Independent versus Collective Expertise
Online Appendix

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1 Heterogeneous experts case

Consider a setting with two experts and suppose they have different prior abilities: ρ_1 and $\rho_2 < \rho_1$. We will show that, unless the heterogeneity is too high, all qualitative results of the model with identical experts hold through. However, the difference between independent and collective expertise diminishes as the heterogeneity grows. At the end we will argue that it is weakly optimal to make the stronger expert the "deputy".

Under independent expertise, since the strategies of the experts are not related to each other, the solution is obviously as follows:

- for $p \leq \rho_2$ there is full information revelation
- for $p \in (\rho_2, \rho_1]$ only expert 1 reveals her signal
- for $p > \rho_1$ no information is revealed.

Let is now turn to collective expertise and keep assuming that expert 1 is the deputy expert, for now. Rather that performing a full equilibrium analysis, we will focus on how the heterogeneity between the experts affects the existence of a fully revealing equilibrium and whether the equilibrium $(m^0, (1,1))$ becomes easier or more difficult to sustain. Note that, with heterogeneous experts, we cannot take truthtelling by expert 2 to expert 1 "for granted", we will have to verify expert 2's incentive compatibility constraints as well.

Let us start from the equilibrium $(m^0, (1,1))$.

Lemma 15 *The equilibrium* $(\{(0,0),(0,1),(1,0)\},\{(1,1)\})$ *exists if and only if* $p \in [\max\{1/2, \underline{p}'\}, \overline{p}]$*, where* $\overline{p}$ *is determined by the incentive compatibility constraint of signal-type (1,1) and* $\underline{p}'$ *– by the condition that expert 2 with* $\sigma_2 = 0$ *is willing to reveal his signal to expert 1. The differences* $\overline{p} - \rho_1$ *and* $\overline{p} - \underline{p}'$ *are decreasing in* ρ_1 *and increasing in* ρ_2 *and become zero for sufficiently large* ρ_1 *or sufficiently small* ρ_2*.*

Proof. *See section 4 of the online appendix.* ■

As in Section 4 of the paper, threshold $\overline{p}$ results from the no-deviation condition for signal-type $(1,1)$. In contrast to the identical experts case, signal-type $(1,0)$ now may want to deviate, as $\Pr(\omega = 1|\sigma = (1,0))$ may exceed $1/2$. Her incentive compatibility constraint would yield threshold $\underline{p}$ (see formula (1) in the proof). However, this condition turns out to be never binding, because there appears a stronger constraint: the no-lying condition of expert 2 who received $\sigma_2 = 0$, yielding threshold $\underline{p}' > \underline{p}$.

The logic is as follows. If $\sigma_1 = 1$ and $\sigma_2 = 0$, expert 2, being the weaker expert, suffers more from message m^0 compared to the strong expert: For any realization of the state, the decision-maker will rationally assign a higher probability to the weak expert receiving a wrong signal, compared to the strong one – we label this effect "shifting the blame". Therefore, expert 2 has a higher temptation to induce deviation by expert 1 to reporting $(1,1)$ compared to the temptation of expert 1 herself to deviate to reporting $(1,1)$. This can be achieved by misreporting σ_2 when $\sigma_2 = 0$. Notice that such misreporting will induce the deputy to report $(1,1)$ if and only if $\sigma_1 = 1$, for if $\sigma_1 = 0$ the deputy will report m^0 irrespective of expert 2's signal (as $\sigma = (0,1)$ makes the deputy believe that $\omega = 0$ is more likely).

There are two effects of ρ_1 and ρ_2 on the incentive compatibility constraint of signal-type $(1,1)$, i.e., on $\overline{p}$. One effect is through confidence about the state: higher either ρ_1 or ρ_2 raises the deputy's belief that $\omega = 1$. The other one is the "shifting the blame" effect: Similarly to raising the temptation of expert 2 to deviate from m^0 to $(1,1)$, a rise in ρ_1 or a decline in ρ_2 increases the temptation of expert 1 to deviate from $(1,1)$ to m^0. Formally, $\Pr(\sigma_1 = 0|\omega = 0, \sigma \in m^0)$ and $\Pr(\sigma_1 = 1|\omega = 1, \sigma \in m^0)$ both go up with ρ_1 and go down with ρ_2 (the expressions can be found in the proof).

