NATIONAL RESEARCH UNIVERSITY

Лекция по эконометрике № 9

3 модуль

Модели множественного выбора (дополнительная тема)

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План лекции

• Модели упорядоченного множественного выбора

• Мультиномиальные модели

Напоминание о моделях бинарного выбора

$$Y_i^* = \beta_0 + \beta_1 X_{1i} + \ldots + \beta_k X_{ki} + \varepsilon_i,$$

$$E(\varepsilon_i) = 0, \ \operatorname{var}(\varepsilon_i) = \sigma_\varepsilon^2, \quad i = 1, \ldots, n,$$

$$F - cumulative \quad function \quad of \quad \varepsilon/\sigma_\varepsilon,$$

$$f = F' \ is \quad a \quad symmetric \quad function.$$

$$\begin{cases} Y_i = 1, if \ Y_i^* \geq 0 \\ Y_i = 0, if \ Y_i^* < 0 \end{cases}$$

Отношение шансов (odd ratio) для логит модели

$$OR = \frac{\Pr(Y = 1)}{\Pr(Y = 0)}$$

$$\ln(OR) = \beta_0 + \beta_1 X_1 + \ldots \beta_k X_k$$

Если X_j изменится на 1 то OR изменится в exp(β$_j$) раз.

Пример 1. Ответ на вопрос: «Насколько Вы доверяете правительству страны?»

Варианты ответов:

- **1 – полностью доверяю**
- **2 – скорее доверяю**
- **3 – скорее не доверяю**
- **4 – полностью не доверяю**

Модели упорядоченного множественного выбора

Пример 2. Состояние здоровья

Варианты ответов:

- **1 – fair**
- **2 – good**
- **3- excellent**

Пример 3. Состояние на рынке труда

Варианты ответов:

- **1 – не работает**
- **2 – частичная занятость (неполный рабочий день)**
- **3 – полная занятость (полный рабочий день)**

Модели упорядоченного множественного выбора

$$Y_i = 1,2,\dots,m$$

$$Y_i^* = x_i'\beta + \varepsilon_i,$$

$$Y_i = j \quad if \quad c_{j-1} < Y_i^* < c_j, \quad j = 1,\dots,m,$$

$$c_0 = -\infty, \quad \dots, \quad c_m = \infty,$$

$$P(Y_i = j) = F(c_j - x_i'\beta) - F(c_{j-1} - x_i'\beta),$$

$$L = \prod_{j=1}^{m} \prod_{i:Y_i=j} (F(c_j - x_i'\beta) - F(c_{j-1} - x_i'\beta)) \to \max_{\beta,c}$$

Оценка параметров с помощью метода максимального правдоподобия

Проверка гипотезы о параллельности

$$P(Y_i = j) = F(c_j - x_i'\beta) - F(c_{j-1} - x_i'\beta),$$

$$\Rightarrow P(Y_i \le k \mid X) = F(c_k - x_i'\beta),\ k = 1, \ldots, m$$

(parallel regression assumption)

Brant test

Детали теста Бранта можно найти в W.Greene, 7 edition, p. 791

Отношение шансов для упорядоченной логит модели

$$\frac{P(Y_i \le k|X)}{P(Y_i > k|X)} = \exp(c_k - x_i'\beta), \; k = 1, \dots, m$$

$$\Rightarrow \frac{P(Y_i \le k\big|X)(X, x_j + 1)}{P(Y_i > k\big|X)(X, x_j + 1)} = \exp(-\beta_j)$$

$$\frac{P(Y_i > k\big|X)(X, x_j + 1)}{P(Y_i \le k\big|X)(X, x_j + 1)} = \exp(\beta_j)$$

Предельные эффекты

$$\frac{\partial P(Y_i = 1)}{\partial X_k} = -\beta_k f(c_1 - (X\beta)),$$

$$\frac{\partial P(Y = j)}{\partial X_k} = -\beta_k [f(c_j - (X\beta)) -$$

$$- f(c_{j-1} - (X\beta))], \qquad j = 1,\ldots, m-1$$

$$\Rightarrow \frac{\partial P(Y = j)}{\partial X_k} = \beta_k f(c_{m-1} - (X\beta))$$

