). In their setup, committee members' losses from type I and type II errors are different from those of the principal designing a committee. The authors show that banning deliberation can benefit the principal if she can choose the equilibrium the committee members play at the voting stage or if she can use non-monotone or non-anonymous social choice rules.

2 Model with two experts

A decision-maker chooses an action from a set consisting of three elements: $a \in \{0, s, 1\}$. Her payoff from the action depends on the unknown state of nature $\omega \in \{0, 1\}$ in the following way:

$$u_D(a, \omega) = \begin{cases} 1, \text{ if } a = \omega; \\ 0, \text{ if } a = 1 - \omega; \\ k \in (0, 1), \text{ if } a = s, \ \forall \omega \end{cases}$$

That is, the decision-maker wants to match her action to the state. In addition, she has a *safe* option, s, with a state-independent payoff which is higher than the payoff from a wrong action but lower than that from the optimal action in any given state. Depending on the real life applications, the safe action can be interpreted as the option to wait until the realization of uncertainty (which involves a cost of delay), costly investment in learning the state immediately, or implementing a safe project with a low return.

Before taking her decision, the decision-maker can consult two experts. The experts are ex-ante identical, and each of them can be of two types, Good and Bad with commonly known prior probability $\Pr(t_i = G) = q \in (0, 1)$, $\forall i \in \{1, 2\}$.[5] The experts' types are uncorrelated and unknown to anyone, including the experts themselves. Each expert receives a private non-verifiable signal $\sigma_i \in \{0, 1\}$. Independently of the state, an expert's signal is correct with probability either g or $b < g$, depending on his/her type:

$$g := \Pr(\sigma_i = \omega | t_i = G) > b := \Pr(\sigma_i = \omega | t_i = B) \geq 1/2.$$

Conditional on the state, the experts' signals are independent. We denote $\sigma := (\sigma_1, \sigma_2)$.

[5] In the online appendix, Section 1, we argue that the results of this section are robust to introducing heterogeneity between the experts, in terms of prior ability.

The expected precision of an expert's signal is denoted by

$$\rho := qg + (1-q)b$$

There is a common prior about the state of nature:

$$p := \Pr(\omega = 0)$$

Without loss of generality, we assume that $p > 1/2$.[6]

Each expert cares only about his/her reputation, which is modelled as the decision-maker's ex-post belief about the expert's type.

The timing of the game is as follows:

1. The nature draws the state ω and the types of the experts.

2. The experts receive their private signals.

3. The experts communicate their information to the decision-maker, according to an *expertise scheme.*

4. The decision-maker takes an action.

5. The state is revealed and the players receive their payoffs.

The focus of our work is the expertise scheme employed in stage 3. Under *independent expertise*, each expert sends a non-contractible binary message, $m_i \in \{0, 1\}$, to the decision-maker. Under *collective expertise*, expert 2 (he) first sends a non-contractible message $m_2 \in \{0, 1\}$ to expert 1 (she), and then expert 1 sends a non-contractible message $m \in \{(0,0), (0,1), (1,0), (1,1)\}$ to the decision-maker. Expert 1 can then be called a *deputy* expert. In the online appendix, Section 2, we argue that, with ex-ante identical experts, the particular way in which communication under collective expertise is organized is not important for the results of the model. In particular, we show that our equilibria remain equilibria (even after applying meaningful refinements) in a model that allows *both* experts to send a message to the decision-maker after talking to each other.

[6] We exclude $p = 1/2$ from consideration, as a trivial degenerate case: Under $p = 1/2$, reputation concerns create no misreporting incentives, and there is full information revelation under either expertise scheme.

An expert's payoff is then:

$$u_i(message, \omega) = \Pr(t_i = G|message, \omega),\ \forall i \in \{1, 2\},$$

where $message$ is either m_i or m depending on the expertise scheme.

We will use the term "signal-type (σ_1, σ_2)" to call expert 1 with signal σ_1 to whom expert 2 truthfully revealed σ_2.

3 Independent expertise

Under independent expertise, an expert's reporting behavior does not depend on the reporting strategy of the other expert. This is because (1) the experts learn nothing about each others' signals prior to reporting, and (2) the state is eventually revealed, thus making the other expert's report redundant in forming the decision-maker's belief about an expert's type.

Hence, each expert behaves as if he/she were a single expert. Consequently, we can just apply Lemma 1 from Ottaviani and Sørensen (2001), which deals precisely with the case of a single expert in a setup with two states, two expert types and a binary expert's signal. Given our notation and the assumption that $p > 1/2$, their lemma can be re-formulated as follows:

Lemma 1 *Under independent expertise, the following is true:*
- When $p \leq \rho$, the experts report their true signals in the most informative equilibrium.
- When $p > \rho$, there exists no equilibrium with informative reporting.

The intuition is simple. An expert wants to maximize the decision-maker's posterior belief that he/she received the signal equal to the state. Since $p > 1/2$, an expert with signal 0 always believes that $\omega = 0$ is more likely. An expert with signal 1 believes that $\omega = 1$ is more likely exactly when $p < \rho$, and considers $\omega = 0$ more likely otherwise. Therefore, when $p < \rho$, reporting the true signal is the natural equilibrium. In contrast, when $p > \rho$, there is a strong temptation to "herd" on the prior, which destroys any informative communication.

4 Collective expertise

Suppose expert 2 has truthfully revealed his signal to expert 1. When will the latter truthfully reveal both expert 2's and her own signal, regardless of her information? It turns out that, under collective expertise, full information revelation is impossible. To make sure, since the experts are *identical*, for "full revelation" we only require truthful reporting of the aggregate number of 0's received by the experts, i.e., we do not require to report who exactly received which signal.

Lemma 2 *Under collective expertise, a fully revealing equilibrium does not exist*

Proof. The direct proof of this lemma can be found in the online appendix. Alternatively, the reader can refer to the proof of Lemma 8 which covers the more general case of $n \geq 2$ identical experts. ■

The intuition behind Lemma 2 is straightforward. Two contradictory signals leave the deputy's belief at the prior, that is, believing that state 0 is more likely. Revealing that $\sigma \in \{(0,1),(1,0)\}$ implies that one of the experts is correct and the other one is wrong irrespective of the realized state. Hence, for any state realization, it induces the belief that the deputy is correct with probability $1/2$, whereas a deviation to reporting $(0,0)$ results in "guessing" the state with probability strictly higher than $1/2$. Any partial or full separation between signal-types $(0,1)$ and $(1,0)$ would damage the expected reputation of the deputy expert with $\sigma_1 = 1$ (given that $\sigma_2 = 0$) even more, thus creating an even stronger deviation incentive for her.

Lemma 2 immediately implies that, for $p \leq \rho$, independent expertise *always* provides the decision-maker with more information relative to collective expertise. But what happens for $p > \rho$?

To answer this question we need to explore what informative equilibria exist under collective expertise. For simplicity (without any effect on the qualitative results) we restrict ourselves to analyzing equilibria in which (1) the non-deputy expert truthfully reveals his signal to the deputy, and (2) pairs of signals $(0,1)$ and $(1,0)$ trigger the same distribution over messages ("anonymous" equilibria).

The first feature is natural, as the experts do not compete with each other (they are "in the same boat"). Given the first feature, the second one is natural too, as the two pairs of signals generate the same belief about the state.

An equilibrium satisfying both requirements always exists, as there is always an equilibrium in which expert 1 babbles to the decision-maker (and then expert 2 has no incentive to deviate from truthtelling). Moreover, the following lemma is true:

Lemma 3 *Suppose the deputy believes that the non-deputy expert always reports his true signal to her. Then, for any deputy's equilibrium behavior satisfying anonymity, truthtelling by the non-deputy expert is weakly optimal for the latter.*

Proof. Suppose expert 2, after receiving σ_2, deviates from truthtelling and lies to expert 1. For any σ_1, such lying makes expert 1 play weakly suboptimally, given the experts' true information. Since we consider only "anonymous" reporting strategies of expert 1, a message to the decision-maker generates the same reputation for both experts. Hence, the induced behavior of expert 1 will be weakly suboptimal for expert 2 as well, for any σ_1. ■

Lemma 3 means that we can ignore incentive compatibility constraints of the non-deputy expert. This reduces the whole problem to considering only the incentives of the deputy, who has learned both her own and the other expert's signal. Equivalently, given the restriction to anonymous equilibria, one can simply consider both experts as a team which collectively decides what to report, given its signal profile (σ_1, σ_2).

Now we are ready to examine possible equilibria. First, it can be shown that any equilibrium partitions the set of signal profiles into at most two *ordered* subsets (possibly with a common boundary), equivalent to only two messages being sent to the decision-maker. "Ordered" means that any profile of signals in one of the subsets contains a weakly higher number of zero signals than any profile in the other subset. We relegate the proof of this result to Section 6, where we consider the more general n-experts case (Proposition 3). A message can then be interpreted as a statement that the signal profile belongs to a certain element of the bipartition,.with the qualification that a threshold profile can randomize between the two messages.[7] Then, if we consider equilibria without such randomization, there arise two possibilities.

- partition $\{(0,0),(0,1),(1,0)\},\{(1,1)\}$;

- partition $\{(0,0)\},\{(0,1),(1,0),(1,1)\}$.

Let us denote message $\{(0,0),(0,1),(1,0)\}$ by m^0 and message $\{(0,1),(1,0),(1,1)\}$ by m^1.

In addition, there can be equilibria with mixing between partition elements:

- one in which signal-type $(1,1)$ randomizes between reporting the truth and reporting m^0;

- one in which signal-type $(0,0)$ randomizes between reporting the truth and reporting m^1;

- one in which signal-type (i,i) always reports the truth, while signal-types $(0,1)$ and $(1,0)$ mix between reporting $(0,0)$ and reporting $(1,1)$.

Let us first examine the equilibrium $(m^0,(1,1))$.

Lemma 4 *The equilibrium* $(\{(0,0),(0,1),(1,0)\},\{(1,1)\})$ *exists if and only if* $p \le \overline{p}$*, where* $\overline{p} = \dfrac{\rho^2(2-\rho)}{1-\rho+\rho^2} > \rho$*. Moreover, when* $p = \overline{p}$*,* $\Pr(\omega = 1|\sigma = (1,1)) > 1/2$*.*

Proof. See the Appendix. ■

Signal-type $(0,1)$ (or $(1,0)$) would never want to deviate to reporting $(1,1)$: As she believes that $\omega = 0$ is more likely, she would not want to be perceived as having received signal 1.

In contrast, signal-type $(1,1)$ may want to deviate to reporting $m^0 \equiv \{(0,0),(0,1),(1,0)\}$, if the prior is sufficiently biased towards $\omega = 0$. She will clearly do so when the prior is so strong that $\Pr(\omega = 0|\sigma = (1,1)) > 1/2$: As she considers $\omega = 0$ more likely, she does not want to be perceived as having

[7] One may wonder whether there can exist multiple *equilibrium* messages that generate exactly the same reputation for the experts in any state of nature ("reputation-equivalent" messages) but provide different information about the state to the decision-maker. It can be shown that, in the setting with two experts, this is impossible: Any two reputation-equivalent messages are also information-equivalent, meaning that they can be considered as the same message. The proof is available upon request.

received signal 1. When $\Pr(\omega = 0|\sigma = (1,1)) < 1/2$, expert 1 has a trade-off. By revealing her signal, she will essentially "bet" on the more likely state. However, deviating to m^0 does not imply "betting" on the less likely state, because m^0 does not imply that expert 1 necessarily received signal 0. This "imprecise" message has the advantage of "favorable" state-contingent interpretation by the decision-maker. When the realized state is 1, the decision-maker assigns a higher probability to (the experts having received) $(0,1)$ or $(1,0)$, compared to when the realized state is 0. Similarly, when the realized state is 0, the decision-maker assigns a higher probability to $(0,0)$ compared to when the realized state is 1.

As a result, the value of the prior at which expert 1 is indifferent between deviating and not, $\overline{p}$, is below p that makes $\Pr(\omega = 0|\sigma = (1,1)) = 1/2$. In other words, at $p = \overline{p}$ signal-type $(1,1)$ still believes that $\omega = 1$ is more likely.

The crucial thing is that $\overline{p} > \rho$. Two same signals combined are stronger than one. This allows to eliminate herding-on-the-prior incentives of the experts, whenever both signals are 1, for a range of parameters where each expert separately would herd.

Let us now consider the equilibrium $((0,0), m^1)$.

Lemma 5 *The equilibrium* $(\{(0,0)\}, \{(0,1),(1,0),(1,1)\})$ *exists if and only if* $p \leq \dfrac{1+\rho}{3}$, *which is strictly below* ρ.

Proof. See the Appendix. ■

Here the threshold on p is determined by the incentive compatibility of signal-type $(0,1)$ (or $(1,0)$). Given that the prior is biased towards $\omega = 0$, signal-type $(0,0)$ is very confident that $\omega = 0$, and, thus, would never want to lie. In contrast, signal-type $(0,1)$ (or $(1,0)$) has a trade-off similar to the trade-off of signal type $(1,1)$ in the equilibrium of Lemma 4: betting on the more likely state by sending $m = (0,0)$ versus playing the "safer" strategy of staying pooled with the other two signal-types. Note that the threshold on p provided by Lemma 5, $\dfrac{1+\rho}{3}$, is smaller than ρ.

Finally, let us consider equilibria with mixing between partition elements.

