Лекция 7.12.20, часть 1

Прогнозирование по регрессионной модели и его точность

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Prediction

$$Y_i = \beta_1 + \beta_2 X_{2i} + \ldots + \beta_k X_{ki} + \varepsilon_i,\ i = 1, \ldots, n$$

$$x_{n+1}^{'} = (1\ X_{2n+1} \ldots X_{kn+1})'$$

$$\hat{Y}_{n+1} = \hat{\beta}_1 + \hat{\beta}_2 X_{2n+1} + \ldots + \hat{\beta}_k X_{n+1}$$

$$Y_{n+1} = x_{n+1}^{'} \beta + \varepsilon_{n+1}$$

$$\varepsilon_{n+1} \sim N(0, \sigma_\varepsilon^2 I_n),\ \mathrm{cov}(\varepsilon_{n+1}, \varepsilon_i) = 0,\ i = 1, \ldots, n$$

Individual forecast error

$$\hat{\varepsilon}_{n+1} = Y_{n+1} - \hat{Y}_{n+1} = x'_{n+1}\beta + \varepsilon_{n+1} - x'_{n+1}\hat{\beta} =$$

$$= \varepsilon_{n+1} - x'_{n+1}(\hat{\beta} - \beta) =$$

$$= \varepsilon_{n+1} - x'_{n+1}(X'X)^{-1}X'\varepsilon$$

$$\mathrm{var}(\hat{\varepsilon}_{n+1}) = \mathrm{var}(\varepsilon_{n+1}) +$$

$$+ x'_{n+1}(X'X)^{-1}X'\mathrm{var}(\varepsilon)X(X'X)^{-1}x_{n+1} =$$

$$= \sigma_{\varepsilon}^{2}(1 + x'_{n+1}(X'X)^{-1}x_{n+1})$$

Confidence interval for the individual prediction

$$1-\alpha = P\{x_{n+1}^{'}\hat{\beta} - t_{\alpha/2}(n-k)\hat{\sigma}_{\varepsilon}\sqrt{1+x_{n+1}^{'}(X'X)^{-1}x_{n+1}} <$$

$$< Y_{n+1} < x_{n+1}^{'}\hat{\beta} - t_{\alpha/2}(n-k)\hat{\sigma}_{\varepsilon}\sqrt{1+x_{n+1}^{'}(X'X)^{-1}x_{n+1}}$$

Mean forecast error

$$\hat{\varepsilon}_{n+1} = E(Y_{n+1}) - \hat{Y}_{n+1} = -x_{n+1}^{'}(X'X)^{-1}X'\varepsilon$$

$$\operatorname{var}(\hat{\varepsilon}_{n+1}) = x_{n+1}^{'}(X'X)^{-1}X'\operatorname{var}(\varepsilon)X(X'X)^{-1}x_{n+1} =$$

$$= \sigma_{\varepsilon}^{2}x_{n+1}^{'}(X'X)^{-1}x_{n+1}$$

Confidence interval for the mean prediction

$$1-\alpha = P\{x_{n+1}^{'}\hat{\beta} - t_{\alpha/2}(n-k)\hat{\sigma}_{\varepsilon}\sqrt{x_{n+1}^{'}(X'X)^{-1}x_{n+1}} <$$

$$< E(Y_{n+1}) < x_{n+1}^{'}\hat{\beta} - t_{\alpha/2}(n-k)\hat{\sigma}_{\varepsilon}\sqrt{x_{n+1}^{'}(X'X)^{-1}x_{n+1}}$$

Prediction for the simple model

$$Y = \beta_1 + \beta_2 X + \varepsilon$$

Individual prediction

$$1-\alpha = P\{\hat{\beta}_2 X_{n+1} - t_{\alpha/2}(n-k)\hat{\sigma}_{\varepsilon}\sqrt{\frac{1}{n}+\frac{(X_{n+1}-\bar{X})^2}{\sum_{i=1}^{n}(X_i-\bar{X})^2}} <$$

$$< Y_{n+1} < \hat{\beta}_2 X_{n+1} - t_{\alpha/2}(n-k)\hat{\sigma}_{\varepsilon}\sqrt{\frac{1}{n}+\frac{(X_{n+1}-\bar{X})^2}{\sum_{i=1}^{n}(X_i-\bar{X})^2}}$$

Prediction for the simple model

$$Y = \beta_1 + \beta_2 X + \varepsilon$$

Mean prediction

$$1-\alpha = P\{\hat{\beta}_2 X_{n+1} - t_{\alpha/2}(n-k)\hat{\sigma}_\varepsilon \sqrt{1+\frac{1}{n}+\frac{(X_{n+1}-\bar{X})^2}{\sum_{i=1}^{n}(X_i-\bar{X})^2}} <$$

$$< Y_{n+1} < \hat{\beta}_2 X_{n+1} - t_{\alpha/2}(n-k)\hat{\sigma}_\varepsilon \sqrt{1+\frac{1}{n}+\frac{(X_{n+1}-\bar{X})^2}{\sum_{i=1}^{n}(X_i-\bar{X})^2}}$$

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Thank you
for your attention!

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