If we increase ρ_1 and decrease ρ_2 in such a way that $\Pr(\omega = 1|\sigma = (1,1))$ stays constant, only the "shifting the blame" effect remains, and, thus, $\overline{p}$ goes down. In general, if ρ_1 increases sufficiently fast relative to a decrease in ρ_2 (so that $\Pr(\omega = 1|\sigma = (1,1))$

rises), $\overline{p}$ may go up. However, we also need to look at the other threshold, $\underline{p}'$. Naturally, it rises as ρ_1 increases or ρ_2 decreases. This is because such changes in ρs: (1) increase $\Pr(\omega = 1|\sigma = (1,0))$ and (2) enhance the "shifting the blame" effect.

In general, $\underline{p}'$ may be below 1/2, but when ρ_1 becomes sufficiently large or ρ_2 becomes sufficiently small, it exceeds 1/2 and eventually hits $\overline{p}$, at which point the equilibrium of the lemma ceases to exist. This dynamics is shown in Figure 2.

Note also that $\overline{p} - \rho_1$ also decreases as ρ_1 goes up or ρ_2 goes down, which is illustrated in Figure 1. Thus, even if segment $\left[\underline{p}', \overline{p}\right]$ is non-empty, the range of parameters where collective expertise unambiguously dominates independent expertise shrinks.

Consider now the fully revealing equilibrium under collective reporting. Note that, with heterogeneous experts, full revelation requires separation of signal-types $(0,1)$ and $(1,0)$. In contrast to the identical experts case, this equilibrium becomes possible because, for sufficiently low values of p, expert 1's signal determines what state is more likely regardless of the signal of expert 2. Therefore, she has an incentive to reveal her signal truthfully independently of the weak expert's information. In turn, expert 2, not knowing expert 1's signal, will tell the truth to the deputy, provided that the prior is sufficiently close to 1/2.

Lemma 16 *Under collective expertise, a fully revealing equilibrium exists if and only if* $p \leq \min\{p_{FR}, \rho_2\}$*, where* $p_{FR} = \dfrac{\rho_1(1-\rho_2)}{\rho_1(1-\rho_2) + \rho_2(1-\rho_1)}$*. The value of* p_{FR} *is increasing in* ρ_1 *and decreasing in* ρ_2*, takes value* $1/2$ *for* $\rho_1 = \rho_2$ *and hits* ρ_2 *for sufficiently high* ρ_1 *or sufficiently small* ρ_2*.*

Proof. The value of p_{FR} is determined by the condition $\Pr(\omega = 0|\sigma = (1,0)) = 1/2$, from which it is straightforward to derive the explicit expression for p_{FR}. For $p > p_{FR}$, $\sigma_2 = 0$ makes expert 1 believe that $\omega = 0$ is more likely even when she has got $\sigma_1 = 1$, and, hence, truthtelling by expert 1 is destroyed – she would deviate to pretending that $\sigma_1 = 0$.

For $p \leq p_{FR}$, expert 1's own signal always determines which state she believes is more likely. Then, following Ottaviani and Sørensen (2001), it is straightforward to show that expert 1 always prefers to be perceived as having received the signal corresponding to the state she considers more likely rather than the opposite signal. Thus, she will always truthfully reveal her signal (in the most informative equilibrium). In addition, she does not lose anything from disclosing the weak expert's message as well. Thus, if the latter tells the truth to expert 1, full information revelation will occur in equilibrium. Since expert 2 does not observe the signal of expert 1 when sending his message, his truthtelling incentives (anticipating that his message will be disclosed) are the same as under independent reporting, i.e., expert 2 tells the truth iff $p \leq \rho_2$.

It is straightforward to verify that p_{FR} is increasing in ρ_1 and decreasing in ρ_2, takes value 1/2 for $\rho_1 = \rho_2$ and becomes equal to ρ_2 when $\frac{\rho_2^2}{(1-\rho_2)^2} = \frac{\rho_1}{1-\rho_1}$. ■

Naturally, an increase in the competence of expert 1 or a decrease in the competence of expert 2 expands the set of p ($p \leq p_{FR}$) for which expert 1's signal alone determines what state is more likely, i.e., for which $\Pr(\omega = 0|\sigma = (1,0)) \leq 1/2$. As a result, truthtelling by expert 1 becomes possible for a wider range of priors under collective expertise. At the same time, given truthtelling by expert 1, truthful revelation by expert 2 is not affected by the expertise scheme. Thus, independent expertise gradually loses its advantage for $\rho < \rho_2$ until segment $[p_{FR}, \rho_2]$ shrinks to zero, see Figure 1.