Lemma 6 *Equilibria with mixing between partition elements do not exist for* $p > \overline{p}$

Proof. See the Appendix. ■

Thus, such equilibria do not expand the set of priors where partial information revelation occurs under collective expertise. The analysis of this section implies the following fundamental result:

Proposition 1 *Irrespectively of which informative equilibrium is played under collective expertise (when an informative equilibrium exists), the following is true. When $p \leq \rho$, independent expertise results in more information transmitted to the decision-maker. When $p \in (\rho, \overline{p}]$, collective expertise results in more information transmitted to the decision-maker. For $p > \overline{p}$, both modes of expertise result in zero information transmission.*

The potential benefit of signal-sharing between the experts is alleviation of the herding-on-the-prior incentives, when both experts receive a signal contradicting the prior. This benefit materializes when each expert's signal is weaker than the prior ($\rho < p$, so that herd behavior results under independent expertise), but two same signals combined are sufficiently stronger than the prior (at $p = \overline{p}$ signal-type $(1, 1)$ believes that $\omega = 1$ is more likely).

The potential cost of signal-sharing is that it aggravates herding incentives, when the experts receive opposite signals. In such a case, the experts' belief remains at the prior, which implies herding on the prior *regardless* of its strength. As a result, only partial information revelation is possible under collective expertise.

5 Welfare analysis: Effects of the prior and the value of the safe action

In the previous section, we have seen that, in terms of information provision, independent expertise dominates collective expertise for $p \leq \rho$, and vice versa for $p \in (\rho, \overline{p}]$. Greater informativeness, however, implies a higher decision-maker's welfare only if it affects his choice of actions – this is the question we turn to in this section.

The important thing to notice is that message $m^0 \equiv \{(0,0),(0,1),(1,0)\}$ pools the signal profiles that, if taken separately, predict that $\omega = 0$ is more likely. At the same time, $\sigma = (1,1)$ implies that $\omega = 1$ is more likely for all $p \leq \overline{p}$ (according to Lemma 4). Hence, if equilibrium $(m^0, (1,1))$ is played, collective expertise correctly predicts which state is more likely conditional on the experts' signals for *any* $p \in [1/2, \overline{p}]$. In contrast, under independent expertise, such information is lost for $p \in (\rho, \overline{p}]$, because no information is transmitted.

Thus, the following lemma is true:

Lemma 7 *Consider the range of p from $1/2$ to $\overline{p}$ and assume that equilibrium $(m^0, (1,1))$ is played under collective expertise. Then, for any pair of the experts' signals, collective expertise correctly predicts which state is more likely for all $p \in [1/2, \overline{p}]$, whereas independent expertise does so only for $p \in [1/2, \rho]$.*

We are now almost ready to state the key result of this section. In formulating it, we will assume that the decision-maker's preferred equilibrium is played under collective expertise.[8] However, as follows intuitively from Proposition 1, the qualitative conclusion will not change if we assume a different equilibrium selection under collective expertise.

To avoid the uninteresting case when the safe action is so attractive that it is always taken in any informative equilibrium under any expertise scheme, we make the following assumption.

Assumption 1 $k < \Pr(\omega = 0 | \sigma = (0,0))|_{p=\rho} =: \overline{k}$

If $k > \overline{k}$, the decision-maker strictly prefers the safe action for $p = \rho$, even after learning that $\sigma = (0,0)$. As we show in the proof of the following proposition, this implies that the safe action will always be taken, regardless of the expertise scheme, for all $p \leq \overline{p}$.

Now we can state the main result of this section:

[8] Equilibrium $(m^0, (1,1))$ is not necessarily the best for the decision-maker. Suppose the optimal signal-contingent policy is to take the safe action whenever $\sigma \in \{(0,1),(1,0),(1,1)\}$ and take action 0 otherwise (this could well be the case, because $(0,0)$ generates less uncertainty than $(1,1)$). Then, equilibrium $((0,0), m^1)$ is the best one (provided it exists).

Proposition 2 *The decision-maker is weakly better off under independent expertise for any $p \in (1/2, \rho]$ and weakly better off under collective expertise for any $p \in (\rho, \overline{p}]$. Moreover, there exists a threshold $\widehat{k} < \overline{k}$, such that:*

i) when $k \leq 1/2$, the decision-maker is equally well off under both expertise schemes for any $p \in (1/2, \rho]$ and ***strictly*** *better off under collective expertise for any $p \in (\rho, \overline{p}]$;*

ii) when $k \in (1/2, \widehat{k})$, the decision-maker is ***strictly*** *better off under independent expertise for p in a positive measure subset of $(1/2, \rho]$ and* ***strictly*** *better off under collective expertise for p in a positive measure subset of $(\rho, \overline{p}]$;*

iii) when $k \in (\widehat{k}, \overline{k})$, the decision-maker is ***strictly*** *better off under independent expertise for p in a positive measure subset of $(1/2, \rho]$ and is equally well off under both expertise schemes for any $p \in (\rho, \overline{p}]$;*

Proof. See the Appendix. ■

When k is below $1/2$, the safe action is never taken because betting on the more likely state is always optimal. In such a case, the only relevant thing is whether an expertise correctly predicts which state is more likely, given the experts' signals. Then, statement (i) of Proposition 2 immediately follows from Lemma 7.

For higher values of k, the safe action is sometimes optimal and sometimes not. Thus, not only which state is more likely, but also how likely is the more likely state (conditional on the experts' information) becomes relevant, and only independent expertise is capable of revealing all experts' information for $p \in (1/2, \rho]$.

For example, when k exceeds $1/2$ but is sufficiently small, and p is not too far from $1/2$, the optimal signal-contingent policy is as follows: take the safe action when $\sigma \in \{(0,1),(1,0)\}$, and take the action suggested by the signals otherwise. This can only be achieved under independent expertise. Under collective expertise, $\{(0,1),(1,0)\}$ is always pooled (at least partially) with either $(0,0)$ or $(1,1)$. As a result, the mapping from the experts' signals into the decision-maker's actions will inevitably be suboptimal: either the safe action will sometimes be taken when it should not be taken or vice versa.[9]

As we raise k (yet keeping it below $\overline{k}$) and hold p close to 1/2, the safe action eventually becomes optimal for all pairs of signals. However, if we consider the values of p slightly below ρ, the safe action will be optimal when $\sigma \in \{(0,1),(1,0)\}$ and not optimal when $\sigma = (0,0)$. Then, to achieve the optimal signal-contingent policy under collective expertise, equilibrium $((0,0), m^1)$ needs to be played, but, by Lemma 5, it does not exist for p close to ρ.

At the same time, for $p \in (\rho, \overline{p}]$, collective expertise has an advantage as long as different messages result in different actions. This remains to be the case only for k below a certain value, denoted $\widehat{k}$: For $k > \widehat{k}$, the safe action is always taken in equilibrium for all $p \in (\rho, \overline{p}]$. Hence, for $k \in (\widehat{k}, \overline{k})$, collective expertise never dominates independent one, whereas the latter is better than the former for a subset of p belonging to $(1/2, \rho]$.

Proposition 2 implies that a higher value of the safe action makes independent expertise more likely to be optimal. Of course, for any fixed p and ρ, either one or the other scheme is weakly preferred for all k. Imagine, however, that the decision-maker needs to set up an expertise scheme before he learns the prior or the experts' ex-ante quality, or cannot condition the choice of the scheme on these parameters for some reason. Then the optimal choice of the scheme will depend on k, and Proposition 2 implies the following:

Corollary 1 *The higher the value of the safe action is, the more likely independent expertise is to be optimal.*

6 More than two experts

We now ask: How does the communication between experts and decision-maker change when the number of (identical) experts is higher than two? The good news is that, qualitatively, it does not change. Under independent reporting, the

[9] Another example is when p is sufficiently close to ρ and k is high enough (yet below $\overline{k}$) so that both $\{(0,1),(1,0)\}$ and $(1,1)$ call for the safe action, while $(0,0)$ calls for $a = 0$. Then, to achieve the optimal signal-contingent policy under collective expertise, equilibrium $((0,0), m^1)$ needs to be played, but, by Lemma 5, it does not exist for p close to ρ.

experts will report truthfully if $p \leq \rho$ and herd on the prior otherwise. Under collective expertise, roughly speaking, they will only communicate to the decision-maker which state is more likely, but this partial information transmission can be achieved also for values of p up to a threshold $\widehat{p} > \rho$, which tends to 1 as the number of experts goes to infinity. Moreover, under collective expertise, as the number of experts grows, the aggregate information about the state becomes more and more accurate, and asymptotically the decision-maker learns the true state (for any value of p).

Under independent expertise, the behavior of each expert does not depend on the number of other experts and, thus, is fully described by Lemma 1.

We keep assuming that, under collective expertise, there is a deputy expert who reports to the decision-maker "on behalf" of all experts. Prior to her report, other experts are assumed to report to the deputy simultaneously.

Under collective expertise, the non-existence of a fully revealing equilibrium continues to hold:

Lemma 8 *For any number of experts, under collective expertise, a fully revealing equilibrium does not exist*

Proof. See the Appendix. ■

To examine partially revealing equilibria, like in the two experts case, we look for equilibria in which the non-deputy experts truthfully reveal their signals to the deputy, and any two profiles of signals that contain the same number of zeros generate the same distribution over messages (anonymous equilibria). Lemma 3 straightforwardly extends to the case of more than two experts. Thus, we can ignore incentive compatibility constraints of non-deputy experts or just treat the experts as a team which collectively decides what to report, given its signal profile $(\sigma_1, ...\sigma_n)$.

We are going to show now (Proposition 3) that any equilibrium under collective expertise is essentially equivalent to the experts reporting just whether they have received more or less than l zeros (with a possible randomization for l zeros). To establish this result we need several preliminary steps.

We say that two messages m, m' are *reputation-equivalent* if, for each $\omega = 0, 1$, $\Pr(G|\omega, m) = \Pr(G|\omega, m')$. Otherwise we will say that the messages are *reputationally distinct*. We first establish the following lemma:

Lemma 9 *Every equilibrium satisfies the following property: For any two equilibrium messages which are not reputation-equivalent, there exists a number of zeros k such that one of them is never sent when the experts have received less than k zeros and the other one is never sent when the experts have received more than k zeros.*

Proof. See the Appendix. ■

The lemma basically says that equilibrium messages are "ordered": It cannot be the case that an equilibrium message is sent under two different numbers of zeros, k' and k'', and a different, *not reputation-equivalent*, message is sent for some k between k' and k''. The intuition is as follows. Take two *equilibrium* messages, m^1 and m^2, which are not reputation-equivalent. Suppose m^1 generates a higher reputation than m^2 if $\omega = 0$ is revealed. This automatically means that m^1 results in a lower reputation than m^2 if the revealed state is $\omega = 1$, otherwise m^2 would never be sent in equilibrium. Consequently, the attractiveness of m^1 relative to m^2 grows with the perceived probability of $\omega = 0$. Now, suppose m^1 is weakly preferred to m^2 by experts with k zeros. Then, it will be strictly preferred to m^2 for *any* higher number of zeros, because of a higher likelihood of $\omega = 0$.

Any number of reputation-equivalent messages can obviously be coalesced into one message that gives exactly the same reputation as the original messages under each state, thus that satisfies the experts' equilibrium incentives as well. That is, if equilibrium messages m^1, $m^2, ...m^J$ are reputation-equivalent, than, if m^1 is sent instead of $m^2, ...m^J$, the resulting communication strategy remains to be an equilibrium, as m^1 would yield exactly the same state-contingent reputation as $m^2, ...m^J$ in the original equilibrium.

Therefore, without loss of generality, we can consider equilibria with only reputationally distinct messages to prove the following result: no equilibrium can have more than two reputationally distinct groups of reputation-equivalent messages.

We call equilibria with exactly two such groups *bipartitional*, as, due to Lemma 9, they will be characterized by a cutoff number of zeros that separates signal-types that send reputationally distinct messages (with the qualification that there can be one boundary type that randomizes between the two groups of reputationally distinct messages).

We need three preparatory lemmas, which are proven in the Appendix. The discussion of Proposition 3 below provides some key intuitions behind these lemmas.

We will call the set of all "numbers of zeros" received by the experts for which they send message m with positive probability by the *support* of message m.

Lemma 10 *Suppose an equilibrium message* m *has support* $\{l, ..., r\}$, *where* $0 \leq l < r \leq n$. *Then, at least one of the following holds:*

1. *experts who received* r *zeros weakly prefer revealing it to sending* m, *and then they consider state* 0 *strictly more likely;*

2. *experts who received* l *zeros weakly prefer revealing it to sending* m, *and then they consider state* 1 *strictly more likely.*

Proof. See the Appendix. ■

Lemma 11 *Suppose the experts received exactly* k *signals equal to* $\overline{\omega}$ *and consider state* $\overline{\omega}$ *strictly more likely. If there is an equilibrium message* m *which is never sent when the number of signals* $\overline{\omega}$ *in the profile is below* k *and is sent with a positive probability when it is above* k, *they strictly prefer sending* m *to revealing the true number of signals* $\overline{\omega}$ *they received.*

Proof. See the Appendix. ■

Lemma 12 *Suppose the experts consider the two states equally likely. If there is an equilibrium message* m *that is sent with positive probability under more than one number of zeros, they strictly prefer sending* m *to revealing the true number of zeros they received.*

Proof. See the Appendix. ■

We are now in the position to state and prove the proposition.

Proposition 3 *Every equilibrium is at most bipartitional.*

Proof. Let k be the number of zero signals in σ. Suppose by contradiction that the equilibrium has more than two reputationally distinct messages. Then, by Lemma 9, there must be an "intermediate" equilibrium message with support $\{l, ..., r\}$, where $0 \leq l \leq r \leq n$, and at least two *other* equilibrium messages, m^0 and m^1, such that m^0 is never sent for $k < r$, and m^1 is never sent for $k > l$. Take the intermediate message. Suppose first that $l < r$. Then, by Lemma 10, either the experts with $k = l$ or the experts with $k = r$ would prefer to reveal their exact number of zeros rather than sending the message, and they consider the corresponding state (1 or 0) strictly more likely. But then such profile of experts would deviate to m^1 or m^0, either because l or r is the support of the corresponding message (in which case the deviation is equivalent to revealing k) or by Lemma 11.

