R-модели

$W \neq \varnothing \qquad S, R \subseteq \underset{\text{транз.}}{U} \subseteq W \times W$

$R \circ S$

$$\begin{tikzcd} x \arrow[r, "R"] \arrow[rr, bend right, "R \circ S"'] & y \arrow[r, "S"] & z \end{tikzcd}$$

$$S/R = \{(x,y) \in U \mid \forall (y,z) \in R \quad (x,z) \in S\}$$

$$\{(x,y)\} \circ R \subseteq S$$

$R \backslash S$

$$B \subseteq A \backslash C \iff A \circ B \subseteq C \iff A \subseteq C/B$$

Исчисление Ламбека L $\quad A \to B \qquad \subseteq$

$$A \to A \qquad (A \circ B) \circ C \leftrightarrow A \cdot (B \cdot C)$$

$$\frac{\dfrac{B \to A \backslash C}{A \circ B \to C}}{A \to C/B} \qquad \frac{A \to B \quad B \to C}{A \to C}$$

Теор. (Andrèka, Mikulàs) 1994. $\quad \vdash_L A \to B \iff A \to B$ общезн. в R-мод.

$$\mathcal{H} \vdash_L A \to B \iff \mathcal{H} \models_{\text{R-мод.}} A \to B$$

$\mathcal{H}$ — мн-во утв.-й (гипотез)

Пример. $(p\backslash p)\backslash p \to p$

общ. при $U = W\times W$ — квадратные модели

но $\nvdash_L$

$L\cap$

$\cup$, $\cap$ ← полнота сохраняется

$$A\cap B \to A \qquad A\cap B\to B \qquad \frac{A\to B \quad A\to C}{A\to B\cap C}$$

L_1 $\quad 1\cdot A \leftrightarrow A \leftrightarrow A\cdot 1$

$L_1 \vdash (p\backslash p)\backslash p \to p$

Теор. (Andréka, Mikulás '94) $\mathcal{H} \models_{\substack{R\text{-мод.}\\ \text{квадр.}}} A\to B \iff \mathcal{H}\vdash_{L_1} A\to B$

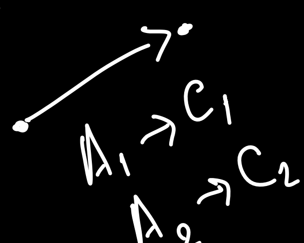
$\mathcal{H}, A, B$ - без 1. $\quad L^*$, L^Λ

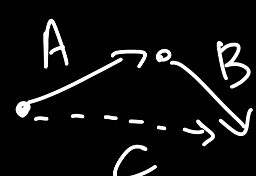
$$\models_{\substack{\text{квадр.}\\ R\text{-мод.}}} 1/(p/p) \to \left(1/(p/p)\right)\cdot\left(1/(p/p)\right)$$

$1 = \delta = \{(x,x) \mid x\in W\}$

? Полнота $L^\Lambda\cap$

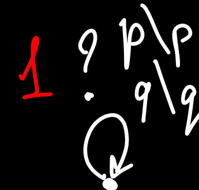
$A \to C_1\cap C_2$





$(x,y)\in v(C)$ сущ. $A\in \mathrm{label}(x,y)$ $\mathcal{H}\vdash A\to C$

$M_{\mathcal{H}}$



$1 \to B$

Теор. (Mikulás '15) $\vdash_{L^\Lambda\cap} A\to B \iff \models_{\substack{R\text{-мод. кв.}\\ \text{с } \cap}} A\to B.$

Нестанд. квадр. модели с 1:

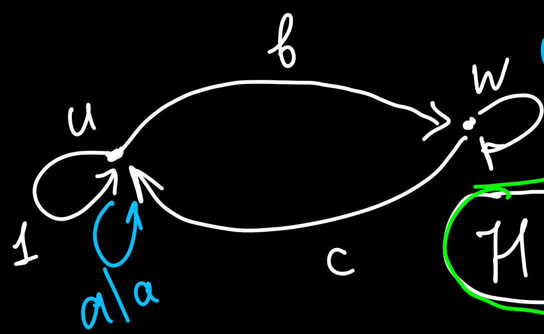
W, $\mathfrak{A}\subseteq \mathcal{P}(W\times W)$ замкн. отн. $\backslash, /, \cdot$, $\quad 1_{\mathfrak{A}}$ - ед. эл-т в $\mathfrak{A}$

$U = W\times W$

Теор. (К. '21) $\vdash_{L_1\cap} A \to B \iff \models A \to B$.

