HSE Masters Exam

1. Andrew wrote 5 binaries: $11000_2, 11110_2, 1111_2, 10011_2, 11011_2$. He converted four of them to decimals (in some order) and obtained $15_{10}, 24_{10}, 27_{10}, 30_{10}$. Write the remaining binary number that Andrew did not convert to decimal.

2. A robot is placed in the maze below in A1 position. It is programmed to move in four directions $\uparrow$ (up), $\downarrow$ (down), $\rightarrow$ (right) and $\leftarrow$ (left). If the robot can't make a move in a programmed direction, then it stays in the same place to make the next move. Where will the robot stop after executing the program $\downarrow\leftarrow\downarrow\rightarrow\downarrow\rightarrow\uparrow\rightarrow\rightarrow\rightarrow\leftarrow\uparrow\downarrow\downarrow$? Give your answer as a pair of a letter and a number, e.g. B4.

3. There are two operations: A: multiply a given number by 3; B: add 5 to a given number. How to get 74 from 1 in the minimal number of steps using operations A and B? Write your answer as a sequence of symbols A and B. (Example: ABB turns 1 to 13).

4. John goes up the stairs. He can hop either one step or two steps at a time. How many ways are there to get to the 8th step? E.g. there are 2 ways to get to the second step: to make two single steps or to hop to the second at once.

5. For a given binary string we consider two operations:
A: replace its fragment 10 with 01 (if there are many such fragments in the string, we replace only one of them);
B: change all its 0's to 1's and vice versa (e. g., 00110 turns into 11001).
What is the minimum number of operations required to obtain 111000 from 101010?

6. Find the value of the expression:

$$(1 - \log_2 12)\,(1 - \log_6 12)$$

7. Simplify:

$$\frac{2}{a} - \left(\frac{a+1}{a^3-1} - \frac{1}{a^2+a+1} - \frac{2}{1-a}\right) \div \frac{a^3+a^2+2a}{a^3-1}$$

8. For a positive real x, find $x^4 + \frac{1}{x^4}$ whenever $x + \frac{1}{x} = 5$.

9. Find the largest root of the equation:

$$\frac{(x-9)^2 \cdot (x-5) \cdot (x-3)}{x-9} = 0$$

10. Solve the equation:

$$\log_2 x - \log_2(x-1) = 3$$

11. Find the last two digits of the number $98^2 + 97^2$. (Example: for the number 2021, the answer is 21)

12. Find the smallest integer a such that the roots of the quadratic equation $-x^2+(8-a)x+4a-12 = 0$ are of opposite signs.

13. Solve the inequality:

$$\frac{\log_2(32x) - 1}{\log_2^2 x - \log_2 x^5} \geq -1$$

14. Solve the system of linear equations:

$$\begin{cases} 2y - 2x = -3x + 2 \\ x + 3y = 2y + 4 \end{cases}$$

15. Solve the system of linear inequalities:

$$\begin{cases} 2^x + 6 \cdot 2^{-x} \leq 7 \\ \frac{2x^2-6x}{x-4} \leq x. \end{cases}$$

16. Two years ago John made a deposit in a bank that pays 10% of interest once a year. Today he has 605$ at the account. How many dollars did John deposit two years ago?

17. There is a new hotel in a city. Each guest brings 40$ income per day, and daily expenses for taxes and rent are 300$. Find the smallest number of guests per day for which the hotel will earn at least 1000$ per day.

18. A polynomial $p(x) = \left(x^4 - 3x^3 - x + 1\right)^9$ is written in standard form. Find the sum of all its coefficients.

19. Given $f(x) = 2x^2 - 3x + 1$, find $f'(2)$.

20. Find $\frac{d}{dx}\left(\arctan\left(\frac{1}{1+\ln x}\right)\right)$