Suppose now that $l = r =: \widehat{k}$. Obviously, neither m^0 nor m^1 can have $\widehat{k}$ as its support, for otherwise it would be reputation-equivalent to the intermediate message. Then, if the experts with $\widehat{k}$ zeros consider one of the two states more likely, Lemma 11 applies. If $n = 2$, then $\widehat{k} = 1$ and the experts do consider a state (state 0) more likely. If $n > 2$, it could be the case that the experts with $\widehat{k}$ zeros consider both states equally likely. In such a case, if there are more than 3 equilibrium messages, the arguments above can be applied to a "neighbouring" but still intermediate message. If there are 3 messages, then either m^0 or m^1 (or both) is sent with positive probability under more than one number zeros. Hence, by Lemma 12, the experts would deviate to such message. ■

Thus, even when the experts have access to the full vector of their signals, they still take a stand towards one of the two states, despite a richer signal structure in hands. Proposition 3 is reminiscent of Proposition 4 of Ottaviani and Sørensen (2006a), but it is not a consequence of that proposition and is of a different nature in general. Ottaviani and Sørensen consider a "reputational cheap talk" game with a *single* expert. They show that all informative equilibria are binary for the class of "multiplicative linear" signal structures, defined by the following form of the conditional pdf of the expert's signal:

$$f(s|x,t) = tg(s|x) + (1-t)h(s) = t\frac{(1+sx)}{2} + (1-t)\frac{1}{2} = \frac{1}{2}(1+stx),$$

where x is the state, s is the signal, and t is the expert's type.

As we demonstrate below, if the expert receives a vector of binary signals instead, there can exist equilibria with *more* than a bipartition. In contrast, when each of these signals is collected by a different expert and then pooled, Proposition 3 guarantees that every equilibrium is (at most) bipartitional. This difference is due to a new driving force that we highlight: "sharing responsibility" for mistakes with other experts, as opposed to being responsible for all the collected information, makes each expert "more brave" when it comes to taking a stand towards one of the two states.

To illustrate our point, consider a setting with a single expert receiving a vector of independent (conditional on the state) binary signals. Suppose an expert with prior reputation $q = 3/4$ receives two binary signals, each correct with probability $g = 3/4$ if the expert is good, and pure noise ($b = 1/2$) if the expert is bad. Suppose the expert has received signals 0 and 1. In a fully informative equilibrium, no matter the realized state, his reputation will be

$$\frac{g(1-g)q}{g(1-g)q + b(1-b)(1-q)} = \frac{9}{13}.$$

If the expert pretends to have received two 0 signals, instead, his expected reputation (given that his belief stays at the prior) will be

$$p\frac{g^2 q}{g^2 q + b^2(1-q)} + (1-p)\frac{(1-g)^2 q}{(1-g)^2 q + (1-b)^2(1-q)} = \frac{27}{31}p + (1-p)\frac{3}{7}.$$

The first expression is bigger than the second one whenever $p < 0.596$. Hence, if the expert is a priori sufficiently uncertain about the state, there does exist a fully informative equilibrium, in which the expert with mixed evidence "abstains" regarding the state (i.e., reveals $(0,1)$).[10] This happens because the expert wants to "defend" a rather high initial reputation, and he can do so by abstaining because also good experts are sufficiently likely to receive mixed (hence partially wrong) evidence.[11] Seen from another angle, relative to a single mistake, the downfall of reputation in the case of a double mistake dominates the potential increase in reputation in case of a double correct guess ($9/13 - 3/7 > 27/31 - 9/13$). This kind of "multiplicative effect" of mistakes arises because the expert is responsible for all wrong signals. This makes him risk-averse with respect to the number of mistakes, and, thus, cautious.

In contrast, in our basic setup, each expert is responsible only for one signal and only takes a share of the blame for an incorrect collective advice. As a result, an expert's reputation is *linear* in the number of mistakes. With two experts, it is x, $(1/2)x + (1/2)y$, y, under zero, one and two mistakes respectively, where $x := \Pr(t = G|\sigma_i = \omega)$; $y := \Pr(t = G|\sigma_i \neq \omega)$. More generally, with n experts making r mistakes, the reputation of each of them is $\frac{n-r}{n}x + \frac{r}{n}y$. This linearity is exploited by Lemma 11 for the bang-bang solution of Proposition 3.

Note however that, while the linearity argument suffices for the two experts case, in presence of more experts (hence of more signals) an intermediate message can be imprecise about the shares of signals, and the expected shares in the eyes of the decision-maker change depending on the realized state. In particular, if a "wide" intermediate message is sent, the expected shares will be skewed towards zero-signals in case of state 0 and towards one-signals in case of state 1. Therefore, there is a positive bias induced by large messages that may help "abstention" to be preferred over taking a stand by sufficiently uncertain experts. Yet, Lemma 10 shows that this does not happen. The benefit of increasing the size of the message is of second order with respect to the increasing confidence in a state of one of the

[10] To be sure, one can verify that an expert with $(1,1)$ will prefer not to report $(1,0)$ whenever $p < 0.767$, which is a weaker constraint compared to the incentive compatibility constraint of signal-type $(1,0)$.

[11] This sounds like the story of a Nobel prize in economics being asked to make a macroeconomic prediction.

two threshold signal profiles.

Now we will show that a bipartitional equilibrium exists up to a value of p that would preclude informative communication under independent reporting. We also argue that this threshold asymptotically goes to 1. To establish these results, we can obviously keep considering equilibria with reputationally distinct messages only, without loss of generality.

We need two preparatory lemmas.

Lemma 13 *There exists $\overline{p} \in (\rho, 1)$ tending to 1 as $n \to \infty$ such that the experts with all ones (i.e., $\sigma = \overrightarrow{1}$), weakly prefer to reveal themselves rather than sending the complementary message (i.e., $\sigma \neq \overrightarrow{1}$) if and only if $p \leq \overline{p}$. Moreover, for $p = \overline{p}$, the experts with all ones consider $\omega = 1$ strictly more likely.*

Proof. See the Appendix. ■

Lemma 14 *Let $\overline{\overline{p}}$ be the value of p such that the experts with all ones consider the two states equally likely. There exists $\widehat{p} \in \left[\overline{p}, \overline{\overline{p}}\right]$ such that a bipartitional equilibrium exists if and only if $p \in (1/2, \widehat{p}]$.*

Proof. See the Appendix. ■

Proposition 4 *For every $n \geq 2$, there exists $\widehat{p} > \rho$ tending to 1 as $n \to \infty$ such that a bipartitional equilibrium exists if and only if $p \in \left(\frac{1}{2}, \widehat{p}\right]$.*

Proof. Immediate from Lemma 14 and Lemma 13. ■

Proposition 4 extends the results of the two experts case by showing that (i) for every number of experts, there are values of p where collective expertise dominates independent expertise, and (ii) this region expands asymptotically with the number of experts.

How informative is an equilibrium bipartition? Although monotonicity of the equilibrium informativeness with respect to n is generally not guaranteed for all n, we show that any given "precision of communication" can be achieved by picking a large enough n, and asymptotically the decision-maker learns the true state in equilibrium. Effectively, as n goes to infinity, the experts virtually learn the state, and a message in the equilibrium bipartition becomes virtually equivalent to communicating the true state to the decision-maker.

Proposition 5 *For every* $p \in \left(\frac{1}{2}, 1\right)$ *and* $\mu \in \left[\frac{1}{2}, 1\right)$*, there exists* $\overline{n} > 1$ *such that for every* $n \geq \overline{n}$*, in any bipartitional equilibrium* (m, m')*,* $\Pr(\omega = 1|m) > \mu$*,* $\Pr(\omega = 0|m') > \mu$*.*

Proof. See the Appendix. ■

Propositions 4 and 5 imply that, as the number of expert grows large enough, the advantage of collective expertise over independent expertise for $p > \rho$ increases, whereas its disadvantage for $p \leq \rho$ shrinks, as the loss of information under collective expertise diminishes and tends to zero at the limit.

Imagine the expertise scheme is to be established before the public belief p is realized, and there is some prior distribution over p. Then Propositions 4 and 5 entail that the expected advantage (disadvantage) of collective expertise over independent expertise increases (decreases) as the number of experts grows large enough.[12]

7 Conclusion

In this paper we study optimal organization of expertise with multiple experts. The only friction in the model is the experts' reputation concerns, which generate incentives to herd on the prior belief about the state of nature. Our key question is: shall the experts be allowed to talk to each other before providing advice to the decision-maker? Information-sharing between the experts alleviates their herding-on-the-prior incentives when their receive similar signals. However, it aggravates herding when the experts receive signals opposing each other, as disagreement tends to leave their beliefs close to the prior. As a result, the experts tend to hide

[12] For reasons outside of our model, the expertise scheme may need to be designed as an institution to be applied in all circumstances (for example, think of the refereeing process in an academic journal).

disagreement and herd on the prior instead. Thus, some information is inevitably lost (for the decision-maker) under collective expertise.

As a result, collective expertise is beneficial when the prior uncertainty is not too high, so that independent reporting would lead to herding on the prior. However, when the prior uncertainty becomes very high, independent reporting becomes fully informative. In such a case, it is better to keep the experts unaware of their potential disagreement (by not allowing them to talk) in order to prevent them from herding on the prior.

Although some information is always lost under collective expertise, it correctly predicts the more likely state, conditional on the experts' information, for a wider range of parameters, compared to independent expertise. Therefore, if the decision-maker just needs to know which state is more likely, collective expertise is always weakly better. However, if the decision-maker also needs to know how likely the more likely state is, independent expertise is better, provided it induces no herding (i.e., when the prior uncertainty is sufficiently large). Thus, if the decision-maker, in addition to "betting on the more likely state", has a valuable enough "safe" option, which is optimal whenever there is a high enough residual uncertainty about the state, independent expertise is more likely to be optimal.

Finally, collective expertise is more likely to be optimal as the number of experts grows. This is because any loss of information arising under collective expertise becomes less important (as the experts' aggregate information becomes more precise), whereas the set of parameters under which collective expertise results in information transmission expands.

8 Appendix

8.1 Preliminaries to proofs

Denote:

$$x := \Pr(t = G|\sigma_i = \omega);\ y := \Pr(t = G|\sigma_i \neq \omega)$$

– the expected reputation of expert i conditional on having received correct and incorrect own signal respectively.

Let m be the message sent to the decision-maker and I – the information available to the expert. Then, the expert's expected reputation from message m conditional on I is:

$$\begin{aligned} R_i(m,I) &= \Pr(\omega=0|I)[\Pr(\sigma_i=0|\omega=0,m)\cdot x+\Pr(\sigma_i=1|\omega=0,m)\cdot y] \\ &+\Pr(\omega=1|I)[\Pr(\sigma_i=0|\omega=1,m)\cdot y+\Pr(\sigma_i=1|\omega=1,m)\cdot x] \\ &= \alpha(m,I)\cdot x+\beta(m,I)\cdot y, \end{aligned}$$

where

$$\alpha(m,I) := \Pr(\omega=0|I)\Pr(\sigma_i=0|\omega=0,m)+\Pr(\omega=1|I)\Pr(\sigma_i=1|\omega=1,m),$$
$$\beta(m,I) := \Pr(\omega=0|I)\Pr(\sigma_i=1|\omega=0,m)+\Pr(\omega=1|I)\Pr(\sigma_i=0|\omega=1,m).$$

It is easy to see that $\alpha+\beta=1$. It is also straightforward to derive

$$x=\frac{qg}{\rho}>y=\frac{q(1-g)}{(1-\rho)}$$

Therefore, all comparisons of expected reputations are equivalent to comparing values of $\alpha(m,I)$:

$$R_i(m',I)>R_i(m'',I)\Leftrightarrow\alpha(m',I)>\alpha(m'',I),\text{ for any } I \text{ and any } m' \text{ and } m'' \quad (1)$$

8.2 Proofs

Proof of Lemma 4. We need to check the incentive compatibility constraints of signal-types $(1,1)$ and $(1,0)$. (Since signal type $(0,1)$'s belief about the state is the same as that of $(1,0)$, her incentive compatibility constraint is identical to that of $(1,0)$). There is no need to check that for $(0,0)$: If $(1,0)$ does not gain from deviating to reporting $(1,1)$, then neither does $(0,0)$, as the latter assigns an even lower probability to $\omega=1$. As we have shown in "Preliminaries to proofs", comparing expected reputations is equivalent to comparing values of $\alpha(m, I)$, as defined above.