Figures 1 and 2 summarize the effects of the experts' heterogeneity, presented in Lemmas 15 and 16. Start with $\rho_1 = \rho_2$ and gradually decrease ρ_2 and/or increase ρ_1. The zone $(\rho_1, \overline{p}]$, where collective expertise dominates, shrinks (though $\overline{p}$ does not necessarily have to decrease as depicted), whereas p_{FR} increases. At some point, $\overline{p}$ hits ρ_1 and p_{FR} hits ρ_2 (one can show that both things happen simultaneously). At this point, collective expertise completely loses its advantage for $\rho \geq \rho_1$, and it also loses its *dis*advantage for $\rho < \rho_2$.

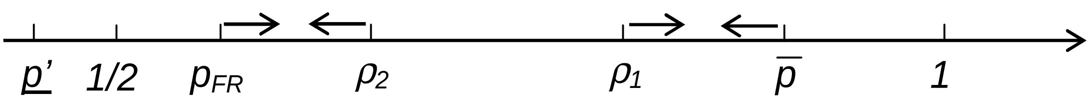

Figure 1. Effects of heterogeneity, when the difference in the abilities is not too high.

Collective expertise, however, may still be preferred for $[\max\{\underline{p}', p_{FR}\}, \overline{p}]$, as equilibrium $(m^0, (1,1))$ may be preferred to just expert 1 revealing her signal (which de facto corresponds to partition $(\{(0,0),(0,1)\}, \{(1,0),(1,1)\})$. (To be sure, for $\rho \in [\rho_2, \rho_1]$, collective expertise can never do worse than the independent one, because, under collective expertise, there is always an equilibrium in which expert 2 babbles to expert 1, and expert 1 truthfully reports her own signal).

However, according to Lemma 15, a further decrease in ρ_2 and/or increase in ρ_1 reduces the zone where equilibrium $(m^0, (1,1))$ exists (i.e., segment $[\underline{p}', \overline{p}]$; one can show that, for $\overline{p} < \rho_1$, $\underline{p}'$ exceeds 1/2.) until, at some point, it disappears completely (when $\underline{p}'$ and $\overline{p}$ become equal) – see Figure 2. Moreover, as the gap between ρ_2 and ρ_1 widens, it becomes less likely that $(m^0, (1,1))$ is preferred to $(\{(0,0),(0,1)\}, \{(1,0),(1,1)\})$, because knowing expert 1's signal becomes "on average" more important relative to learning whether both experts received signals 1 or not.

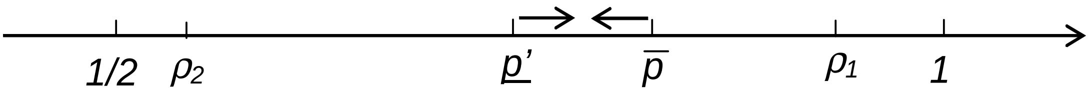

Figure 2. Effects of heterogeneity, when the difference in the abilities is very high.

Overall, the above analysis implies the following proposition:

Proposition 6 *When heterogeneity between the experts is small enough, all the qualitative results of the model with ex-ante identical experts hold through. However, as the heterogeneity grows, the choice of the expertise scheme becomes less and less relevant, as collective expertise loses its advantage for high priors, and independent expertise loses its advantage for low priors.*

Remark on the optimality of expert 1 as the deputy: Suppose the roles of the experts are inverted: expert 2 is the deputy, and expert 1 has to report to expert 2. It is easy to show that Lemma 15 remains intact. This is because the no-lying conditions of a non-deputy expert are equivalent to his/her incentive compatibility conditions once he/she becomes a deputy. For example, expert 2, being a non-deputy, can influence the message of expert 1 to the decision-maker only if the latter received $\sigma_1 = 1$. Thus, by considering whether to lie to expert 1 or not, he considers a deviation from m^0 to $(1,1)$ when $\sigma = (1,0)$ and a deviation from $(1,1)$ to m^0 when $\sigma = (1,1)$, as if he actually were the deputy. The same is true for expert 1. Thus, regardless of who is assigned the role of the deputy, the same four constraints are the necessary and sufficient conditions for the equilibrium $(m^0, (1,1))$ to exist: $\forall i \in \{1,2\}$, expert i must not be willing to deviate to $(1,1)$ when $\sigma = (1,0)$ and to m^0 when $\sigma = (1,1)$

However, two of these conditions are always stronger than the other two: the no-deviation condition of expert 2 when $\sigma = (1, 0)$ (yielding $\underline{p}'$) and the no-deviation condition of expert 1 when $\sigma = (1, 1)$ (yielding $\overline{p}$).