неот. кв. R-мод

$\mathcal{H} = \{ 1 \to b \cdot c \}$ b, c – перем. $\delta \leq 1_\Omega$



$\cancel{1} \cap (c \cdot b)$

$(u,u) \in v(1)$
$(u,u) \in v(b\cdot c)$

$$\boxed{\mathcal{H} \models 1 \to b \cdot (1 \cap (c \cdot b)) \cdot c}$$

$$\boxed{a\backslash a \to b \cdot c \models d \to d \cdot b \cdot ((a \backslash a) \cap (c \cdot b)) \cdot c}$$

$\nvdash_{L_1\cap}$

$a := 1$
$d := 1$

Секвенциальное исчисление для $L_1\cap$.

$\Pi \to B \qquad \Pi = A_1, \ldots, A_n$

$$\frac{}{A \to A}\,(ax) \qquad \frac{}{\to 1}\,(\to 1) \qquad \frac{\Gamma, \Delta \to B}{\Gamma, 1, \Delta \to B}\,(1 \to)$$

$$\frac{\Gamma, A, B, \Delta \to C}{\Gamma, A \cdot B, \Delta \to C}\,(\cdot \to) \qquad \frac{\Pi \to A \quad \Delta \to B}{\Pi, \Delta \to A \cdot B}\,(\to \cdot)$$

$$\text{cut}\ \frac{\Pi \to A \quad \Gamma, A, \Delta \to B}{\Gamma, \Pi, \Delta \to B}$$

$$\frac{\Pi \to A \quad \Gamma, B, \Delta \to C}{\Gamma, \Pi, A \backslash B, \Delta \to C}\,(\backslash \to) \qquad \frac{A, \Pi \to B}{\Pi \to A \backslash B}\,(\to \backslash)$$

$$\frac{\Pi \to A_1 \quad \Pi \to A_2}{\Pi \to A_1 \cap A_2}\,(\to \cap) \qquad \frac{\Gamma, A_i, \Delta \to B}{\Gamma, A_1 \cap A_2, \Delta \to B}\,(\cap \to)$$

$\rightarrow b \cdot c \quad \nvdash \quad \rightarrow b \cdot (1 \cap (c \cdot b)) \cdot c$

$$\frac{\Gamma, b, c, \Delta \rightarrow D}{\Gamma, \Delta \rightarrow D}\ (bc)$$

Ут. В исч. $L_1 \cap + (bc)$ устранимо (cut).

$\nvdash_{L_1 \cap + (bc)} \rightarrow b \cdot (1 \cap (c \cdot b)) \cdot c$

$$\frac{\dfrac{b \rightarrow b \quad \Psi \rightarrow 1 \cap (c \cdot b)}{\Phi_1 \rightarrow b \cdot (1 \cap c \cdot b)} \qquad \overset{= c}{\Phi_2} \rightarrow c}{\Phi_1, \Phi_2 \rightarrow (b \cdot (1 \cap (c \cdot b))) \cdot c}$$

$\vdots$

$\rightarrow b \cdot (1 \cap (c \cdot b)) \cdot c$

$\underbrace{b\Gamma'}_{c} \underbrace{c}_{b} \rightarrow c \cdot b$

$bc \rightarrow c$

$b \rightarrow c$

$b.. \rightarrow c$

$\Psi \rightarrow 1$

пустая

b, c

пол-ть $b\Gamma' c$

$\Gamma \rightarrow c \cdot b$

$\vdots$

$\Psi \rightarrow c \cdot b$