Incentive compatibility of signal-type $(1,1)$:

First, compute α of signal-type $(1,1)$ if she does not deviate.

$$\begin{aligned}\Pr(\omega &= 0|\sigma=(1,1)) = \frac{\Pr(\sigma=(1,1)|\omega=0)\Pr(\omega=0)}{num. + \Pr(\sigma=(1,1)|\omega=1)\Pr(\omega=1)} \\ &= \frac{(1-\rho)^2 p}{(1-\rho)^2 p + \rho^2(1-p)} \qquad (2)\end{aligned}$$

$$\Pr(\sigma_1 = 0|\omega=0, \sigma=(1,1)) = 0,\ \Pr(\sigma_1 = 1|\omega=1, \sigma=(1,1)) = 1$$

Thus,

$$\begin{aligned}\alpha(m &= (1,1), \sigma=(1,1)) \\ &= \Pr(\omega=0|\sigma=(1,1))\cdot 0 + \Pr(\omega=1|\sigma=(1,1))\cdot 1 \\ &= \frac{\rho^2(1-p)}{(1-\rho)^2 p + \rho^2(1-p)} \qquad (3)\end{aligned}$$

Now, compute α of signal-type $(1,1)$ if she deviates to m^0.

$$\Pr(\sigma_1 = 0|\omega=0, \sigma\in m^0) = \frac{\Pr(\sigma_1 = 0 \cap \sigma \in m^0|\omega=0)}{\Pr(\sigma\in m^0|\omega=0)}$$

$$= \frac{\Pr(\sigma_1=0\cap\sigma\in(0,0)|\omega=0)+\Pr(\sigma_1=0\cap\sigma\in(0,1)|\omega=0)+\Pr(\sigma_1=0\cap\sigma\in(1,0)|\omega=0)}{\Pr(\sigma\in m^0|\omega=0)}$$

$$= \frac{\rho^2+\rho(1-\rho)}{\rho^2+2\rho(1-\rho)} = \frac{1}{2-\rho}$$

$$\Pr(\sigma_1 = 1|\omega=1, \sigma\in m^0) = \frac{\Pr(\sigma_1 = 1 \cap \sigma \in m^0|\omega=1)}{\Pr(\sigma\in m^0|\omega=1)}$$

$$= \frac{\Pr(\sigma_1=1\cap\sigma\in(0,0)|\omega=1)+\Pr(\sigma_1=1\cap\sigma\in(0,1)|\omega=1)+\Pr(\sigma_1=1\cap\sigma\in(1,0)|\omega=1)}{\Pr(\sigma\in m^0|\omega=1)}$$

$$= \frac{\rho(1-\rho)}{(1-\rho)^2+2(1-\rho)\rho} = \frac{\rho}{1+\rho}$$

Thus,

$$\begin{aligned}
\alpha(m &= m_0, \sigma = (1,1)) \\
&= \Pr(\omega = 0|\sigma = (1,1)) \cdot \frac{1}{2-\rho} + \Pr(\omega = 1|\sigma = (1,1)) \cdot \frac{\rho}{1+\rho} \\
&= \frac{(1-\rho)^2 p}{(1-\rho)^2 p + \rho^2(1-p)} \cdot \frac{1}{2-\rho} + \frac{\rho^2(1-p)}{(1-\rho)^2 p + \rho^2(1-p)} \cdot \frac{\rho}{1+\rho}
\end{aligned}$$

The expert will not deviate whenever

$$\alpha(m = (1,1), \sigma = (1,1)) \geq \alpha(m = m^0, \sigma = (1,1)),$$

which yields

$$p \leq \frac{\rho^2(2-\rho)}{1-\rho+\rho^2} =: \overline{p}.$$

It is straightforward to show that $\overline{p} > \rho$, given that $\rho > 1/2$.

Let us show now that at $p = \overline{p}$, $\Pr(\omega = 1|\sigma = (1,1)) > 1/2$. Using (2),

$$\Pr(\omega = 1|\sigma = (1,1)) = \frac{\rho^2(1-p)}{\rho^2(1-p) + (1-\rho)^2 p}$$

$$\frac{\rho^2(1-\overline{p})}{\rho^2(1-\overline{p}) + (1-\rho)^2\overline{p}} > 1/2 \Leftrightarrow \frac{\rho^2}{(1-\rho)^2} > \frac{\overline{p}}{1-\overline{p}}$$

Substituting the expression for $\overline{p}$ into the last inequality yields $\rho > 1/2$ which is always true (as $g > b \geq 1/2$ and $q > 0$ by the assumptions of the model).

Incentive compatibility of signal-type $(1,0)$:

First, compute α of signal-type $(1,0)$ if she does not deviate.

$$\begin{aligned}
\Pr(\omega &= 0|\sigma = (1,0)) = \frac{\Pr(\sigma = (1,0)|\omega = 0)\Pr(\omega = 0)}{num. + \Pr(\sigma = (1,0)|\omega = 1)\Pr(\omega = 1)} \\
&= \frac{(1-\rho)\rho p}{(1-\rho)\rho p + \rho(1-\rho)(1-p)}
\end{aligned}$$

Using the expressions for $\Pr(\sigma_1 = 0|\omega = 0, \sigma \in m^0)$ and $\Pr(\sigma_1 = 1|\omega = 1, \sigma \in$

m^0) derived above, we obtain:

$$\begin{aligned}\alpha(m &= m^0, \sigma = (1,0)) = \frac{(1-\rho)\rho p}{(1-\rho)\rho p + \rho(1-\rho)(1-p)} \cdot \frac{1}{2-\rho} \\ &+ \frac{\rho(1-\rho)(1-p)}{(1-\rho)\rho p + \rho(1-\rho)(1-p)} \cdot \frac{\rho}{1+\rho}\end{aligned}$$

Now, compute α of signal-type $(1,0)$ if she deviates to $(1,1)$.

$$\Pr(\sigma_1 = 0|\omega = 0, \sigma = (1,1)) = 0, \;\Pr(\sigma_1 = 1|\omega = 1, \sigma = (1,1)) = 1$$

Thus,

$$\begin{aligned}\alpha(m &= (1,1), \sigma = (1,0)) = \Pr(\omega = 0|\sigma = (1,0)) \cdot 0 + \Pr(\omega = 1|\sigma = (1,0)) \cdot 1 \\ &= \frac{\rho(1-\rho)(1-p)}{(1-\rho)\rho p + \rho(1-\rho)(1-p)}\end{aligned}$$

The expert will not deviate whenever

$$\alpha(m = m^0, \sigma = (1,0)) \geq \alpha(m = (1,1), \sigma = (1,1)),$$

which yields

$$(1+\rho)p \geq (2-\rho)(1-p)$$

or

$$p \geq \frac{2-\rho}{3}$$

As $\rho > 1/2$, the right-hand side is always below $1/2$. Thus, the incentive compatibility condition of signal-type $(1,0)$ is always satisfied. ■

Proof of Lemma 5. Since the equilibrium is symmetric to $(m^0, (1,1))$, the incentive compatibility conditions are exactly the same as in the proof of the previous lemma, with the only difference that p has to be substituted with $1-p$.

Thus, the no-deviation condition of signal-type $(0,0)$ is

$$1 - p \leq \frac{\rho^2(2-\rho)}{1-\rho+\rho^2} \equiv \overline{p} \Leftrightarrow p \geq 1 - \overline{p}$$

Since $\overline{p} > \rho > 1/2$, the condition is satisfied for all $p > 1/2$ (which is an assumption of the model).

The no-deviation condition of signal-type $(0, 1)$ is

$$1 - p \geq \frac{2 - \rho}{3} \Leftrightarrow p \leq \frac{1 + \rho}{3},$$

which is smaller than ρ, as $\rho > 1/2$. ■

Proof of Lemma 6. First, consider the equilibrium in which signal-type $(1, 1)$ randomizes between reporting the truth (with probability μ) and reporting m^0 (with probability $1 - \mu$). It must be the case that $\alpha(m = (1, 1), \sigma = (1, 1)) = \alpha(m = m_0, \sigma = (1, 1))$.

Since, in such an equilibrium, $m = (1, 1)$ implies $\sigma = (1, 1)$ with certainty, (3) applies:

$$\alpha(m = (1, 1), \sigma = (1, 1)) = \frac{\rho^2(1 - p)}{(1 - \rho)^2 p + \rho^2(1 - p)}$$

Let us now compute

$$\begin{aligned} &\alpha(m = m^0, \sigma = (1, 1)) \\ &\equiv \Pr(\omega = 0|\sigma = (1, 1)) \cdot \Pr(\sigma_1 = 0|\omega = 0, m = m^0) \\ &+ \Pr(\omega = 1|\sigma = (1, 1)) \cdot \Pr(\sigma_1 = 1|\omega = 1, m = m^0) \end{aligned}$$

From (2) we have

$$\Pr(\omega = 0|\sigma = (1, 1)) = \frac{(1 - \rho)^2 p}{(1 - \rho)^2 p + \rho^2(1 - p)}$$

Next,

$$\Pr(\sigma_1 = 0|\omega = 0, m = m^0) = \frac{\Pr(m = m^0|\sigma_1 = 0, \omega = 0) \Pr(\sigma_1 = 0|\omega = 0)}{num. + \Pr(m = m^0|\sigma_1 = 1, \omega = 0) \Pr(\sigma_1 = 1|\omega = 0)}$$

Since m^0 is always sent when $\sigma_1 = 0$,

$$\Pr(m = m^0|\sigma_1 = 0, \omega = 0) = 1$$

$$
\begin{aligned}
&\Pr(m = m^0|\sigma_1 = 1, \omega = 0)\\
&= \Pr(\sigma = (0,0)|\sigma_1 = 1, \omega = 0) + \Pr(\sigma \in \{(0,1),(1,0)\}|\sigma_1 = 1, \omega = 0)\\
&+ \Pr(\sigma = (1,1)|\sigma_1 = 1, \omega = 0)(1-\mu)\\
&= 0 + \Pr(\sigma_2 = 0|\omega = 0) + \Pr(\sigma_2 = 1|\omega = 0)(1-\mu)\\
&= \rho + (1-\rho)(1-\mu)
\end{aligned}
$$

Thus,

$$
\Pr(\sigma_1 = 0|\omega = 0, m = m^0) = \frac{\rho}{\rho + [\rho + (1-\rho)(1-\mu)](1-\rho)}
$$

Analogously,

$$
\Pr(\sigma_1 = 1|\omega = 1, m = m^0) = \frac{\Pr(m = m^0|\sigma_1 = 1, \omega = 1)\Pr(\sigma_1 = 1|\omega = 1)}{num. + \Pr(m = m^0|\sigma_1 = 0, \omega = 1)\Pr(\sigma_1 = 0|\omega = 1)},
$$

$$
\Pr(m = m^0|\sigma_1 = 0, \omega = 1) = 1,
$$

$$
\Pr(m = m^0|\sigma_1 = 1, \omega = 1) = 1 - \rho + \rho(1-\mu),
$$

yielding

$$
\Pr(\sigma_1 = 1|\omega = 1, m = m^0) = \frac{[1-\rho+\rho(1-\mu)]\rho}{[1-\rho+\rho(1-\mu)]\rho + 1 - \rho}
$$

Thus,

$$
\begin{aligned}
\alpha(m \;\; = \;\; & m_0, \sigma = (1,1)) = \frac{(1-\rho)^2 p}{(1-\rho)^2 p + \rho^2(1-p)} \cdot \frac{\rho}{\rho + [\rho + (1-\rho)(1-\mu)](1-\rho)}\\
&+ \frac{\rho^2(1-p)}{(1-\rho)^2 p + \rho^2(1-p)} \cdot \frac{[1-\rho+\rho(1-\mu)]\rho}{[1-\rho+\rho(1-\mu)]\rho + 1 - \rho}
\end{aligned}
$$

Solving $\alpha(m = (1,1), \sigma = (1,1)) = \alpha(m = m_0, \sigma = (1,1))$ yields

$$
p = \frac{\rho[1-(1-\rho)^2\mu]}{1-\rho\mu(1-\rho)}
$$

The right-hand side is increasing in μ and takes values ρ and $\overline{p}$ at $\mu = 0$ and $\mu = 1$ respectively. Thus, the equilibrium in which signal-type $(1,1)$ randomizes between reporting the truth and reporting m^0 does not exist for $p > \overline{p}$.

The equilibrium in which signal-type $(0,0)$ randomizes between reporting the truth and reporting m^1 is symmetric. The indifference condition for signal-type $(0,0)$ thus yields

$$p = 1 - \frac{\rho[1-(1-\rho)^2\mu]}{1-\rho\mu(1-\rho)},$$

which ranges from $1-\overline{p}$ at $\mu = 1$ to $1-\rho$ at $\mu = 0$. Both values are below $1/2$, meaning that such an equilibrium does not exist in our setup.