In contrast, the fully revealing equilibrium never exists when expert 2 is the deputy. Since expert 1's signal is stronger than that of expert 2, there cannot be a situation in which expert 2's signal determines which state is more likely regardless of expert 1's signal. In particular, if expert 1 reveals $\sigma_1 = 0$ to expert 2, the latter will fail to reveal $\sigma_2 = 1$ to the decision-maker, as he believes that $\omega = 0$ is more likely.

Thus, making expert 2 the deputy is weakly suboptimal.

2 Robustness to the communication protocol

All our equilibria for the collective expertise scenario rely on the equilibrium selection hypothesis that the experts share their signals truthfully. How is this robust to different assumptions about the nature of their interaction?

For simplicity, let us look at a setting with two experts. Consider an alternative game in which both experts send a private message to the decision-maker after talking simultaneously with each other. We are going to show that when our equilibrium of interest exists, i.e. the one where the deputy communicates $(1, 1)$ or m^0, it exists also in the alternative game, in the sense that the experts report truthfully to each other and both communicate to the decision-maker $(1, 1)$ or m^0.

We are going to show that it also survives an adaptation of neologism-proofness and of a strengthening of the intuitive criterion to this double-cheap-talk game with multiple senders whenever independent expertise does not generate (full) information transmission.

Let us consider a candidate equilibrium where the experts report truthfully to each other, both report truthfully to the decision-maker whether their declared profile is $(1, 1)$ or not, and the decision-maker believes that the true signal profile is $(1, 1)$ whenever at least one of the two experts reports $(1, 1)$, and has the other equilibrium belief (i.e. that the experts' signals belong to $\{(0, 0), (0, 1), (1, 0)\}$) otherwise.

It is immediate to check that this is an equilibrium. But is it plausible that the decision-maker believes that signals are $(1, 1)$ when just one expert, say expert 2, reports so? It could also be the case that expert 1 received 0 but lied to expert 2: if expert 2 receives signal 0, the lie is inconsequential; if expert 2 receives signal 1, expert 2 is tricked into reporting $(1,1)$ and expert 1 can "admit the lie" with the decision-maker to convince her that he got signal 0. Provided that the decision-maker is convinced by expert 1, it can be shown that such "tricking strategy" will indeed be a profitable deviation for expert 1 with signal 0 whenever $p > p_{cheat}$, where $p_{cheat} \in (1/2, \rho)$. That is, for such values of p, he prefers revealing his signal 0 rather than sending m^0, regardless of expert 2's signal.[1]

Net of obvious out-of-the-model considerations that make admission of lie unattractive, we are going to check whether this alternative interpretation of incongruent reports by the decision-maker is the only reasonable one according to neologism-proofness, and if our equilibrium interpretation is reasonable according to the spirit of the intuitive criterion.

Since for $p \leq p_{cheat}$ the "tricking strategy" is unprofitable anyway, we will only consider the case $p > p_{cheat}$. Remember also that we are only interested in $p \leq \overline{p}$, as for higher values of p, the equilibrium of interest does not exist.

Let us start from neologism-proofness. In neologism-proofness, deviations are interpreted under the view that the sender believes that equilibrium messages are interpreted as expected, while neologisms are expected to be taken at face value. We are going to apply this idea as well.

To begin with, *fix* expert 2's equilibrium strategy and consider a deviation to a "neologism" by expert 1. For $p \in (p_{cheat}, \overline{p}]$, claiming $\sigma_1 = 0$ after tricking expert 2 into reporting $(1,1)$ is a credible neologism: Indeed, for $p \leq \overline{p}$, $\sigma = (1,1)$ implies that $\omega = 1$ is more likely. This means that expert 1 with $\sigma_1 = 1$, after hearing message 1 from expert 2 (who is supposed to report truthfully to expert 1 in equilibrium) would not want to be perceived as having received $\sigma_1 = 0$. However, we actually have two senders in our model, who are both strategic and *both* can consider deviations to neologisms. Thus, a legitimate question arises: when considering a deviation to a "neologism", do we want to force expert 1 to believe that expert 2 never lies to him? Such a restriction seems unjustified: An expert who schemes as above can also conceive that the other expert schemes in the same way.