Пример оценки моделей упорядоченного множественного выбора

Пример позаимствован у Ani Katchova,
https://www.youtube.com/watch?v=c9kvqeLFF8U

Зависимая переменная: Y = healthstatus
Значения: 1 – fair, 2 – good, 3- excellent

tab healthstatus
health status (fair, good, excellent)

	Freq.	Percent	Cum.
Fair	523	9.38	9.38
good	2,034	36.49	45.87
Excellent	3,017	54.13	100.00
Total	5,574	100.00	

Пример оценки моделей упорядоченного множественного выбора

Независимые переменные

sum age logincome numberdiseases

Variable	Obs	Mean	Std. Dev.	Min	Max
age	5,574	25.57613	16.73011	.0253251	63.27515
logincome	5,574	8.696929	1.220592	0	10.28324
numberdiseases	5,574	11.20526	6.788959	0	58.6

Результаты оценки

```
. ologit healthstatus age logincome numberdiseases

Iteration 0:   log likelihood = -5140.0463
Iteration 1:   log likelihood =  -4776.008
Iteration 2:   log likelihood = -4769.8693
Iteration 3:   log likelihood = -4769.8525
Iteration 4:   log likelihood = -4769.8525

Ordered logistic regression                     Number of obs     =      5,574
                                                LR chi2(3)        =     740.39
                                                Prob > chi2       =     0.0000
Log likelihood = -4769.8525                     Pseudo R2         =     0.0720
```

healthstatus	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
age	-.0292944	.001681	-17.43	0.000	-.0325891	-.0259996
logincome	.2836537	.0231098	12.27	0.000	.2383593	.3289481
numberdiseases	-.0549905	.0040692	-13.51	0.000	-.0629661	-.047015
/cut1	-1.39598	.2061301			-1.799987	-.9919722
/cut2	.9513097	.2054301			.5486741	1.353945

Предельные эффекты

```
. margins, dydx(*) atmeans predict(outcome(1))

Conditional marginal effects                    Number of obs     =      5,574
Model VCE    : OIM

Expression   : Pr(healthstatus==1), predict(outcome(1))
dy/dx w.r.t. : age logincome numberdiseases
at           : age             =    25.57613 (mean)
               logincome       =    8.696929 (mean)
               numberdise~s    =    11.20526 (mean)
```

	dy/dx	Delta-method Std. Err.	z	P>|z|	[95% Conf.	Interval]
age	.002058	.0001333	15.44	0.000	.0017969	.0023192
logincome	-.0199278	.0017344	-11.49	0.000	-.0233272	-.0165284
numberdiseases	.0038633	.0003056	12.64	0.000	.0032643	.0044623

Предельные эффекты

```
. margins, dydx(*) atmeans predict(outcome(2))

Conditional marginal effects                    Number of obs     =      5,574
Model VCE    : OIM

Expression   : Pr(healthstatus==2), predict(outcome(2))
dy/dx w.r.t. : age logincome numberdiseases
at           : age             =    25.57613 (mean)
               logincome       =    8.696929 (mean)
               numberdise~s    =    11.20526 (mean)
```

	dy/dx	Delta-method Std. Err.	z	P>|z|	[95% Conf.	Interval]
age	.0052244	.0003258	16.04	0.000	.0045859	.0058629
logincome	-.0505872	.0043054	-11.75	0.000	-.0590256	-.0421489
numberdiseases	.0098071	.000768	12.77	0.000	.0083018	.0113124

Предельные эффекты

```
. margins, dydx(*) atmeans predict(outcome(3))

Conditional marginal effects                    Number of obs     =      5,574
Model VCE    : OIM

Expression   : Pr(healthstatus==3), predict(outcome(3))
dy/dx w.r.t. : age logincome numberdiseases
at           : age             =    25.57613 (mean)
               logincome       =    8.696929 (mean)
               numberdise~s    =    11.20526 (mean)
```

	dy/dx	Delta-method Std. Err.	z	P>|z|	[95% Conf.	Interval]
age	-.0072824	.0004179	-17.43	0.000	-.0081014	-.0064634
logincome	.070515	.0057527	12.26	0.000	.05924	.0817901
numberdiseases	-.0136704	.0010126	-13.50	0.000	-.015655	-.0116858