Finally, consider the equilibrium in which signal-types $(0,0)$ and $(1,1)$ always report the truth, while signal-type $\{(0,1),(1,0)\}$ mixes between reporting $(0,0)$ (with probability μ) and reporting $(1,1)$ (with probability $1-\mu$).

$$\begin{aligned}
&\alpha(m=(0,0),\sigma=(0,1)) \\
&\equiv \Pr(\omega=0|\sigma=(0,1))\cdot\Pr(\sigma_1=0|\omega=0,m=(0,0)) \\
&+\Pr(\omega=1|\sigma=(0,1))\cdot\Pr(\sigma_1=1|\omega=1,m=(0,0))
\end{aligned}$$

$$\Pr(\omega=0|\sigma=(0,1)) = p$$

$$\Pr(\sigma_1=0|\omega=0,m=(0,0)) = \frac{\Pr(m=(0,0)|\sigma_1=0,\omega=0)\Pr(\sigma_1=0|\omega=0)}{num. + \Pr(m=(0,0)|\sigma_1=1,\omega=0)\Pr(\sigma_1=1|\omega=0)}$$

$$\begin{aligned}
\Pr(m &= (0,0)|\sigma_1=0,\omega=0) = \Pr(\sigma=(0,0)|\sigma_1=0,\omega=0) \\
+\Pr(\sigma &= (0,1)|\sigma_1=0,\omega=0)\mu \\
&= \Pr(\sigma_2=0|\omega=0)+\Pr(\sigma_2=1|\omega=0)\mu = \rho+(1-\rho)\mu \\
\Pr(m &= (0,0)|\sigma_1=1,\omega=0) = \Pr(\sigma=(1,0)|\sigma_1=1,\omega=0)\mu \\
&= \Pr(\sigma_2=0|\omega=0)\mu = \rho\mu
\end{aligned}$$

Thus,

$$\Pr(\sigma_1=0|\omega=0,m=(0,0)) = \frac{[\rho+(1-\rho)\mu]\rho}{[\rho+(1-\rho)\mu]\rho+\rho\mu(1-\rho)} = \frac{\rho+(1-\rho)\mu}{\rho+2(1-\rho)\mu}$$

Analogously, it is straightforward to derive that

$$\begin{aligned}\Pr(\sigma_1 &= 1|\omega = 1, m = (0,0)) \\ &= \frac{\Pr(m=(0,0)|\sigma_1=1,\omega=1)\Pr(\sigma_1=1|\omega=1)}{num. + \Pr(m=(0,0)|\sigma_1=0,\omega=1)\Pr(\sigma_1=0|\omega=1)} \\ &= \frac{(1-\rho)\mu\rho}{(1-\rho)\mu\rho + [(1-\rho)+\rho\mu](1-\rho)} = \frac{\rho\mu}{2\rho\mu+1-\rho}\end{aligned}$$

Hence,

$$\alpha(m=(0,0),\sigma=(0,1)) = p\cdot\frac{\rho+(1-\rho)\mu}{\rho+2(1-\rho)\mu} + (1-p)\cdot\frac{\rho\mu}{2\rho\mu+1-\rho}$$

By symmetry, to obtain $\alpha(m = (1,1), \sigma = (0,1))$ we just need to replace μ with $1-\mu$ and p with $1-p$:

$$\alpha(m=(1,1),\sigma=(0,1)) = (1-p)\cdot\frac{\rho+(1-\rho)(1-\mu)}{\rho+2(1-\rho)(1-\mu)} + p\cdot\frac{\rho(1-\mu)}{2\rho(1-\mu)+1-\rho}$$

Equation $\alpha(m = (0,0), \sigma = (0,1)) = \alpha(m = (1,1), \sigma = (0,1))$ can be rewritten as

$$\frac{p}{1-p} = f(\mu),$$

where the derivative of $f(\cdot)$ can be shown to have the sign of

$$\left(1-3\rho+2\rho^2\right)\left(12\mu^2\rho+12\mu\rho^2-12\mu^2\rho^2-12\mu\rho+4\mu-4\mu^2+3\rho-3\rho^2+2\right)$$

The first bracket is negative for all $\rho \in (1/2, 1)$ (which is always true by the assumptions of the model). The second bracket can be rewritten as $3\rho(1-\rho)(1-2\mu)^2 + 4\mu(1-\mu) + 2$, which is always positive. Thus,$f'(\mu) < 0$, and, hence, p decreases with μ. It is then straightforward to derive that p ranges from $\frac{2-\rho}{3}$ for $\mu = 1$ to $\frac{1+\rho}{3}$ for $\mu = 0$. Hence, the equilibrium under consideration does not exist for $p > \frac{1+\rho}{3}$, which is below ρ. ■

Proof of Proposition 2. The first sentence of the proposition is an obvious consequence of Proposition 1.

When $k \leq 1/2$, betting on the more likely state is always better than the safe action. Then, statement (i) of the proposition immediately follows from Lemma 7.

Denote by $\overline{k}$ the minimum value of k for which the safe action is *always* taken under *independent* expertise for *all* $p \in (1/2, \rho)$. This value must make the decision-maker indifferent between taking the safe action and betting on the more likely state under the strongest possible belief about a state that can arise under independent expertise for $p \in (1/2, \rho)$:

$$\overline{k} := \Pr(\omega = 0 | \sigma = (0,0))|_{p=\rho}$$

It is obvious that for all $k > \overline{k}$ and $p \in (1/2, \rho)$ the safe action is always taken under collective expertise as well.

Similarly, denote by $\widehat{k}$ the minimum value of k for which the safe action is *always* taken under *collective* expertise for *all* $p \in (\rho, \overline{p})$. From Lemmas 4, 5 and the proof of Lemma 6, only two types of informative equilibria exist for $p \in (\rho, \overline{p})$: $(m^0, (1,1))$ and the one in which $(1,1)$ randomizes between reporting m^0 and $(1,1)$. The former is unambiguously more informative than the latter. Moreover, it is easy to show that $\Pr(\omega = 0 | \sigma \in \{(0,0),(0,1),(1,0)\}) > \Pr(\omega = 1 | \sigma = (1,1))$ for any $p \geq \rho$. Thus, $\widehat{k}$ is determined by

$$\widehat{k} := \Pr(\omega = 0 | \sigma \in \{(0,0),(0,1),(1,0)\})|_{p=\overline{p}}$$

It is obvious that for all $k > \widehat{k}$ and $p \in (\rho, \overline{p})$ the safe action is always taken under independent expertise as well: Since the safe action is preferred to betting on $\omega = 0$ conditional on $\sigma \in \{(0,0),(0,1),(1,0)\}$, so it is unconditionally, because $p < \Pr(\omega = 0 | \sigma \in \{(0,0),(0,1),(1,0)\})$.

Suppose $\overline{k} > \widehat{k}$, later we will show that this is indeed the case. Then, as follows from the above, for all $k > \overline{k}$ and $p \in (1/2, \overline{p})$ the safe action is always taken under either type of expertise. Assumption 1 rules out this situation.

Consider now $k \in (1/2, \overline{k})$ and $p \in (1/2, \rho)$. From the definition of $\overline{k}$ it follows that $\Pr(\omega = 0|\sigma = (0,0))|_{p=\rho} > k$ for any $k < \overline{k}$. Thus, by continuity, for any given $k < \overline{k}$ there exists a set (p', ρ) such that, for all p belonging to this set, $\Pr(\omega = 0|\sigma = (0,0)) > k$, implying that taking $a = 0$ is optimal for such values of p if $\sigma = (0,0)$.

There are two possible subcases to consider. Suppose first that $\Pr(\omega = 0|\sigma \in \{(0,1),(1,0)\})|_{p=\rho} \leq k$, which means that, for any $p < \rho$, the safe action is optimal if $\sigma \in \{(0,1),(1,0)\}$. Then, when $p \in (p', \rho)$, collective expertise can achieve the optimal signal-contingent policy only if equilibrium $((0,0), m^1)$ is realized. But, due to Lemma 5, it exists only for $p \leq \frac{1+\rho}{3} < \rho$. Hence, for $p \in \left(\max\left\{\frac{1+\rho}{3}, p'\right\}, \rho\right)$, collective expertise cannot achieve the optimal signal-contingent policy, whereas independent expertise can.

Suppose now $\Pr(\omega = 0|\sigma \in \{(0,1),(1,0)\})|_{p=\rho} > k$. Since $\Pr(\omega = 0|\sigma \in \{(0,1),(1,0)\})|_{p=\rho} = \rho$ and $\Pr(\omega = 0|\sigma = (0,0))|_{p=1/2} > \rho$, we have $\Pr(\omega = 0|\sigma = (0,0))|_{p=1/2} = \Pr(\omega = 1|\sigma = (1,1))|_{p=1/2} > k$. Then, for p sufficiently close to $1/2$, the safe action is optimal if and only if $\sigma \in \{(0,1),(1,0)\}$. But this can be achieved only under independent expertise, as collective expertise inevitably pools (fully or partially) $\{(0,1),(1,0)\}$ with either $(0,0)$ or $(1,1)$ or both, depending on the equilibrium.

Thus, for all $k \in (1/2, \overline{k})$, there exists a positive measure subset of p belonging to $(1/2, \rho)$, in which independent expertise strictly dominates collective expertise. Thus, we have proven statements (ii) and (iii) in relation to $p \in (1/2, \rho]$.

Consider now $p \in (\rho, \overline{p})$. For $k \in (\widehat{k}, \overline{k})$, the safe action is taken for any $p \in (\rho, \overline{p})$ under either expertise scheme. In contrast, by definition of $\widehat{k}$ and continuity, for $k \in (1/2, \widehat{k})$ there must be a positive measure subset of p belonging to $(\rho, \overline{p})$ on which the safe action is not taken under collective expertise after message m^0. Moreover, the optimal action will be message-contingent there, as $\Pr(\omega = 1|\sigma = (1,1))|_{p=\overline{p}} > 1/2$. Thus, when $k \in (1/2, \widehat{k})$, collective expertise strictly dominates independent one for a positive measure subset of p belonging to $(\rho, \overline{p})$. This completes the proof of statements (ii) and (iii).

Finally, let us show that $\overline{k} > \widehat{k}$, as we conjectured. Preliminarily, notice that, for any collection of independent signals $\sigma_1, \sigma_2, ...\sigma_n$, the following is true:

$$\Pr(\omega = 0|\sigma_1, ...\sigma_n)|_{p=p'} = \Pr(\omega = 0|\sigma_{m+1}, ...\sigma_n)|_{p=p''},$$
$$\text{where } p'' = \Pr(\omega = 0|\sigma_1, ...\sigma_m)|_{p=p'}$$

It is straightforward to show that $\Pr(\omega = 0|\sigma = 0)|_{p=1/2} = \rho$. It follows then that $\overline{k} \equiv \Pr(\omega = 0|\sigma = (0,0))|_{p=\rho} = \Pr(\omega = 0|\sigma = (0,0,0))|_{p=1/2}$.

Next, since $\Pr(\omega = 1|\sigma = (1,1))|_{p=\overline{p}} > 1/2$ (according to Lemma 4), we have $\Pr(\omega = 1|\sigma = (1,1))|_{p=1/2} > \overline{p}$, and, by symmetry, $\Pr(\omega = 0|\sigma = (0,0))|_{p=1/2} > \overline{p}$ This implies that $\Pr(\omega = 0|\sigma = 0)|_{p=\overline{p}} < \Pr(\omega = 0|\sigma = (0,0,0))|_{p=1/2}$. Thus, the following chain of relationships is true:

$$\widehat{k} \equiv \Pr(\omega = 0|\sigma \in \{(0,0),(0,1),(1,0)\})|_{p=\overline{p}} < \Pr(\omega = 0|\sigma \in \{(0,0),(0,1)\})|_{p=\overline{p}}$$
$$= \Pr(\omega = 0|\sigma_1 = 0)|_{p=\overline{p}} < \Pr(\omega = 0|\sigma = (0,0,0))|_{p=1/2} = \overline{k}$$

■

Proof of Lemma 8. Suppose all non-deputy experts have truthfully revealed their signals to the deputy. Let message m' be a message sent with a *positive* probability by the deputy when she is the only one with signal 1. Full revelation requires that, for any vector of signals, the number of zeros is truthfully revealed. This implies that, in a fully revealing equilibrium, message m' can only be sent when the number of zeros is $n-1$.

We would like to show that it is profitable for signal-type $(1, 0...0)$ (i.e., the deputy when she is the only one with signal 1) to deviate to reporting $(0...0)$, that is,
$\alpha(m = (0,...0), \sigma = (1,0...0)) > \alpha(m = m', \sigma = (1,0...0))$.

Denote $\Pr(\omega = 0|\sigma = (1,0...0)) := \pi > 1/2$.

Since $\Pr(\sigma_1 = 0|m = (0...0)) = 1$ in a fully revealing equilibrium, irrespective of the realized state, we have

$$\begin{aligned}\alpha(m &= (0,...0), \sigma = (1,0...0)) \\ &= \Pr(\omega = 0|\sigma = (1,0...0)) \cdot 1 + \Pr(\omega = 1|\sigma = (1,0...0)) \cdot 0 = \pi\end{aligned}$$

Now compute $\alpha(m = m', \sigma = (1,0...0))$.

Denote the set of all vectors of the experts' signals containing only one signal 1 by Σ.

Denote:

$$\begin{aligned}\mu &:= \Pr(m = m'|\sigma = (1,0...0)) \\ \nu &:= \Pr(m = m'|\sigma \in \Sigma\backslash(1,0...0))\end{aligned}$$

Then, using the facts that, in a fully revealing equilibrium, message m' can only be sent by signal-types from Σ, all vectors in Σ are equally likely, and there are n vectors in Σ, we can derive:

$$\Pr(\sigma_1 = 0|\omega = 0, m = m') = \frac{\Pr(\sigma_1 = 0 \cap m = m'|\omega = 0)}{num. + \Pr(\sigma_1 = 1 \cap m = m'|\omega = 0)}$$
$$= \frac{\nu(n-1)\rho^{n-1}(1-\rho)}{\nu(n-1)\rho^{n-1}(1-\rho) + \mu(1-\rho)\rho^{n-1}} = \frac{\nu(n-1)}{\nu(n-1)+\mu} =: \gamma$$

$$\Pr(\sigma_1 = 1|\omega = 1, m = m') = \frac{\Pr(\sigma_1 = 1 \cap m = m'|\omega = 1)}{num. + \Pr(\sigma_1 = 0 \cap m = m'|\omega = 1)}$$
$$= \frac{\mu\rho(1-\rho)^{n-1}}{\mu\rho(1-\rho)^{n-1} + \nu(n-1)(1-\rho)^{n-1}\rho} = \frac{\mu}{\nu(n-1)+\mu}$$
$$= 1-\gamma$$

Hence,

$$\alpha(m = m', \sigma = (1,0...0)) = \pi\gamma + (1-\pi)(1-\gamma)$$

Since, by assumption, $\mu > 0$, $1-\gamma > 0$, and, thus, $\alpha(m = m', \sigma = (1,0...0)) < \pi \equiv \alpha(m = (0,...0), \sigma = (1,0...0))$. ■

Proof of Lemma 9. Consider two non-reputation-equivalent equilibrium messages, m^1 and m^2. Assume, without loss of generality, that $\Pr(G|\omega = 0, m^1) > \Pr(G|\omega = 0, m^2)$. This implies that $\Pr(G|\omega = 1, m^2) > \Pr(G|\omega = 1, m^1)$, otherwise the experts would always strictly prefer to send m^1 instead of m^2. Let k' be the smallest number of zeros for which the experts send m^1 with positive probability. It must be that

$$\begin{aligned} \Pr(G|\omega &= 0, m^1) \Pr(\omega = 0|k') + \Pr(G|\omega = 1, m^1) \Pr(\omega = 1|k') \\ &\geq \Pr(G|\omega = 0, m^2) \Pr(\omega = 0|k') + \Pr(G|\omega = 1, m^2) \Pr(\omega = 1|k'). \end{aligned}$$

or, rearranging terms,

$$\begin{aligned} \Pr(\omega &= 0|k')[\Pr(G|\omega = 0, m^1) - \Pr(G|\omega = 0, m^2)] \\ &\geq \Pr(\omega = 1|k')[\Pr(G|\omega = 1, m^2) - \Pr(G|\omega = 1, m^1)]. \end{aligned}$$

For any $k'' > k'$ we have $\Pr(\omega = 0|k'') > \Pr(\omega = 0|k')$. Hence,

$$\begin{aligned} \Pr(\omega &= 0|k'')[\Pr(G|\omega = 0, m^1) - \Pr(G|\omega = 0, m^2)] \\ &> \Pr(\omega = 1|k'')[\Pr(G|\omega = 1, m^2) - \Pr(G|\omega = 1, m^1)]. \end{aligned}$$

Thus, the experts with $k'' > k'$ zeros never send m^2. Recall that, by definition of k', the experts with $k'' < k'$ zeros never send m^1. Hence, k' is the desired k. ■

Proof of Lemma 10. As we consider equilibria with reputationally distinct messages only, then, due to Lemma 9, for numbers of zeros strictly between l and r, the supported message must be sent with probability 1.