So, we first formulate the following adaptation of neologism-proofness (we specify it

[1] Let us remark that if experts' signals were verifiable among them, the above concern would be irrelevant, as lying among experts would be infeasible. Note that signals verifiability among experts is not at odds with being unverifiable for the decision maker, signals typically need specialized expertise to be understood.

without loss of generality for the case above):

> When the decision-maker receives message $(1,1)$ from expert 2 and a different message from expert 1, she must conclude that expert 1 received signal 0 if the following condition is satisfied. Expert 1 may prefer to send the alternative message when he truly received signal 0 but not when he received signal 1 under the following hypotheses:
>
> (i) he expects the alternative message to be interpreted as evidence that he got signal 0;
>
> (ii) he expects $(1,1)$ to be interpreted as evidence that he received signal 1;
>
> (iii) he expects expert 2 to behave rationally under (i) and (ii) as well.

Let us suppose that expert 1 got signal 1, reports signal 1 to and hears 1 from expert 2, but thinks that the expert 2 always reports 1 even when his true signal is 0, to then admit his lie in front of the decision-maker when he hears 1 from expert 1. As depicted for expert 1 with signal 0 above, this is a rational behavior for expert 2, so (iii) is satisfied for $p > p_{cheat}$. If expert 1, conditional on his signal 1, considers state 1 more likely, he will not change his mind after hearing 1 from expert 2, which he regards as babbling. So, for $p < \rho$, expert 1 prefers not to send $(1,1)$ under hypotheses (i) and (ii) only when he truly gets signal 0. Thus, adapted neologism-proofness has bite and our equilibrium of interest does not survive this refinement. But, if expert 1, conditional on his signal 1, still considers state 0 more likely, he will prefer not to send $(1,1)$ as well, given that hearing 1 from expert 2 does not change his mind. Therefore, for $p \geq \rho$, the decision-maker needs not conclude that expert 1 truly received 0. So, adapted neologism-proofness has no bite.

Now, we take the opposite perspective, which is typical of the intuitive criterion. Is our off-path belief reasonable under the following view: The sender would never make a deviation that, for any belief, cannot yield at least the expected payoff from the equilibrium messages under the equilibrium interpretation? In cheap talk games, the original intuitive criterion has no bite because messages are costless. Thus, we strengthen it and argue that our equilibrium survives the modified criterion as well. Specifically, to obtain refinement power, we impose that senders would never deviate if they do not expect a *strict* improvement of their payoff. In our case of interest, this can be expressed as follows:

When the decision-maker receives message $(1, 1)$ from expert 2 and a different message from expert 1, she must *exclude* that expert 1 received signal 1 if the following condition is satisfied. Expert 1 may *strictly* prefer to send the alternative message when he received signal 0 but not when he received signal 1 under the following hypotheses:

(i) he expects $(1, 1)$ to be interpreted as evidence that he received signal 1;

(ii) he expects expert 2 to behave rationally under (i).

When expert 1 with signal 1 believes that also expert 2 has received signal 1, he cannot improve his expected payoff by sending a message different from $(1, 1)$ (given that $p \leq \overline{p}$) regardless of what the decision-maker believes about his signal after this message. However, expert 1 may well believe that expert 2, despite declaring having received 1, has actually received 0 but is trying to deceive expert 1, exactly as discussed for neologism-proofness. Then, for $p > \rho$, expert 1 with signal 1 will still consider state 0 more likely and, thus, will strictly prefer to claim he received 0 under the decision-maker's belief that he indeed received 0. Hence, given expert 1's described belief about the behavior of expert 2, expert 1 with signal 1 can strictly improve his expected payoff by claiming he received 0, under some decision-maker's belief following this claim. Therefore, our off-path belief is not ruled out by this adaptation of the intuitive criterion to our multiple senders context.

3 Proof of Lemma 2

As we have shown in section "Preliminaries to proofs" of the main Appendix, expert i's expected reputation from message m conditional on his/her information I can be written as

$$R_i(m, I) = \alpha(m, I) \cdot x + (1 - \alpha(m, I)) \cdot y,$$

where

$$x := \Pr(t = G|\sigma_i = \omega) > y := \Pr(t = G|\sigma_i \neq \omega),$$

$$\alpha(m, I) := \Pr(\omega = 0|I) \Pr(\sigma_i = 0|\omega = 0, m) + \Pr(\omega = 1|I) \Pr(\sigma_i = 1|\omega = 1, m).$$

Hence, all comparisons of expected reputations are equivalent to comparing values of $\alpha(m, I)$.