Предсказанные значения зависимой переменной

```
. predict p1ologit p2ologit p3ologit, pr

. summarize p1ologit p2ologit p3ologit
```

Variable	Obs	Mean	Std. Dev.	Min	Max
p1ologit	5,574	.0946903	.0843148	.0233629	.859022
p2ologit	5,574	.3651672	.0946158	.1255265	.5276064
p3ologit	5,574	.5401425	.1640575	.0154515	.7999009

```
. tab healthstatus
```

health status (fair, good, excellent)	Freq.	Percent	Cum.
1	523	9.38	9.38
2	2,034	36.49	45.87
3	3,017	54.13	100.00
Total	5,574	100.00	

Пример 1. Отношение шансов

```
. ologit healthstatus age logincome numberdiseases, or

Iteration 0:   log likelihood = -5140.0463
Iteration 1:   log likelihood =  -4776.008
Iteration 2:   log likelihood = -4769.8693
Iteration 3:   log likelihood = -4769.8525
Iteration 4:   log likelihood = -4769.8525

Ordered logistic regression                     Number of obs     =      5,574
                                                LR chi2(3)        =     740.39
                                                Prob > chi2       =     0.0000
Log likelihood = -4769.8525                     Pseudo R2         =     0.0720
```

healthstatus	Odds Ratio	Std. Err.	z	P>|z|	[95% Conf.	Interval]
age	.9711306	.0016325	-17.43	0.000	.9679362	.9743355
logincome	1.327973	.0306892	12.27	0.000	1.269165	1.389506
numberdiseases	.9464941	.0038515	-13.51	0.000	.9389753	.9540731
/cut1	-1.39598	.2061301			-1.799987	-.9919722
/cut2	.9513097	.2054301			.5486741	1.353945

Пример 1. Тест Бранта

```
. brant

Brant Test of Parallel Regression Assumption
```

Variable	chi2	p>chi2	df
All	17.45	0.001	3
age	0.18	0.672	1
logincome	17.40	0.000	1
numberdise~s	0.20	0.659	1

```
A significant test statistic provides evidence that the parallel
regression assumption has been violated.
```

Мультиномиальные модели

Пример 1. Сфера деятельности

Варианты ответов:

- 1 – бизнес
- 2 – образование
- 3 – некоммерческая организация

Пример 2. Способ рыбалки

Варианты ответов:

- 1 – с берега
- 2 – с причала
- 3 – с собственной лодки
- 4 – с арендованной лодки

Мультиномиальные модели

Зависимая переменная не является упорядоченной,

Существует выбор между альтернативами j = 1, 2,…, M,

U_{ij} – полезность j-ой альтернативы для i-го индивида.

$P\{Y_i = j\} = P\{U_{ij} = \max\{U_{i1},\ldots, U_{iM}\}\}$,

$U_{ij} = x'_i\beta_j + \varepsilon_{ij}$.

Задача допускает аналитическое решение, если ε_{ij} независимы и имеют функцию распределения

$F(x) = \exp\{-\exp(-x)\}$

Мультиномиальные логит модели

$$P(Y_i = 1) = \frac{1}{1 + \exp(x'_i\beta_2) + \ldots + \exp(x'_i\beta_m)},$$

$$P(Y_i = j) = \frac{exp(x'_i\beta_j)}{1 + \exp(x'_i\beta_2) + \ldots + \exp(x'_i\beta_m)}$$

$$\Rightarrow \frac{P(Y_i = j)}{P(Y_i = k)} = \frac{exp(x'_i\beta_j)}{\exp(x'_i\beta_k)} = \exp(x'_i(\beta_j - \beta_k))$$