We will first consider the case when signal-types l and r send m with probability 1 as well; then we will generalize the argument by allowing for randomization by the threshold types.

For notational convenience, a profile of signals will be identified with the number of zeros it contains and a set of profiles will be identified with a message that communicates it. As we have shown in "Preliminaries to proofs", comparing expected reputations boils down to comparing $\alpha(m, I)$. When the experts have received k zeros and the deputy sends message m, for each expert i we have:

$$\alpha(m,k) = \Pr(\omega = 0|k)\Pr(\sigma_i = 0|\omega = 0, m) + \Pr(\omega = 1|k)\Pr(\sigma_i = 1|\omega = 1, m)$$

We will first consider how the expected reputation of the expert changes when the experts have received $k = r$ or $k = l$ zeros and the opposite boundary of the message is "marginally" increased from l to $l + 1$ in the first case and decreased from r to $r - 1$ in the second case. Then we will argue that if an expert's expected reputation weakly rises after a given marginal change, it will weakly increase following the next marginal change, up to when the message coincides with revealing r in the first case and revealing l in the second case.

Consider an alternative message m' which is interpreted as "the experts have received between $l + 1$ and r zeros" or "the experts have received between l and $r - 1$ zeros." Denote by t the number of zeros left out by a given marginal cut, and let k be the intact boundary. Then, $t = l$ and $k = r$ in the first case, and $t = r$ and $k = l$ in the second case. Hence, for both cases, $\alpha(m, k)$ can be rewritten as:

$$\begin{aligned}
&\Pr(\omega = 0|k)(\Pr(\sigma_i = 0|t)\Pr(t|\omega = 0, m) + \Pr(\sigma_i = 0|\omega = 0, m')\Pr(m'|\omega = 0, m)) \\
&+\Pr(\omega = 1|k)(\Pr(\sigma_i = 1|t)\Pr(t|\omega = 1, m) + \Pr(\sigma_i = 1|\omega = 1, m')\Pr(m'|\omega = 1, m)) \\
&= \Pr(\omega = 0|k)(\Pr(\sigma_i = 0|t)\Pr(t|\omega = 0, m) + \Pr(\sigma_i = 0|\omega = 0, m')(1 - \Pr(t|\omega = 0, m)) \\
&+\Pr(\omega = 1|k)(\Pr(\sigma_i = 1|t)\Pr(t|\omega = 1, m) + \Pr(\sigma_i = 1|\omega = 1, m')(1 - \Pr(t|\omega = 1, m)) \\
&= \Pr(\omega = 0|k)\Pr(t|\omega = 0, m)(\Pr(\sigma_i = 0|t) - \Pr(\sigma_i = 0|\omega = 0, m')) \\
&+ \Pr(\omega = 1|k)\Pr(t|\omega = 1, m)(\Pr(\sigma_i = 1|t) - \Pr(\sigma_i = 1|\omega = 1, m')) + \alpha(m', k).
\end{aligned}$$

Hence, we have $\alpha(m, k) \leq \alpha(m', k)$ if and only if

$$\begin{aligned}
&\Pr(\omega = 0|k)\Pr(t|\omega = 0, m)(\Pr(\sigma_i = 0|t) - \Pr(\sigma_i = 0|\omega = 0, m')) \\
&\quad + \Pr(\omega = 1|k)\Pr(t|\omega = 1, m)(\Pr(\sigma_i = 1|t) - \Pr(\sigma_i = 1|\omega = 1, m')) \leq 0.
\end{aligned}$$

First, note that in the first case

$$\Pr(\sigma_i = 0|l) - \Pr(\sigma_i = 0|\omega = 0, m') = 1 - \Pr(\sigma_i = 1|l) - \Pr(\sigma_i = 0|\omega = 0, m')$$
$$\leq 1 - \Pr(\sigma_i = 1|l) - (1 - \Pr(\sigma_i = 1|\omega = 1, m')) = \Pr(\sigma_i = 1|\omega = 1, m') - \Pr(\sigma_i = 1|l) < 0,$$

and in the second case

$$\Pr(\sigma_i = 1|r) - \Pr(\sigma_i = 1|\omega = 1, m') = 1 - \Pr(\sigma_i = 0|r) - \Pr(\sigma_i = 1|\omega = 1, m')$$
$$\leq 1 - \Pr(\sigma_i = 0|r) - (1 - \Pr(\sigma_i = 0|\omega = 0, m')) = \Pr(\sigma_i = 0|\omega = 0, m') - \Pr(\sigma_i = 0|r) < 0,$$

So, in the first case, we have $\alpha(m, k) \leq \alpha(m', k)$ if

$$\Pr(\omega = 0|r) \Pr(l|\omega = 0, m) \geq \Pr(\omega = 1|r) \Pr(l|\omega = 1, m)$$
$$\Leftrightarrow$$
$$\frac{\Pr(r|\omega = 0) \Pr(\omega = 0)}{\Pr(r)} \frac{\Pr(l|\omega = 0)}{\Pr(m|\omega = 0)} \geq \frac{\Pr(r|\omega = 1) \Pr(\omega = 1)}{\Pr(r)} \frac{\Pr(l|\omega = 1)}{\Pr(m|\omega = 1)} \tag{4}$$
$$\Leftrightarrow \frac{\Pr(m|\omega = 1)}{\Pr(m|\omega = 0)} \geq \frac{\Pr(l|\omega = 1) \Pr(r|\omega = 1) \Pr(\omega = 1)}{\Pr(l|\omega = 0) \Pr(r|\omega = 0) \Pr(\omega = 0)}$$

In the second case, we have $\alpha(m, k) \leq \alpha(m', k)$ if

$$\Pr(\omega = 0|l) \Pr(r|\omega = 0, m) \leq \Pr(\omega = 1|l) \Pr(r|\omega = 1, m)$$
$$\Leftrightarrow$$
$$\frac{\Pr(l|\omega = 0) \Pr(\omega = 0)}{\Pr(l)} \frac{\Pr(r|\omega = 0)}{\Pr(m|\omega = 0)} \leq \frac{\Pr(l|\omega = 1) \Pr(\omega = 1)}{\Pr(l)} \frac{\Pr(r|\omega = 1)}{\Pr(m|\omega = 1)} \tag{5}$$
$$\Leftrightarrow \frac{\Pr(m|\omega = 1)}{\Pr(m|\omega = 0)} \leq \frac{\Pr(l|\omega = 1) \Pr(r|\omega = 1) \Pr(\omega = 1)}{\Pr(l|\omega = 0) \Pr(r|\omega = 0) \Pr(\omega = 0)}$$

Clearly, at least one of the two must be true. Without loss of generality, suppose now that the first is true, i.e. that

$$\frac{\Pr(m|\omega = 1) \Pr(l|\omega = 0)}{\Pr(m|\omega = 0) \Pr(l|\omega = 1)} \geq \frac{\Pr(r|\omega = 1) \Pr(\omega = 1)}{\Pr(r|\omega = 0) \Pr(\omega = 0)}.$$

Now we want to show that if the passage from l to $l + 1$ weakly increases expert i's expected reputation, so does the passage from $l+1$ to $l+2$. By induction, this will imply that so does *any* increase in l. So, we want to show that $\alpha(m',k) \leq \alpha(m'',k)$, where m'' is the message which is interpreted as "the experts have received between $l+2$ and r zeros." We just need to show that the above inequality holds for $l+1$. Note that

$$\frac{\Pr(m|\omega=1)\Pr(l|\omega=0)}{\Pr(m|\omega=0)\Pr(l|\omega=1)} = \frac{\frac{\sum_{t=l}^{r}\Pr(t|\omega=1)}{\Pr(l|\omega=1)}}{\frac{\sum_{t=l}^{r}\Pr(t|\omega=0)}{\Pr(l|\omega=0)}}$$

$$= \frac{\sum_{t=l}^{r}\left(\frac{1-\rho}{\rho}\right)^{t-l}\frac{\binom{n}{t}}{\binom{n}{l}}}{\sum_{t=l}^{r}\left(\frac{\rho}{1-\rho}\right)^{t-l}\frac{\binom{n}{t}}{\binom{n}{l}}} = \frac{\sum_{t=l}^{r}\left(\frac{1-\rho}{\rho}\right)^{t-l}\binom{n}{t}}{\sum_{t=l}^{r}\left(\frac{\rho}{1-\rho}\right)^{t-l}\binom{n}{t}}. \tag{6}$$

We just need to show that raising l by 1 increases the above ratio. This can be shown through the following chain of relations.

$$\frac{\sum_{t=l+1}^{r}\left(\frac{1-\rho}{\rho}\right)^{t-l-1}\binom{n}{t}}{\sum_{t=l+1}^{r}\left(\frac{\rho}{1-\rho}\right)^{t-l-1}\binom{n}{t}} = \frac{\sum_{t=l}^{r-1}\left(\frac{1-\rho}{\rho}\right)^{t-l}\binom{n}{t+1}}{\sum_{t=l}^{r-1}\left(\frac{\rho}{1-\rho}\right)^{t-l}\binom{n}{t+1}} = \frac{\sum_{t=l}^{r-1}\left(\frac{1-\rho}{\rho}\right)^{t-l}\binom{n}{t}\frac{n-t}{t+1}}{\sum_{t=l}^{r-1}\left(\frac{\rho}{1-\rho}\right)^{t-l}\binom{n}{t}\frac{n-t}{t+1}}$$

$$= \frac{\sum_{t=l}^{r-1}\left(\frac{1-\rho}{\rho}\right)^{t-l}\binom{n}{t}\frac{n-t}{t+1} + \left(\frac{1-\rho}{\rho}\right)^{r-l}\cdot 0}{\sum_{t=l}^{r-1}\left(\frac{\rho}{1-\rho}\right)^{t-l}\binom{n}{t}\frac{n-t}{t+1} + \left(\frac{\rho}{1-\rho}\right)^{r-l}\cdot 0} > \frac{\sum_{t=l}^{r}\left(\frac{1-\rho}{\rho}\right)^{t-l}\binom{n}{t}}{\sum_{t=l}^{r}\left(\frac{\rho}{1-\rho}\right)^{t-l}\binom{n}{t}} \tag{7}$$

The equalities in this formula are obvious, while the inequality in the second line is due to the following argument. Consider a class of ratios of the form

$$\frac{a_1x_1 + a_2x_2 + ... + a_mx_m}{a_1y_1 + a_2y_2 + ... + a_my_m}, \tag{8}$$

such that

$$\begin{aligned} x_1 &> x_2 > ... > x_m, \\ y_1 &< y_2 < ... < y_m, \\ a_t &> 0 \forall t \in \{1, ...m-1\},\ a_m \geq 0 \end{aligned}$$

The left-hand side ratio in the inequality belongs to this class, with $x_t = \left(\frac{1-\rho}{\rho}\right)^{t-l}$, $y_t = \left(\frac{\rho}{1-\rho}\right)^{t-l}$, $a_t = \begin{pmatrix} n \\ t \end{pmatrix} \frac{n-t}{t+1}$ for $t = l, ...r-1$, $a_r = 0$, and $m = r$.

Consider a transformation to coefficients a_t such that each subsequent coefficient is multiplied by a higher number. Let us call the new coefficients b_t; b_t/a_t grows with t. The transformed ratio is

$$\frac{b_1x_1 + b_2x_2 + ... + b_mx_m}{b_1y_1 + b_2y_2 + ... + b_my_m},$$

The right-hand side ratio in the inequality is obtained through exactly this type of transformation: for $t = l, ...r-1$ each term in the left-hand side is multiplied by $\dfrac{t+1}{n-t}$, which grows with t, and the last term is multiplied by ∞.

Our argument then consists of two steps:

1. Any such transformation of the whole ratio can be achieved by a sequence of transformations of the form

$$\frac{a_1x_1 + ... + a_sx_s + c(a_{s+1}x_{s+1} + ... + a_mx_m)}{a_1y_1 + ... + a_sy_s + c(a_{s+1}y_{s+1} + ... + a_my_m)}, \tag{9}$$

where $c > 1$, and c and s are different for each transformation.

2. Any such intermediate transformation reduces the ratio. Hence, the overall transformation reduces the ratio as well.

The first step is trivial. We first need to multiply all coefficients by b_1/a_1 to make the first coefficient b_1, then all coefficients starting from the second one by $\frac{b_2 a_1}{b_1 a_2}$ to make the second coefficient b_2, and so on.