Suppose expert 2 has truthfully revealed his signal to expert 1. Let message m' be a message sent with a *positive* probability by signal-type $(1,0)$. Full revelation requires that, for any vector of signals, the number of zeros is truthfully revealed. This implies that, in a fully revealing equilibrium, message m' can only be sent by either signal-type $(1,0)$ or $(0,1)$.

We would like to show that it is profitable for signal-type $(1,0)$ to deviate to reporting $(0,0)$, that is, $\alpha(m = (0,0), \sigma = (1,0)) > \alpha(m = m', \sigma = (1,0))$.

Since $\Pr(\sigma_1 = 1 | m = (0,0)) = 0$ in a fully revealing equilibrium, irrespective of the realized state, we have

$$\alpha(m = (0,0), \sigma = (1,0)) = p$$

Now compute $\alpha(m = m', \sigma = (1,0))$.

Denote:

$$\mu \ : \ = \Pr(m = m' | \sigma = (1,0))$$
$$\nu \ : \ = \Pr(m = m' | \sigma = (0,1))$$

Using the fact that message m' is never sent by signal-types $(0,0)$ and $(1,1)$, we can derive:

$$\Pr(\sigma_1 = 0 | \omega = 0, m = m') = \frac{\Pr(\sigma_1 = 0 \cap m = m' | \omega = 0)}{num. + \Pr(\sigma_1 = 1 \cap m = m' | \omega = 0)}$$
$$= \frac{\Pr(\sigma = (0,1) \cap m = m' | \omega = 0)}{num. + \Pr(\sigma = (1,0) \cap m = m' | \omega = 0)} = \frac{\rho(1-\rho)\nu}{\rho(1-\rho)\nu + (1-\rho)\rho\mu} = \frac{\nu}{\nu + \mu}$$

$$\Pr(\sigma_1 = 1 | \omega = 1, m = m') = \frac{\Pr(\sigma_1 = 1 \cap m = m' | \omega = 1)}{num. + \Pr(\sigma_1 = 0 \cap m = m' | \omega = 1)}$$
$$= \frac{\Pr(\sigma = (1,0) \cap m = m' | \omega = 1)}{num. + \Pr(\sigma = (0,1) \cap m = m' | \omega = 1)} = \frac{\rho(1-\rho)\mu}{\rho(1-\rho)\mu + (1-\rho)\rho\nu} = \frac{\mu}{\nu + \mu}$$

Hence,

$$\alpha(m = m', \sigma = (1,0)) = p \cdot \frac{\nu}{\nu + \mu} + (1-p) \cdot \frac{\mu}{\nu + \mu}$$

Since, by assumption, $\mu > 0$, $\alpha(m = m', \sigma = (1,0)) < p = \alpha(m = (0,0), \sigma = (1,0))$. ■

4 Proof of Lemma 15

As we have shown in section "Preliminaries to proofs" of the main Appendix, expert i's expected reputation from message m conditional on his/her information I can be written as

$$R_i(m, I) = \alpha(m, I) \cdot x + (1 - \alpha(m, I)) \cdot y,$$

where

$$x := \Pr(t = G|\sigma_i = \omega) > y := \Pr(t = G|\sigma_i \neq \omega),$$
$$\alpha(m, I) := \Pr(\omega = 0|I) \Pr(\sigma_i = 0|\omega = 0, m) + \Pr(\omega = 1|I) \Pr(\sigma_i = 1|\omega = 1, m).$$

Hence, all comparisons of expected reputations are equivalent to comparing values of $\alpha(m, I)$.

Assuming that expert 2 tells the truth to expert 1, we will first derive the incentive compatibility constraints of signal-types $(1, 1)$ and $(1, 0)$. Given that $\rho_1 > \rho_2$, there is no need to check those for signal-types $(0, 1)$ and $(0, 0)$: If $(1, 0)$ does not gain from deviating to reporting $(1, 1)$, neither $(0, 0)$ nor $(0, 1)$ does, as those signal-types assign a lower probability to $\omega = 1$ compared to signal-type $(1, 0)$. We will then derive the condition for truthful reporting by expert 2 to expert 1.