IIA – independence from irrelevant alternatives
Test Small-Hsiao

Пример 1

**Пример 1 позаимствован у Ani Katchova,
https://www.youtube.com/watch?v=iqypob4My4o**

Зависимая переменная Y = mode = Способ рыбалки

Значения:

- **1 – beach (с берега)**
- **2 – pier (с причала)**
- **3 – private (с собственной лодки)**
- **4 – charter (с арендованной лодки)**

Пример

```
. mlogit mode income

Iteration 0:   log likelihood = -1497.7229
Iteration 1:   log likelihood = -1477.5265
Iteration 2:   log likelihood = -1477.1514
Iteration 3:   log likelihood = -1477.1506
Iteration 4:   log likelihood = -1477.1506

Multinomial logistic regression                 Number of obs     =      1,182
                                                LR chi2(3)        =      41.14
                                                Prob > chi2       =     0.0000
Log likelihood = -1477.1506                     Pseudo R2         =     0.0137
```

mode	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
beach						
income	.0316399	.0418463	0.76	0.450	-.0503774	.1136571
_cons	-1.341291	.1945167	-6.90	0.000	-1.722537	-.9600457
pier						
income	-.111763	.0439795	-2.54	0.011	-.1979612	-.0255649
_cons	-.5271412	.1777842	-2.97	0.003	-.8755918	-.1786906
private						
income	.1235462	.0279106	4.43	0.000	.0688425	.17825
_cons	-.6023707	.1360964	-4.43	0.000	-.8691147	-.3356267
charter	(base outcome)					

Пример

```
. mlogit mode income, baseoutcome(2)

Iteration 0:   log likelihood = -1497.7229
Iteration 1:   log likelihood = -1477.5265
Iteration 2:   log likelihood = -1477.1514
Iteration 3:   log likelihood = -1477.1506
Iteration 4:   log likelihood = -1477.1506

Multinomial logistic regression                 Number of obs     =      1,182
                                                LR chi2(3)        =      41.14
                                                Prob > chi2       =     0.0000
Log likelihood = -1477.1506                     Pseudo R2         =     0.0137

------------------------------------------------------------------------------
        mode |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
beach        |
      income |   .1434029   .0532884     2.69   0.007     .0389595    .2478463
       _cons |  -.8141503    .228632    -3.56   0.000    -1.262261   -.3660399
-------------+----------------------------------------------------------------
pier         |  (base outcome)
-------------+----------------------------------------------------------------
private      |
      income |   .2353093   .0436681     5.39   0.000     .1497214    .3208971
       _cons |  -.0752295   .1832396    -0.41   0.681    -.4343724    .2839134
-------------+----------------------------------------------------------------
charter      |
      income |    .111763   .0439795     2.54   0.011     .0255649    .1979612
       _cons |   .5271412   .1777842     2.97   0.003     .1786906    .8755918
------------------------------------------------------------------------------
```

Предельные эффекты

```
. margins, dydx(*) atmeans predict(pr outcome(1))

Conditional marginal effects                    Number of obs     =      1,182
Model VCE    : OIM

Expression   : Pr(mode==beach), predict(pr outcome(1))
dy/dx w.r.t. : income
at           : income          =    4.099337 (mean)

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      income |    .000075   .0039337     0.02   0.985    -.0076349    .0077848
------------------------------------------------------------------------------

.
. margins, dydx(*) atmeans predict(pr outcome(2))

Conditional marginal effects                    Number of obs     =      1,182
Model VCE    : OIM

Expression   : Pr(mode==pier), predict(pr outcome(2))
dy/dx w.r.t. : income
at           : income          =    4.099337 (mean)

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      income |  -.0206598   .0048735    -4.24   0.000    -.0302117    -.011108
------------------------------------------------------------------------------
```