To prove the second statement, differentiate (9) with respect to c. We obtain:

$$\frac{(a_{s+1}x_{s+1} + ... + a_m x_m) \cdot denom - (a_{s+1}y_{s+1} + ... + a_m y_m) \cdot num}{denom^2}$$
$$= \frac{(a_{s+1}x_{s+1} + ... + a_m x_m)(a_1 y_1 + ... + a_s y_s) - (a_{s+1}y_{s+1} + ... + a_m y_m)(a_1 x_1 + ... + a_s x_s)}{denom^2}$$

The numerator is the sum of the following terms: $a_i a_j (x_i y_j - x_j y_i)$, where $i > j$. Thus, by the properties of the sequences of x_i and y_i, each of the terms is negative. Thus, the derivative is negative, meaning that the considered multiplication by $c > 1$ reduces the ratio. Since, after each intermediate transformation, the ratio preserves the form of (8), the statement holds for all intermediate transformations.

Let us show now that if the expert's expected reputation when the experts received r zeros is higher by revealing it (message "r") than by sending m, then the expert considers state 0 strictly more likely; and likewise for l with state 1. Suppose not; that is, suppose that $\Pr(\omega = 0|r) \leq 1/2$. Note that

$$\begin{aligned}
\Pr(\sigma_i &= 0|\omega = 0, m) + \Pr(\sigma_i = 1|\omega = 1, m) \\
&> \Pr(\sigma_i = 0|\omega = 0, r) + \Pr(\sigma_i = 1|\omega = 1, r) = 1; \\
\Pr(\sigma_i &= 0|\omega = 0, m) < \Pr(\sigma_i = 0|\omega = 0, r); \\
\Pr(\sigma_i &= 1|\omega = 1, m) > \Pr(\sigma_i = 1|\omega = 1, r).
\end{aligned}$$

Then, $\alpha(m, r) < \alpha(r, r)$, a contradiction.

Finally, let us show that the above proof generalizes to the case when signal-types l and/or r are allowed to send m with probability below 1. First, notice that until formula (4) no derivation relies on r or l playing a pure strategy (m' keeps the meaning of the set of all profiles with the number of zeros between $l + 1$ and r or between l and $r - 1$, inclusive).

Derivations in (4) need to be slightly modified:

$$\begin{aligned}
&\Pr(\omega=0|r)\Pr(l|\omega=0,m) \geq \Pr(\omega=1|r)\Pr(l|\omega=1,m) \\
\Leftrightarrow\ &\frac{\Pr(r|\omega=0)\Pr(\omega=0)}{\Pr(r)}\frac{\Pr(m|\omega=0,l)\Pr(l|\omega=0)}{\Pr(m|\omega=0)} \\
\geq\ &\frac{\Pr(r|\omega=1)\Pr(\omega=1)}{\Pr(r)}\frac{\Pr(m|\omega=1,l)\Pr(l|\omega=1)}{\Pr(m|\omega=1)}
\end{aligned}$$

Since $\Pr(m|\omega,l)$ does not depend on ω, the new terms that appeared in both sides cancel out and, thus, we obtain the same condition as before:

$$\frac{\Pr(m|\omega=1)}{\Pr(m|\omega=0)} \geq \frac{\Pr(l|\omega=1)\Pr(r|\omega=1)\Pr(\omega=1)}{\Pr(l|\omega=0)\Pr(r|\omega=0)\Pr(\omega=0)}$$

The same is true for (5).

Formula (6) changes in the following way. In the summations $\sum_{t=l}^{r}\Pr(t|\omega=1)$ and $\sum_{t=l}^{r}\Pr(t|\omega=0)$ the first and the last term have to be multiplied by $\Pr(m|l)$ and $\Pr(m|r)$ respectively. Equivalently, $\begin{pmatrix} n \\ t \end{pmatrix}$ has to be multiplied by these terms for $t=l$ and $t=r$ in both the numerator and the denominator.

Correspondingly, in all summations in (7) the last term has to be just multiplied by a constant ($\Pr(m|r)$). Additionally, the first term in the summations in the right-hand side of the inequality has to be multiplied by $\Pr(m|l)$. Overall, these modifications can be considered as affecting only coefficients a_t and b_t and not affecting x_t and y_t. Moreover, it is easy to check that the desired property of the transformation of a_t's into b_t's is preserved. Thus, the argument goes through. ■

Proof of Lemma 11. The value of α from revealing k signals equal to $\overline{\omega}$ is

$$\begin{aligned}
\alpha(k,k) &= \Pr(\omega=\overline{\omega}|k)\Pr(\sigma_i=\overline{\omega}|\omega=\overline{\omega},k)+\Pr(\omega\neq\overline{\omega}|k)\Pr(\sigma_i\neq\overline{\omega}|\omega\neq\overline{\omega},k) \\
&= \Pr(\omega=\overline{\omega}|k)\frac{k}{n}+\Pr(\omega\neq\overline{\omega}|k)\frac{n-k}{n}
\end{aligned}$$

If, instead, m is sent,

$$\alpha(m,k)=\Pr(\omega=\overline{\omega}|k)\Pr(\sigma_i=\overline{\omega}|\omega=\overline{\omega},m)+\Pr(\omega\neq\overline{\omega}|k)\Pr(\sigma_i\neq\overline{\omega}|\omega\neq\overline{\omega},m)$$

Since $\Pr(\omega = \overline{\omega}|k) > \Pr(\omega \neq \overline{\omega}|k)$, $\Pr(\sigma_i = \overline{\omega}|\omega = \overline{\omega}, m) > \frac{k}{n}$ and $\Pr(\sigma_i = \overline{\omega}|\omega = \overline{\omega}, m) + \Pr(\sigma_i \neq \overline{\omega}|\omega \neq \overline{\omega}, m) \geq 1 = \frac{k}{n} + \frac{n-k}{n}$, we have

$$\alpha(m, k) > \alpha(k, k)$$

■

Proof of Lemma 12. The value of α from revealing k zeros is

$$\begin{aligned} \alpha(k,k) &= \frac{1}{2}\Pr(\sigma_i = 0|\omega = 0, k) + \frac{1}{2}\Pr(\sigma_i = 1|\omega = 1, k) \\ &= \frac{1}{2}\frac{k}{n} + \frac{1}{2}\frac{n-k}{n} = \frac{1}{2}. \end{aligned}$$

If, instead, m is sent,

$$\alpha(m,k) = \frac{1}{2}\Pr(\sigma_i = 0|\omega = 0, m) + \frac{1}{2}\Pr(\sigma_i = 1|\omega = 1, m).$$

Since message m is sent with positive probability for both k' and $k'' > k'$ zeros, $\Pr(\sigma_i = 1|\omega, m)$ depends on ω, which implies that $\Pr(\sigma_i = 1|\omega = 1, m) > \Pr(\sigma_i = 1|\omega = 0, m)$. Then, since $\Pr(\sigma_i = 0|\omega = 0, m) + \Pr(\sigma_i = 1|\omega = 0, m) = 1$, we have $\Pr(\sigma_i = 0|\omega = 0, m) + \Pr(\sigma_i = 1|\omega = 1, m) > 1$. Hence,

$$\alpha(m, k) > \alpha(k, k).$$

■

Proof of Lemma 13.

For every $p \leq \rho$, the expected reputation of the experts with all ones from revealing themselves is not lower than q:

$$\Pr(G|\sigma = \overrightarrow{1}) = \Pr(\omega = 1|\sigma = \overrightarrow{1})x + \Pr(\omega = 0|\sigma = \overrightarrow{1})y \geq \rho x + (1-\rho)y = q,$$

because, by $p \leq \rho$ and $n > 1$,

$$\begin{aligned}\Pr(\omega &= 1|\sigma = \overrightarrow{1}) = \frac{\rho^n(1-p)}{\rho^n(1-p) + (1-\rho)^n p} \\ &= \rho\left(\rho + (1-\rho)\frac{p}{(1-p)}\left(\frac{1-\rho}{\rho}\right)^{n-1}\right)^{-1} \geq \rho\left(\rho + (1-\rho)\right)^{-1} = \rho.\end{aligned}$$

Instead, the expected reputation from the complementary message, m^0, conditional on having received all ones is strictly lower than q. To see this, note first that the unconditional expected reputation can be written as

$$q = \Pr(\sigma \neq \overrightarrow{1})\Pr(G|\sigma \neq \overrightarrow{1}) + \Pr(\sigma = \overrightarrow{1})\Pr(G|\sigma = \overrightarrow{1})$$

Since, as we have just shown, $\Pr(G|\sigma = \overrightarrow{1}) \geq q$, it must be that $\Pr(G|\sigma \neq \overrightarrow{1}) \leq q$. Now, the expected reputation from message m^0, conditional on having received all ones, is

$$\begin{aligned}&R_i(m^0, \sigma = \overrightarrow{1}) \\ &= \Pr(\omega = 1|\sigma = \overrightarrow{1})\Pr(G|\omega = 1, \sigma \neq \overrightarrow{1}) + \Pr(\omega = 0|\sigma = \overrightarrow{1})\Pr(G|\omega = 0, \sigma \neq \overrightarrow{1}) \\ &< \Pr(\omega = 1|\sigma \neq \overrightarrow{1})\Pr(G|\omega = 1, \sigma \neq \overrightarrow{1}) + \Pr(\omega = 0|\sigma \neq \overrightarrow{1})\Pr(G|\omega = 0, \sigma \neq \overrightarrow{1}) \\ &= R_i(m^0, \sigma \neq \overrightarrow{1}) \equiv \Pr(G|\sigma \neq \overrightarrow{1}),\end{aligned}$$

where the inequality follows from the observation that $\Pr(\omega = 0|\sigma \neq \overrightarrow{1}) > \Pr(\omega = 0|\sigma = \overrightarrow{1})$ and $\Pr(G|\omega = 0, \sigma \neq \overrightarrow{1}) > \Pr(G|\omega = 1, \sigma \neq \overrightarrow{1})$.

Thus, $R_i(m^0, \sigma = \overrightarrow{1}) < q$. This implies that the incentive compatibility constraint of the experts with all ones, $\Pr(G|\sigma = \overrightarrow{1}) \geq R_i(m^0, \sigma = \overrightarrow{1})$, holds as a *strict* inequality when $p = \rho$, and, by continuity, for some $p > \rho$.

The incentive compatibility constraint can be rewritten as

$$\begin{aligned}\Pr(\omega &= 1|\sigma = \overrightarrow{1})(x - \Pr(G|\omega = 1, \sigma \neq \overrightarrow{1})) \\ &\geq \Pr(\omega = 0|\sigma = \overrightarrow{1})(\Pr(G|\omega = 0, \sigma \neq \overrightarrow{1}) - y)\end{aligned}$$

Obviously, $x > \Pr(G|\omega = 1, \sigma \neq \overrightarrow{1})$ and $\Pr(G|\omega = 0, \sigma \neq \overrightarrow{1}) > y$. Thus, since $\Pr(\omega = 1|\sigma = \overrightarrow{1})$ is decreasing in p and $\Pr(G|\omega, \sigma)$ does not depend on p, the left-hand side is decreasing in p, whereas the right-hand side is increasing in p. Furthermore, the condition is violated when $p = 1$, because then $\Pr(\omega = 1|\sigma = \overrightarrow{1}) = 0$. Hence, the existence of $\overline{p} > \rho$ follows; it is determined by $\Pr(G|\sigma = \overrightarrow{1}) = R_i(m^0, \sigma = \overrightarrow{1})$.

For given p, when $n \to \infty$, $\Pr(\omega = 1|\sigma = \overrightarrow{1}) \to 1$, $\Pr(\omega = 0|\sigma = \overrightarrow{1}) \to 0$ and $\Pr(G|\omega = 1, \sigma \neq \overrightarrow{1}) \to q$. The last result follows from the following:

$$q \equiv \Pr(G|\omega = 1)$$
$$= \Pr(G|\omega = 1, \sigma \neq \overrightarrow{1}) \Pr(\sigma \neq \overrightarrow{1}|\omega = 1) + \Pr(G|\omega = 1, \sigma = \overrightarrow{1}) \Pr(\sigma = \overrightarrow{1}|\omega = 1),$$
$$\Pr(G|\omega = 1, \sigma = \overrightarrow{1}) = const > 0,$$
$$\Pr(\sigma = \overrightarrow{1}|\omega = 1) \to 0 \text{ and} \Pr(\sigma \neq \overrightarrow{1}|\omega = 1) \to 1 \text{ as } n \to \infty.$$

From the definition of x and y (in the "Preliminaries to proofs" section of the Appendix), $y < q < x$. Thus, when $n \to \infty$, $\overline{p} \to 1$.

To see that at $\overline{p}$ the experts with all ones consider state 1 strictly more likely, note the following: if they would consider the two states equally likely, by Lemma 12 they would prefer to send the complementary message rather than revealing themselves; if they would consider state 0 strictly more likely, the same would be true by Lemma 11. ■

Proof of Lemma 14. Since all reputation-equivalent messages can be coalesced into one message without affecting the experts' incentives compatibility constraints, we can confine ourselves to considering equilibria with reputationally distinct messages only.

In this proof we will use the following result, derived in the proof of Lemma 9. If message m' generating a higher reputation than message m'' under state $\overline{\omega}$ ($\Pr(G|\omega = \overline{\omega}, m') > \Pr(G|\omega = \overline{\omega}, m'')$) is weakly preferred to m'' by the experts with k signals equal to $\overline{\omega}$, then it is strictly preferred by the experts with a greater number of signals equal to $\overline{\omega}$.