Incentive compatibility of signal-type $(1, 1)$:

First, compute α of signal-type $(1, 1)$ if she does not deviate.

$$\begin{aligned} \Pr(\omega &= 0|\sigma = (1,1)) = \frac{\Pr(\sigma = (1,1)|\omega = 0) \Pr(\omega = 0)}{num. + \Pr(\sigma = (1,1)|\omega = 1) \Pr(\omega = 1)} \\ &= \frac{(1-\rho_1)(1-\rho_2)p}{(1-\rho_1)(1-\rho_2)p + \rho_1\rho_2(1-p)} \end{aligned}$$

$$\Pr(\sigma_1 = 0|\omega = 0, \sigma \in (1,1)) = 0, \ \Pr(\sigma_1 = 1|\omega = 1, \sigma \in (1,1)) = 1$$

Thus,

$$\begin{aligned} \alpha(m &= (1,1), \sigma = (1,1)) \\ &= \Pr(\omega = 0|\sigma = (1,1)) \cdot 0 + \Pr(\omega = 1|\sigma = (1,1)) \cdot 1 \\ &= \frac{\rho_1\rho_2(1-p)}{(1-\rho_1)(1-\rho_2)p + \rho_1\rho_2(1-p)} \end{aligned}$$

Now, compute α of signal-type $(1,1)$ if she deviates to m^0.

$$\Pr(\sigma_1 = 0|\omega = 0, \sigma \in m^0) = \frac{\Pr(\sigma_1 = 0 \cap \sigma \in m^0|\omega = 0)}{\Pr(\sigma \in m^0|\omega = 0)}$$
$$= \frac{\Pr(\sigma_1 = 0 \cap \sigma = (0,0)|\omega = 0) + \Pr(\sigma_1 = 0 \cap \sigma = (0,1)|\omega = 0) + \Pr(\sigma_1 = 0 \cap \sigma = (1,0)|\omega = 0)}{\Pr(\sigma \in m^0|\omega = 0)}$$
$$= \frac{\rho_1\rho_2 + \rho_1(1-\rho_2)}{\rho_1\rho_2 + \rho_1(1-\rho_2) + (1-\rho_1)\rho_2} = \frac{\rho_1}{(1-\rho_1)\rho_2 + \rho_1}$$

$$\Pr(\sigma_1 = 1|\omega = 1, \sigma \in m^0) = \frac{\Pr(\sigma_1 = 1 \cap \sigma \in m^0|\omega = 1)}{\Pr(\sigma \in m^0|\omega = 1)}$$
$$= \frac{\Pr(\sigma_1 = 1 \cap \sigma = (0,0)|\omega = 1) + \Pr(\sigma_1 = 1 \cap \sigma = (0,1)|\omega = 1) + \Pr(\sigma_1 = 1 \cap \sigma = (1,0)|\omega = 1)}{\Pr(\sigma \in m^0|\omega = 1)}$$
$$= \frac{\rho_1(1-\rho_2)}{(1-\rho_1)(1-\rho_2) + (1-\rho_1)\rho_2 + \rho_1(1-\rho_2)} = \frac{\rho_1(1-\rho_2)}{1-\rho_1\rho_2}$$

Thus,

$$\begin{aligned}
\alpha(m &= m_0, \sigma = (1,1)) \\
&= \Pr(\omega = 0|\sigma = (1,1)) \cdot \frac{\rho_1}{(1-\rho_1)\rho_2 + \rho_1} + \Pr(\omega = 1|\sigma = (1,1)) \cdot \frac{\rho_1(1-\rho_2)}{1-\rho_1\rho_2} \\
&= \frac{(1-\rho_1)(1-\rho_2)p}{(1-\rho_1)(1-\rho_2)p + \rho_1\rho_2(1-p)} \cdot \frac{\rho_1}{(1-\rho_1)\rho_2 + \rho_1} \\
&\quad + \frac{\rho_1\rho_2(1-p)}{(1-\rho_1)(1-\rho_2)p + \rho_1\rho_2(1-p)} \cdot \frac{\rho_1(1-\rho_2)}{1-\rho_1\rho_2}
\end{aligned}$$

The expert will not deviate whenever

$$\alpha(m = (1,1), \sigma = (1,1)) \geq \alpha(m = m^0, \sigma = (1,1)),$$

which yields

$$p \leq \frac{\rho_2[(1-\rho_1)\rho_2 + \rho_1]}{1-\rho_2+\rho_2^2} =: \overline{p}.$$

Incentive compatibility of signal-type $(1,0)$:

First, compute α of signal-type $(1,0)$ if she does not deviate.

$$\Pr(\omega = 0|\sigma = (1,0)) = \frac{\Pr(\sigma = (1,0)|\omega = 0)\Pr(\omega = 0)}{num. + \Pr(\sigma = (1,0)|\omega = 1)\Pr(\omega = 1)} = \frac{(1-\rho_1)\rho_2 p}{(1-\rho_1)\rho_2 p + \rho_1(1-\rho_2)(1-p)}$$