Предельные эффекты

```
. margins, dydx(*) atmeans predict(pr outcome(3))

Conditional marginal effects                    Number of obs     =      1,182
Model VCE    : OIM

Expression   : Pr(mode==private), predict(pr outcome(3))
dy/dx w.r.t. : income
at           : income          =    4.099337 (mean)

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      income |   .0325985    .005692     5.73   0.000     .0214424    .0437547
------------------------------------------------------------------------------

.
. margins, dydx(*) atmeans predict(pr outcome(4))

Conditional marginal effects                    Number of obs     =      1,182
Model VCE    : OIM

Expression   : Pr(mode==charter), predict(pr outcome(4))
dy/dx w.r.t. : income
at           : income          =    4.099337 (mean)

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      income |  -.0120137   .0060756    -1.98   0.048    -.0239215   -.0001058
------------------------------------------------------------------------------
```

Предсказанные значения зависимой переменной

```
. predict pmlogit1 pmlogit2 pmlogit3 pmlogit4, pr

.
. summarize pmlogit1 pmlogit2 pmlogit3 pmlogit4
```

Variable	Obs	Mean	Std. Dev.	Min	Max
pmlogit1	1,182	.1133672	.0036716	.0947395	.1153659
pmlogit2	1,182	.1505922	.0444575	.0356142	.2342903
pmlogit3	1,182	.3536379	.0797714	.2396973	.625706
pmlogit4	1,182	.3824027	.0346281	.2439403	.4158273

```
.
. tab mode
```

Fishing mode	Freq.	Percent	Cum.
beach	134	11.34	11.34
pier	178	15.06	26.40
private	418	35.36	61.76
charter	452	38.24	100.00
Total	1,182	100.00	

Пример 2. Продолжение примера про здоровье

```
. mlogit healthstatus age logincome numberdiseases

Iteration 0:   log likelihood = -5140.0463
Iteration 1:   log likelihood = -4826.7356
Iteration 2:   log likelihood = -4772.0141
Iteration 3:   log likelihood = -4766.2058
Iteration 4:   log likelihood = -4766.1902
Iteration 5:   log likelihood = -4766.1902

Multinomial logistic regression                 Number of obs     =      5,574
                                                LR chi2(6)        =     747.71
                                                Prob > chi2       =     0.0000
Log likelihood = -4766.1902                     Pseudo R2         =     0.0727
```

healthstatus	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
fair						
age	.0424728	.0031346	13.55	0.000	.0363292	.0486165
logincome	-.4191076	.0323039	-12.97	0.000	-.4824221	-.3557931
numberdiseases	.0833499	.006825	12.21	0.000	.0699731	.0967266
_cons	-.3537368	.282676	-1.25	0.211	-.9077716	.2002979
good						
age	.025483	.0018533	13.75	0.000	.0218506	.0291155
logincome	-.1473865	.0291931	-5.05	0.000	-.2046039	-.0901692
numberdiseases	.0454721	.004827	9.42	0.000	.0360114	.0549328
_cons	-.2366722	.2592993	-0.91	0.361	-.7448895	.2715451
excellent	(base outcome)					

Пример 2. Продолжение примера про здоровье

```
. mlogit healthstatus age logincome numberdiseases, baseoutcome(2)

Iteration 0:   log likelihood = -5140.0463
Iteration 1:   log likelihood = -4826.7356
Iteration 2:   log likelihood = -4772.0141
Iteration 3:   log likelihood = -4766.2058
Iteration 4:   log likelihood = -4766.1902
Iteration 5:   log likelihood = -4766.1902

Multinomial logistic regression                 Number of obs     =      5,574
                                                LR chi2(6)        =     747.71
                                                Prob > chi2       =     0.0000
Log likelihood = -4766.1902                     Pseudo R2         =     0.0727
```

healthstatus	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
fair						
age	.0169898	.0031035	5.47	0.000	.010907	.0230726
logincome	-.2717211	.0286096	-9.50	0.000	-.327795	-.2156473
numberdiseases	.0378778	.0062911	6.02	0.000	.0255475	.050208
_cons	-.1170646	.2524238	-0.46	0.643	-.6118062	.3776769
good	(base outcome)					
excellent						
age	-.025483	.0018533	-13.75	0.000	-.0291155	-.0218506
logincome	.1473865	.0291931	5.05	0.000	.0901692	.2046039
numberdiseases	-.0454721	.004827	-9.42	0.000	-.0549328	-.0360114
_cons	.2366722	.2592993	0.91	0.361	-.2715451	.7448895