Fix $k \in \{1, ..., n\}$. Let m^1 denote the message "at most $k-1$ zeros" and m^0 the message "at least k zeros". The two incentive compatibility constraints of the threshold profiles, i.e., those with $k-1$ and k zeros read:

$$\begin{aligned} &\Pr(\omega = 1|k-1)\Pr(G|\omega = 1, m^1) + \Pr(\omega = 0|k-1)\Pr(G|\omega = 0, m^1) \\ &\geq \Pr(\omega = 1|k-1)\Pr(G|\omega = 1, m^0) + \Pr(\omega = 0|k-1)\Pr(G|\omega = 0, m^0) \end{aligned} \tag{IC1}$$

and

$$\begin{aligned} &\Pr(\omega = 1|k)\Pr(G|\omega = 1, m^0) + \Pr(\omega = 0|k)\Pr(G|\omega = 0, m^0) \\ &\geq \Pr(\omega = 1|k)\Pr(G|\omega = 1, m^1) + \Pr(\omega = 0|k)\Pr(G|\omega = 0, m^1). \end{aligned} \tag{IC0}$$

Note that by $\Pr(\omega = 1|k-1) > \Pr(\omega = 1|k)$, $\Pr(G|\omega = 1, m^1) > \Pr(G|\omega = 0, m^1)$ and $\Pr(G|\omega = 0, m^0) > \Pr(G|\omega = 1, m^0)$, the following holds:

$$\begin{aligned} &\Pr(\omega = 1|k-1)[\Pr(G|\omega = 1, m^1) + \Pr(G|\omega = 0, m^0) - \Pr(G|\omega = 1, m^0) - \Pr(G|\omega = 0, m^1)] \\ &\geq \Pr(\omega = 1|k)[\Pr(G|\omega = 1, m^1) + \Pr(G|\omega = 0, m^0) - \Pr(G|\omega = 1, m^0) - \Pr(G|\omega = 0, m^1)], \end{aligned}$$

which can be rewritten as the sum of the two IC constraints:

$$\begin{aligned} &\Pr(\omega = 1|k-1)\Pr(G|\omega = 1, m^1) + \Pr(\omega = 0|k-1)\Pr(G|\omega = 0, m^1) \\ &\quad + \Pr(\omega = 1|k)\Pr(G|\omega = 1, m^0) + \Pr(\omega = 0|k)\Pr(G|\omega = 0, m^0) \\ &\geq \Pr(\omega = 1|k-1)\Pr(G|\omega = 1, m^0) + \Pr(\omega = 0|k-1)\Pr(G|\omega = 0, m^0) \\ &\quad + \Pr(\omega = 1|k)\Pr(G|\omega = 1, m^1) + \Pr(\omega = 0|k)\Pr(G|\omega = 0, m^1). \end{aligned}$$

Hence, at least one of the two IC must be satisfied. First, note that for every $p > 1/2$, the experts with n zeros prefer to reveal themselves rather than sending the complementary message. This comes from the fact that, by Lemma 13, the experts with n ones would prefer to reveal themselves for $p < 1/2$ (if $p < 1/2$ were allowed by the assumptions of the model), and the problem of "n zeros" for $p > 1/2$ is identical to that of "n ones" for $p < 1/2$. So, for $k = n$ (IC0) is satisfied.

Second, *suppose* that there is $k' > 0$ such that (IC1) is satisfied for $k = k'$ (i.e., for experts with $k' - 1$ zeros). Due to Lemma 13 this supposition is correct at least up to $p = \overline{p}$. Then, two situations are possible.

If there is $k \in \{k', ..., n\}$ such that both (IC1) and (IC0) are satisfied, then other signal-types will not deviate from their respective messages (m^1 or m^0) either (by the argument presented in the beginning of the proof), and thus we have the desired equilibrium.

If there is no such k, then there must be $k := \widehat{k} \in \{k', ..., n\}$ where the inversion happens, i.e., such that:

1) Only (IC1) is satisfied for $k = \widehat{k}$ (i.e., for experts with $\widehat{k} - 1$ zeros) and only (IC0) is satisfied for $k = \widehat{k} + 1$ (i.e., for experts with $\widehat{k} + 1$ zeros);

2) The experts with $\widehat{k}$ zeros prefer both the message "at most $\widehat{k} - 1$ zeros" to the message "at least $\widehat{k}$ zeros" and the message "at least $\widehat{k} + 1$ zeros" to the message "at most $\widehat{k}$ zeros".

So, calling $\widehat{m}^1$ and $\widehat{m}^0$ two messages sent with probabilities η and $1 - \eta$ by profile $\widehat{k}$ and with probability 1 by the experts with, respectively, less and more than $\widehat{k}$ zeros, we have $\widehat{m}^1 \succ \widehat{m}^0$ for $\eta = 0$ (as it becomes (IC1) for experts with $\widehat{k} - 1$ zeros) and $\widehat{m}^0 \succ \widehat{m}^1$ for $\eta = 1$ (as it becomes (IC0) for experts with $\widehat{k} + 1$ zeros). Therefore, by continuity, there exists η such that profile $\widehat{k}$ is indifferent between $\widehat{m}^1$ and $\widehat{m}^0$, and then the other profiles strictly prefer the message they are supposed to send (by the argument presented in the beginning of the proof). So we have the desired equilibrium.

So far, we have shown that a bipartitional equilibrium definitely exists for all $p \in (1/2, \overline{p}]$. The next step is to show that it definitely does not exist for any $p > \overline{\overline{p}}$. By the definition of $\overline{\overline{p}}$, when $p > \overline{\overline{p}}$, the experts with *any* profile of signals believe that $\omega = 0$ is more likely. Then, due to Lemmas 10 and 11, the experts with the highest number of zeros in the support of $\widehat{m}^1$ would deviate to $\widehat{m}^0$.

What remains to show is the existence of a precise $\widehat{p} \in \left[\overline{p}, \overline{\overline{p}}\right]$ claimed in the Lemma. To establish this, it is enough to show that if a bipartitional equilibrium does not exist for given p, it does not exist for any higher p.

Suppose a bipartitional equilibrium does not exist for given p. This means that there is no k such such that (IC1) is satisfied (otherwise we would be able to construct an equilibrium the way we did above). This, in turn, implies that, for *any* possible partition of the numbers of zeros into two supports of messages $\widehat{m}^1$ and $\widehat{m}^0$, the experts with the highest number of zeros in the support of $\widehat{m}^1$ want to deviate to $\widehat{m}^0$. This is obvious in the case when $\widehat{m}^1$ and $\widehat{m}^0$ are "pure" messages, i.e., when $\eta \in \{0, 1\}$. If $\eta \in (0, 1)$, then, given that for no k (IC1) holds, if the threshold signal-type would want to deviate to $\widehat{m}^1$ instead of $\widehat{m}^0$, we could raise η to the point where she would become indifferent between $\widehat{m}^1$ and $\widehat{m}^0$. But then, contrary to our assumption, we would achieve an equilibrium. Now, if the threshold signal-type of $\widehat{m}^1$ always wants to deviate to $\widehat{m}^0$ for some p, she will clearly do so for a higher p, as a rise in p makes $\widehat{m}^0$ more attractive.

To conclude, for *any* possible partition, the deviation incentive of the threshold signal-type in the support of $\widehat{m}^1$ is preserved with a rise in p. Hence, no bipartitional equilibrium for a given p implies no bipartitional equilibrium for any higher p. This establishes the existence of $\widehat{p}$. ■

Proof of Proposition 5. Let n go to infinity and consider any sequence of bipartitional equilibria $(m(n), m'(n))$. Since $\Pr(\sigma_i = 0|\omega = 0) = \Pr(\sigma_i = 1|\omega = 1)$, $\Pr(\omega|\sigma)$ depends on σ only through the difference between the number of zeros and the number of ones in σ. Thus, if k is the number of zeros in σ, $\Pr(\omega|\sigma)$ can be rewritten as $\Pr(\omega|k - (n - k)) = \Pr(\omega|2k - n)$. Suppose $k(n)$ is some sequence of k depending on n. Clearly, if $2k(n) - n \to \infty$, then $\lim_{n\to\infty} \Pr(\omega = 0|2k(n) - n) = 1$ (likewise for $\Pr(\omega = 1|n - 2k(n))$ when $n - 2k(n) \to \infty$). Hence, in order to keep the threshold type(s) from deviating to the neighboring message, the difference between the number of zeros and the number of ones must stay bounded for these types. Then

$$\lim_{n\to\infty} \Pr(m'(n)|\omega = 1) = 0,$$

because, conditional on $\omega = 1$, by the law of large numbers, the proportion of ones $\frac{n-k(n)}{n} \xrightarrow{P} \rho$, whereas it stays bounded away from ρ as $n \to \infty$ even for the type with the lowest number of zeros sending m'.

This means that

$$\Pr(\omega = 0|m'(n)) = \frac{\Pr(m'(n)|\omega = 0) \cdot p}{\Pr(m'(n)|\omega = 0) \cdot p + \Pr(m'(n)|\omega = 1) \cdot (1-p)} \to 1 \text{ as } n \to \infty$$

By similar reasoning, $\Pr(\omega = 1|m) \to 1$ as $n \to \infty$. ■

References

[1] Ali, S.N., and J.A. Bohren, 2019, "Should Straw Polls be Banned?" *Games and Economic Behavior*, 118, 284–294.

[2] Ambrus, A., and S. Takahashi, 2008, "Multi-sender cheap talk with restricted state spaces," *Theoretical Economics*, 3(1), 1–27.

[3] Austen-Smith, D., and T.J.and Feddersen, 2005, "Deliberation and voting rules," In Social Choice and Strategic Decisions, 269–316, Springer, Berlin, Heidelberg.

[4] Austen-Smith, D., and T.J. Feddersen, 2006, "Deliberation, preference uncertainty, and voting rules," *American Political Science Review*, 100(2), 209–217.

[5] Austen-Smith, D., and T.J. Feddersen, 2009, "Information aggregation and communication in committees," *Philosophical Transactions of the Royal Society B: Biological Sciences*, 364(1518), 763–769.

[6] Bag, P.K., and T. Sharma, 2019, "Sequential expert advice: Superiority of closed-door meetings," *International Economic Review*, fothcoming.

[7] Battaglini, M., 2002, "Multiple referrals and multidimensional cheap talk," *Econometrica*, 70(4), 1379–1401.

[8] Catonini, E., and S. Stepanov, 2019, "Reputation and Information Aggregation," Higher School of Economics WP9/2019/01.

[9] Coughlan, P.J., 2000, "In defense of unanimous jury verdicts: Mistrials, communication, and strategic voting," *American Political Science Review*, 94(2), 375–393.

[10] Crawford, V.P., and J. Sobel, 1982, "Strategic information transmission," *Econometrica*, 50(6), 1431–1451.

[11] Elliott, M., B. Golub, and A. Kirilenko, 2014, "How sharing information can garble experts' advice," *American Economic Review*, 104(5), 463–68.

[12] Fehrler, S., and N. Hughes, 2018, "How transparency kills information aggregation: theory and experiment," *American Economic Journal: Microeconomics*, 10(1), 181–209.

[13] Gerardi, D., and L. Yariv, 2007, "Deliberative voting," *Journal of Economic Theory*, 134(1), 317–338.

[14] Gilligan, T.W., and K. Krehbiel, 1989, "Asymmetric information and legislative rules with a heterogeneous committee," *American Journal of Political Science*, 33(2), 459–490.

[15] Hori, K., 2006, "Organizing information transmission with multiple agents," WIAS Discussion Paper No. 2008-010.

[16] Krishna, V., and J. Morgan, 2001a, "A Model of Expertise," *Quarterly Journal of Economics* 116, 747–775.

[17] Krishna, V., and J. Morgan, 2001b, "Asymmetric Information and Legislative Rules," *American Political Science Review* 95, 435–452.

[18] Levy, G., 2007, "Decision making in committees: Transparency, reputation, and voting rules," *American Economic Review*, 97(1), 150–168.

[19] Li, M., 2010, "Advice from multiple experts: A comparison of simultaneous, sequential, and hierarchical communication," *The BE Journal of Theoretical Economics*, 10(1).

[20] Li, Z., H. Rantakari, and H. Yang, 2016, "Competitive cheap talk," *Games and Economic Behavior*, 96, 65–89.

[21] McGee, A., and H. Yang, 2013, "Cheap talk with two senders and complementary information," *Games and Economic Behavior*, 79, 181–191.

[22] Meade, E.E., and D. Stasavage, 2008. "Publicity of debate and the incentive to dissent: evidence from the US Federal Reserve," *The Economic Journal*, 118(528), 695–717.

[23] Swank, O.H., and B. Visser, 2013, "Is transparency to no avail?" *The Scandinavian Journal of Economics*, 115(4), 967–994.

[24] Ottaviani, M., and P.N. Sørensen, 2001, "Information Aggregation in Debate: Who Should Speak First?" *Journal of Public Economics*, 81(3), 393–421.

[25] Ottaviani, M., and P.N. Sørensen, 2006a, "Professional advice," *Journal of Economic Theory*, 126(1), 120–142.

[26] Ottaviani, M., and P.N. Sørensen, 2006b, "Reputational Cheap Talk," *RAND Journal of Economics*, 37(1), 155–175.

[27] Prat, A., 2005, "The Wrong Kind of Transparency," *American Economic Review*, 95(3): 862–877.

[28] Van Weelden, R., 2008, "Deliberation Rules and Voting," *Quarterly Journal of Political Science*, 3: 83–88.

[29] Visser, B., and O.H. Swank, 2007, "On committees of experts," *The Quarterly Journal of Economics*, 122(1), 337–372.

[30] Wolinsky, A., 2002, "Eliciting information from multiple experts," *Games and Economic Behavior*, 41(1), 141–160.