Using the expressions for $\Pr(\sigma_1 = 0|\omega = 0, \sigma \in m^0)$ and $\Pr(\sigma_1 = 1|\omega = 1, \sigma \in m^0)$ derived above, we obtain:

$$\begin{aligned}\alpha(m &= m^0, \sigma = (1,0)) = \frac{(1-\rho_1)\rho_2 p}{(1-\rho_1)\rho_2 p + \rho_1(1-\rho_2)(1-p)} \cdot \frac{\rho_1}{(1-\rho_1)\rho_2 + \rho_1} \\ &+ \frac{\rho_1(1-\rho_2)(1-p)}{(1-\rho_1)\rho_2 p + \rho_1(1-\rho_2)(1-p)} \cdot \frac{\rho_1(1-\rho_2)}{1-\rho_1\rho_2}\end{aligned}$$

Now, compute α of signal-type $(1,0)$ if she deviates to $(1,1)$.

$$\Pr(\sigma_1 = 0|\omega = 0, \sigma \in (1,1)) = 0,\ \Pr(\sigma_1 = 1|\omega = 1, \sigma \in (1,1)) = 1$$

Thus,

$$\begin{aligned}\alpha(m &= (1,1), \sigma = (1,0)) = \Pr(\omega = 0|\sigma = (1,0)) \cdot 0 + \Pr(\omega = 1|\sigma = (1,0)) \cdot 1 \\ &= \frac{\rho_1(1-\rho_2)(1-p)}{(1-\rho_1)\rho_2 p + \rho_1(1-\rho_2)(1-p)}\end{aligned}$$

The expert will not deviate whenever $\alpha(m = m^0, \sigma = (1,0)) \geq \alpha(m = (1,1), \sigma = (1,0))$, which yields

$$(1-\rho_1\rho_2)\rho_2 p \geq (1-\rho_2)[(1-\rho_1)\rho_2 + \rho_1](1-p)$$

or

$$p \geq \frac{(1-\rho_2)[(1-\rho_1)\rho_2 + \rho_1]}{\rho_2(1-\rho_1\rho_2) + (1-\rho_2)[(1-\rho_1)\rho_2 + \rho_1]} =: \underline{p} \tag{1}$$

Truthtelling by expert 2 to expert 1:

It is easy to show that the no-lying incentives of expert 2 are equivalent to *his* incentives not to deviate if he were the deputy (and the other expert would report truthfully to him). The reason is that expert 2, being a non-deputy, can influence the message of expert 1 to the decision-maker only if the latter received $\sigma_1 = 1$. Thus, by considering whether to lie to expert 1 or not, he considers a deviation from m^0 to $(1,1)$ when $\sigma = (1,0)$ and a deviation from $(1,1)$ to m^0 when $\sigma = (1,1)$, as if he actually were the deputy.

Analyzing the former deviation yields condition $p \geq \underline{p}'$, where $\underline{p}'$ is derived analogously to $\underline{p}$ and equals:

$$\underline{p}' \equiv \frac{\rho_1(1-\rho_2)^2[(1-\rho_1)\rho_2+\rho_1]}{(1-\rho_1)\rho_2^2(1-\rho_1\rho_2)+\rho_1(1-\rho_2)^2[(1-\rho_1)\rho_2+\rho_1]}$$

Analyzing the latter deviation yields condition $p \leq \overline{p}'$, where $\overline{p}'$ is derived analogously to $\overline{p}$ and equals:

$$\overline{p}' \equiv \frac{\rho_1[(1-\rho_1)\rho_2+\rho_1]}{1-\rho_1+\rho_1^2}$$

Given that $\rho_1 > \rho_2$, it is easy to show that $\underline{p}' > \underline{p}$ and $\overline{p}' > \overline{p}$. Thus, there are two, rather than four, relevant no-deviation conditions: $p \geq \underline{p}'$ and $p \leq \overline{p}$.

Simple algebra shows that the differences $\overline{p} - \rho_1$ and $\overline{p} - \underline{p}'$ are decreasing in ρ_1 and increasing in ρ_2. Finally, it is straightforward to derive that, for $\rho_1 = 1$ or $\rho_2 = 1/2$, $\overline{p} < \rho_1$ and $\overline{p} < \underline{p}'$. ■