Пример 2. Предельные эффекты для исхода 1

```
. margins, dydx(*) atmeans predict(pr outcome(1))

Conditional marginal effects                    Number of obs     =      5,574
Model VCE    : OIM

Expression   : Pr(healthstatus==fair), predict(pr outcome(1))
dy/dx w.r.t. : age logincome numberdiseases
at           : age             =    25.57613 (mean)
               logincome       =    8.696929 (mean)
               numberdise~s    =    11.20526 (mean)

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         age |   .0022253   .0001985    11.21   0.000     .0018363    .0026144
   logincome |  -.0249397   .0019742   -12.63   0.000    -.0288091   -.0210703
numberdiseases |   .004497   .0004403    10.21   0.000      .003634    .0053601
------------------------------------------------------------------------------
```

Пример 2. Сравнение предельных эффектов ologit и mlogit для исхода 1

	dy/dx	Delta-method Std. Err.	z	P>|z|	[95% Conf.	Interval]
age	.002058	.0001333	15.44	0.000	.0017969	.0023192
logincome	-.0199278	.0017344	-11.49	0.000	-.0233272	-.0165284
numberdiseases	.0038633	.0003056	12.64	0.000	.0032643	.0044623

	dy/dx	Delta-method Std. Err.	z	P>|z|	[95% Conf.	Interval]
age	.0022253	.0001985	11.21	0.000	.0018363	.0026144
logincome	-.0249397	.0019742	-12.63	0.000	-.0288091	-.0210703
numberdiseases	.004497	.0004403	10.21	0.000	.003634	.0053601

Пример 2. Сравнение предельных эффектов ologit и mlogit для исхода 2

	dy/dx	Delta-method Std. Err.	z	P>|z|	[95% Conf.	Interval]
age	.0052244	.0003258	16.04	0.000	.0045859	.0058629
logincome	-.0505872	.0043054	-11.75	0.000	-.0590256	-.0421489
numberdiseases	.0098071	.000768	12.77	0.000	.0083018	.0113124

	dy/dx	Delta-method Std. Err.	z	P>|z|	[95% Conf.	Interval]
age	.0047914	.0004161	11.52	0.000	.0039759	.0056069
logincome	-.022744	.0064079	-3.55	0.000	-.0353032	-.0101848
numberdiseases	.0083332	.0010648	7.83	0.000	.0062462	.0104203

Пример 2. Сравнение предельных эффектов ologit и mlogit для исхода 3

	dy/dx	Delta-method Std. Err.	z	P>|z|	[95% Conf.	Interval]
age	-.0072824	.0004179	-17.43	0.000	-.0081014	-.0064634
logincome	.070515	.0057527	12.26	0.000	.05924	.0817901
numberdiseases	-.0136704	.0010126	-13.50	0.000	-.015655	-.0116858

	dy/dx	Delta-method Std. Err.	z	P>|z|	[95% Conf.	Interval]
age	-.0070167	.000441	-15.91	0.000	-.0078811	-.0061524
logincome	.0476837	.006889	6.92	0.000	.0341815	.0611859
numberdiseases	-.0128302	.0011574	-11.09	0.000	-.0150987	-.0105618

Пример 2. Предсказанные значения зависимой переменной

```
. predict pmlogit1 pmlogit2 pmlogit3, pr

.
. summarize pmlogit1 pmlogit2 pmlogit3

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
    pmlogit1 |      5,574    .0938285    .0903648   .0168382   .8635994
    pmlogit2 |      5,574    .3649085    .0860006   .1258262   .5486323
    pmlogit3 |      5,574     .541263    .1576133   .0105744   .7683809

.
. tab healthstatus

     health |
     status |
     (fair, |
      good, |
 excellent) |      Freq.     Percent        Cum.
------------+-----------------------------------
       fair |        523        9.38        9.38
       good |      2,034       36.49       45.87
  excellent |      3,017       54.13      100.00
------------+-----------------------------------
      Total |      5,574      100.00
```

Thank you for your attention!

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